Politechnika Lubelska

Jerzy Szerafin

Wooden Engineering Structures

Basics of designing

PODRĘCZNIKI

Lublin 2022

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LUBLIN UNIVERSITY
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Konstrukcje drewniane. Podstawy projektowania

Streszczenie

W podręczniku przedstawione zostały podstawowe zasady wymiarowania drewnianych elementów konstrukcyjnych. Omówiono zasady sprawdzania stanów granicznych zgodnie z wymaganiami aktualnych norm. Szczególną uwagę zwrócono na zagadnienia projektowe, w szczególności wymiarowanie przekrojów, w tym przekrojów złożonych, a także połączeń. Szczegółowo opisano właściwości drewna i istotne czynniki wpływające na wytrzymałość drewna, natomiast zastosowanie w budownictwie drewna litego i klejonego opisano w formie skrótowej. Dla zilustrowania poruszanych zagadnień wykorzystano przykłady zaczerpnięte z własnej praktyki inżynierskiej autora. Książka przeznaczona jest przede wszystkim dla studentów, ale może być również pomocna dla projektantów konstrukcji.

Słowa kluczowe: konstrukcje drewniane, właściwości wytrzymałościowe drewna, wymiarowanie, przekroje złożone

Wooden Engineering Structures. Basics of designing

Abstract

The book presents the basic principles of dimensioning wooden structural elements. The rules of checking the limit states conditions according to the current standards requirements were discussed. Specially attention is paid to design issues, particularly dimensioning of the member's cross-sections including complex cross-sections and connections. The properties of wood and the important factors that influence the strength of wood are described in detail, whereas the use of solid timber and glue laminated timber in buildings are mentioned only briefly. Some examples taken from the author's own engineering practice are used to illustrate the topics discussed. The book is intended especially for students, but also can be helpful for designers of the constructions.

Keywords: timber structures, strength properties of wood, dimensioning, complex cross-section

Most important symbols

Main symbols

A – area
C – compression force
E – modulus of elasticity
F – load
G – shear modulus
I – moment of inertia
K – slip modulus
L – length
M – bending moment
T – tension force
V – shear force

a – distance
b – width of cross-section
d – diameter
f – strength
h – depth of cross-section
k – factor
l – length
s – spacing
t – thickness
u – deflection

γ – partial factor or slip coefficient
λ – slenderness ratio
ρ – density
σ – normal stress
τ – shear stress
ω – moisture content

Subscripts

c – compression
d – designed
def – deformation
ef – effective

fin – final
g – properties of glued laminated timber
h – depth effect or embedment
inst – instantaneous
k – characteristic
m – bending
mod – modification
mean – mean value
rel – relative
ser – serviceability
t – tensile
u – ultimate
y – axis y
v – shear
z – axis z
0 – parallel to the grain
05 – 5 %-fractile
90 – perpendicular to the grain

Examples

fh,k – characteristic value of embedment strength
$f_{t,o,k}$ – characteristic value of tensile strength parallel to the grain
k_{mod} – modification factor
γ_{ser} – slip coefficient for serviceability limit state
$\lambda_{rel,z}$ – relative slenderness ratio for z direction
$\sigma_{m,y,d}$ – design bending stress about y axis
u_{inst} – instantaneous deflection

Introduction

Wood used for building structures is called timber or sawn lumber. These terms refer to wood members produced by being cut directly from a tree trunk.

Wood is an organic material that is not specially designed for building structures, unlike concrete or steel. However, living trees resist many natural loads such as wind pressure, the weight of snow, the weight of branches and leaves, sometimes the weight of big animals climbing a tree etc. Because of that, the strength properties of wood are quite good and suitable for use in manmade structures. Timber is able to transfer tensile, as well as compressive loads. Therefore it is suitable material for almost all kinds of members, including columns and beams. The strength–to-weight ratio is very high, and it resists the influence of hazardous environments better than steel. In this book the properties of wood and the important factors that influence the strength of wood are described in detail.

For a long time, wood was considered a rather niche material and uncompetitive in relation to reinforced concrete and steel, especially in the field of structures requiring larger spans of elements. However, recently there has been a significant increase in interest in wood as a construction material. This is due to the development of joining techniques, the emergence of new types of fasteners, and especially the improvement of wood gluing techniques. Thanks to the use of Glulam and Cross Laminated Timber (CLT), innovative structures for covering large-span objects are created, and architects value this material because of its aesthetic qualities.

However the use of solid timber and glue laminated timber in buildings are mentioned rather briefly in this book. Whereas specially attention is paid to design issues, particularly dimensioning of the member’s cross-sections including complex cross-sections.

The book is addressed primarily to students of building construction. Therefore, the design process is described in detail, including the basics of design and an explanation of the principles of applying the Limit States Method and Partial Safety Factors in particular. Due to the frequent updates of standards necessary for designing, the content of the book includes current standard data and current design principles. Therefore, this book can also be helpful to designers. However, it should be remembered that the long-awaited update of the basic standard EN 1995-1-1 *Design of timber structures. Part 1-1: General – Common rules and rules for buildings,* which also includes new materials such as CLT, is to be released soon.

Designing is related to the responsibility for the safety of the structure. Therefore, the methods and calculation procedures presented in this book are illustrated with examples derived from experimental research and the author's engineering practice. The examples of damaged structural elements shown here are intended to encourage careful designing.

1. Mechanical Properties of Wood

1.1. Structure of wood

The knowledge of the structure of wood is important to better understand wood as a building material. Wood is composed of elongated, hollow cells, arranged parallel to the direction of the tree trunk. The cell walls are made mainly of cellulose with a small amount of extraneous materials and glued together by lignin, which is a fairly weak binder. It means that wood has an anisotropic nature, and its strength depends considerably on the direction of the load.

1.1.1. Cross-section of a tree

The well-known cross-section of a tree trunk is shown in Figure 1.1. In the approximate middle of the cross-section there is the pith, which is the oldest part of the tree. The pith diameter (for pines) is several millimetres. The strength of this feature is low, so it is an undesirable part of timber, especially in thin members.

The new cells are produced in the outer zone of the trunk called the vascular cambium and make subsequent layers around the pith. If a tree lives in a temperate climate these layers are easily visible in the form of concentric rings known as annular or growth rings. The wood from a tropical climate is uniform and forms no rings. The important information for an engineer is the width of the ring, which often indicates the strength of the wood. For example the wood of a pine is stronger when the rings are narrow, but for the wood of an oak this relation is the reverse.

During the spring trees grow at a faster rate, which results in a low density of the fibres. These parts of the annular ring are lighter than those produced during the summer, which form darker areas containing cells with thicker walls. These clearly visible parts are called earlywood (springwood) and latewood (summerwood) respectively. The proportion between them can be used to judge the strength of the wood.

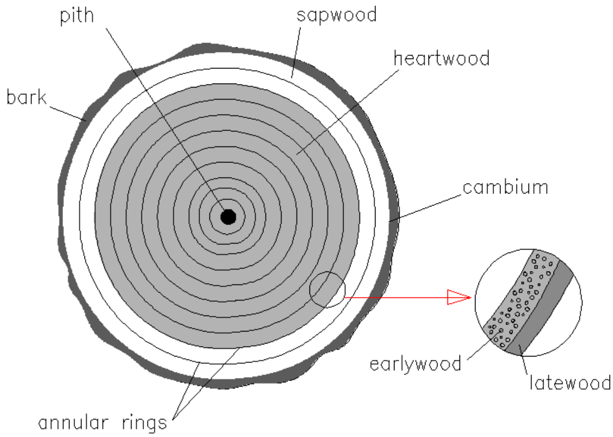


Figure 1.1. Cross-section of the tree trunk

Another important difference in the wood's properties appears between cells located in the inner and outer parts of the tree trunk. The inner part containing the oldest cells is called the heartwood. This area is darker than the outer part called the sapwood. The strength properties of the heartwood and the sapwood are rather similar, however their durability makes a real difference. Heartwood collects many extractives such as waxes, oils and resins, which provide natural resistance to fungi decay. Many historic structures were built without the use of any preservatives. For this reason, members made from the sapwood of softwoods have been damaged so much that they are in a relatively poor state. On the other hand, it is hard to impregnate hardwood with preservatives, unlike sapwood.

1.1.2. Softwood and hardwood

Variations in the characteristics and proportions of cells components and differences in the cellular structure make wood species hard or soft, strong or weak. Generally trees species are divided into two classes: softwoods and hardwoods. Softwoods are mainly evergreens with needle-like leaves, whereas hardwoods are broad-leaved. The structure of hardwoods contains different cells for support and different cells for conducting sap, and it is more complex than that of softwoods. Typically, timber of hardwoods is heavier, stronger and more durable than that of softwoods, but it is not a general rule. However, present-day building structures are made mainly from softwood because of its good handling properties, good strength-to-weight ratio and comparatively low cost of the material. Typical hardwoods for building structures are oak, beech or ash. Whereas the most frequently used softwoods are pine, spruce or larch.

1.2. Factors that influence the strength of wood

The strength of wood depends on some important factors, for example its density, age, and the presence of natural defects. However, the strength is not a constant value that can be assigned to a given piece of wood, but changes during its service as a structural element due to many environmental factors such moisture content, duration of load, temperature etc. The following are the most important factors affecting the strength properties of wood, and are described in subsections.

1.2.1. Moisture content

Moisture has an important influence on wood properties, e.g. it especially influences the strength, elasticity, durability or fire-resistance of the wood. Overall moisture content in wood is expressed by the formula:

$$\omega = \frac{m_{wet} - m_{dry}}{m_{dry}} 100\%\,, \tag{1.1}$$

where

m_{wet} is the mass of wood with water,

m_{dry} is the mass of dry wood material.

The moisture content in a piece of timber is determined according to the rules described in EN 13183-1. All the time moisture exchanges between the wood and air follow the pattern of external temperature and humidity, reaching an equilibrium moisture content. Approximate moisture content depends on the environment as specified in Table 1.

Table 1. Moisture content of the timber in structures, according to Neuhaus (1994)

moisture content, %	description and examples
35–120 (200)	green wood - freshly sawn wood
20–35	wet wood - semi-dried wood
12–20 ab. 18% (±6%) ab. 15% (±3%) ab. 12% (±3%) ≤ 12%	dry wood - structures outside, contact with rain water - structures outside, no contact with rain water - structures inside, without heating - structures inside, with heating

Moisture exists in wood as free water, in the cell lumina or cavities, and bound water, which is held by intermolecular attraction within the cell walls. The water that is chemically held within the cell walls significantly influences the strength of the timber. A general rule can be formulated: the more water is absorbed by the fibres, the lower the strength of timber.

The maximum amount of water that can be chemically absorbed corresponds to the moisture content equalling ab. 30%, and is expressed by the term *fibre saturation point*. If there is more water in the wood it creates free water contained in cavities. Substantially, free water does not affect the strength of wood. The influence of the moisture content on the wood's strength was shown in Figure 1.2.

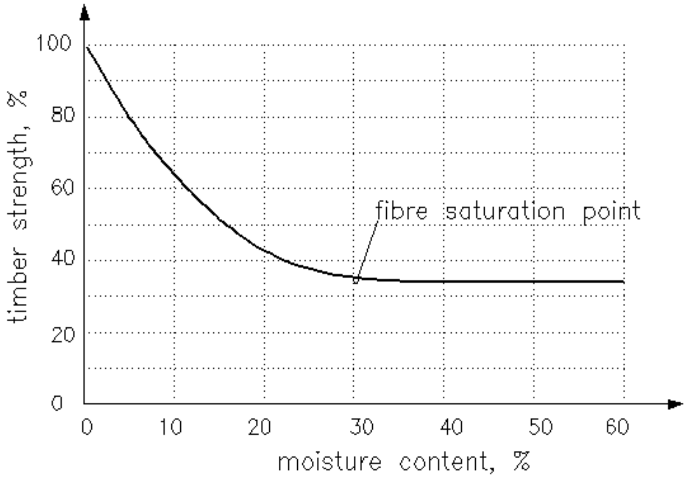


Figure 1.2. Timber strength in relation to the moisture content

1.2.2. Duration of load

The stress that a timber member can safely sustain is a function of the time of load duration. The results of much experimental research clearly show that the wood strength reduces during the lasting action of a load. This relationship as shown in Figure 1.3 is exponential. When the time of load is very short (tends to zero) the load and appropriated strength is called instantaneous. The long-lasting strength of wood is only a certain percentage of its instantaneous one. After a period of 10 years of the permanent action of a load, the load required to produce a failure was about 60% of the standard test load that lasts a few minutes, as was reported in Ross et al (2021). This loss of strength under the long-term action of a load resembles the kind of human fatigue during extended work.

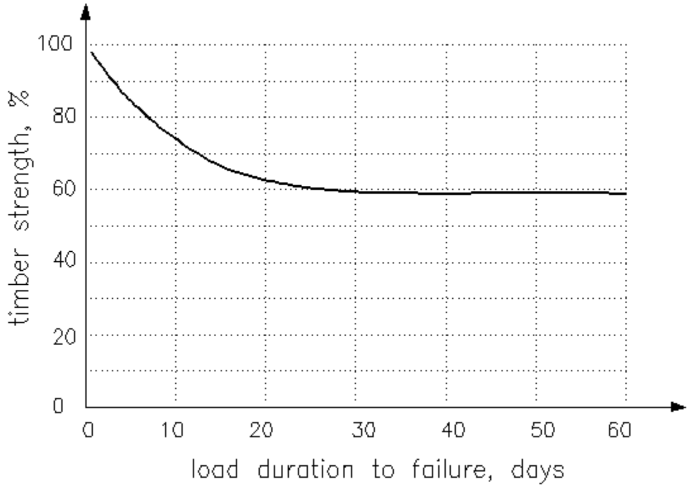


Figure 1.3. Timber strength in relation to the duration of load

The periodically applied and removed intermittent load has a cumulative effect on the strength properties of wood. The cumulative influence of intermittent load duration is approximately equal to the time of permanent load.

1.2.3. Temperature

The influence of temperature on the strength of wood is inversely proportional. An increase in temperature causes an approximate linear decrease in strength. Therefore, some older standards for the designing of wood constructions forbade the use of wooden elements in buildings where the temperature exceeds certain values (for example 55°C in the old polish standard PN-53/B-03150 from 1953).

The effect of the temperature on the strength depends mainly on the moisture content, but also from the heating medium, period of exposure, wood specie and size of tested specimen, and is also different for the compression, tensile and bending strength of wood. The data included in Table 1.2 show the changes in strength of defect-free wood at -50°C and +50°C relative to those of +20°C. It is clearly visible that the effect of temperature increases for a higher moisture content.

Table 1.2. Effects of temperature on the strength of wood at various moisture conditions, according to Ross et al (2021). FSP – fibre saturation point

Strength	**Moisture content, %**	**Relative change in strength from +20°C**	
		-50°C, %	**+50°C, %**
Bending strength	≤4 11–15 18–20 >FSP	+18 +35 +60 +110	-10 -20 -25 -25
Tensile strength parallel to grain	0-12	-	-4
Compressive strength parallel to grain	0 12–45	+20 +50	-10 -25
Shear strength	>FSP	-	-25
Tensile strength perpendicular to grain	4–6 11–16 ≥18	- - -	-10 -20 -30
Compressive strength perpendicular to grain	≥10	-	-35

The changes in wood strength are, in reality, reversible at temperatures below 100°C, if the temperature change is rapid. But if the temperature changes are of a long-term nature, some irreversible effects occur, that means the strength and elasticity properties of wood do not return to the original values after the elevated temperature period. Repeated periods of elevated temperature have a cumulative effect and the losses in strength constantly grow.

1.2.4. Age of wood

There is a relatively small amount of research into the strength properties of wood at a young age. The test results show that wood up to the age of 60 years is about 18% weaker than matured wood. The best technical properties that wood has when aged are shown in Table 1.3.

Table 1.3. The preferred age of the tree, according to Ross et al (2021)

Wood specie	**Age, years**
pine	80–120
spruce	80–100
fir	100 and more
oak	180 and more

The mechanical properties of wood are very stable if it is protected from decay or insect attack. Ross et al (2021), reports that a significant loss in clear wood strength was observed only after several centuries of aging, which proves the good durability of wood.

1.2.5. Specific gravity

The specific gravity of the wood substance itself is almost the same for all wood species and equal to 1.54 g/cm^3 (1540kg/m^3). However, besides its solid substance, wood also contains cell cavities and pores, the volume of which ranges between different wood species and also between individual pieces of wood of the same species. Thus the apparent density measures the amount of solid substance in the wood and therefore is a very good measure of the mechanical properties of clear, defect-free wood. The exemplary values of the apparent density of some common wood species are presented in Table 1.4.

Table 1.4. Apparent density of common wood species at a moisture content of 12–15% (the mean values)

Wood specie	Apparent density, kg/m^3
pine	510
spruce	470
fir	450
oak	690
beach	720

However, in the case of some varieties of tree species these values can be significantly different. For example there are 80 species of pine around the world, in which the apparent density ranges between 330 and 890 kg/m^3.

Within a species, the relationship between apparent density and the strength of the wood is approximately linear, as is shown in Figure 1.4. This conspicuous relationship is intuitively clear, taking into account that the structure of the wood resembles a bunch of fibres.

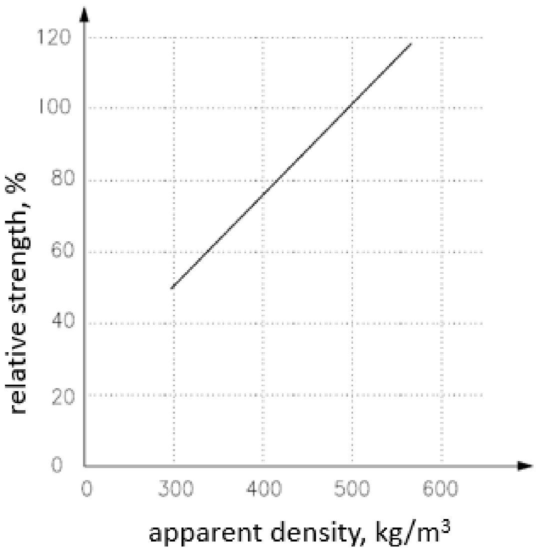


Figure 1.4. The influence of wood apparent density on strength (example data from the author's own research)

1.2.6. Defects

The natural defects in wood negatively affect the mechanical properties of the wood, especially the strength. The strength of clear wood can be drastically reduced by the presence of natural defects. In extreme cases the strength is reduced from the value of several dozen MPa to scarcely a few MPa. Therefore both the knowledge about the influence of these defects on the strength properties, and the ability to recognize them in timber members is important for the evaluation of the timber's usefulness in the structure.

Slope of grain

If the fibre direction is not parallel to the longitudinal axis of the timber, the wood structure is unsettled, and is described by the term "cross grain", which is measured by the slope of the grain. In turn, the measure of the slope of the grain is the angle between these two directions. The slope of the grain always causes a loss of the strength, and is more deleterious with the existence of higher angles. The influence of the slope of the grain on the wood strength is shown in the Figure 1.5.

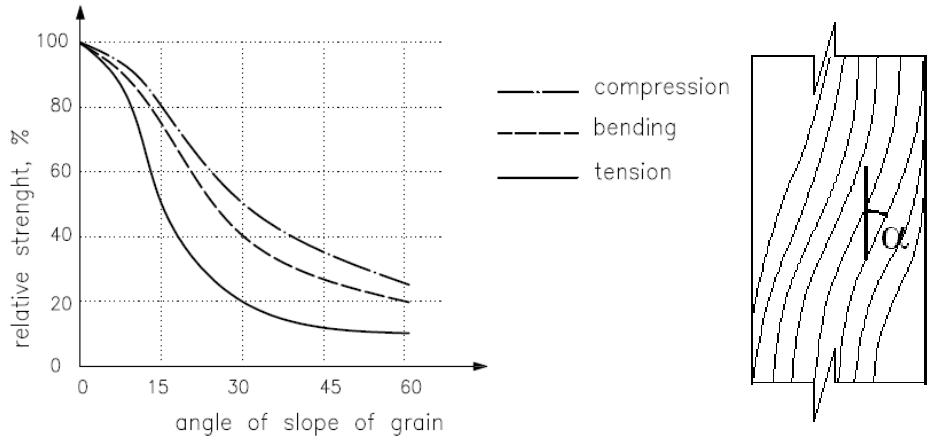


Figure 1.5. Influence of the slope of the grain on wood strength, according to Neuhaus (1994)

In most cases cross grain occurs as a result of the irregular growth of the tree or the presence of knots. Less often it is caused by poor cutting of the timber.

Knots

Knots are the remains of branches that were cut off from a tree trunk during the sawing process. There are many types of knots depending on their size, shape, location in the cross-section of a timber member or integrity with the surrounding wood.

- intergrown knot: forms around the living limb and is also known as a tight knot. The intergrown knot is well connected with the surrounding wood, but usually accompanied by considerable cross-grain;
- encased knot: occurs if growing tree trunk encloses the dead limb. Often there is a ring of bark around the encased knot, therefore it can be easily removed from the timber member;
- dead or loose knot: a knot that is not connected (or this connection is very weak) to the surrounding wood;
- sound knot: healthy knot, with no indication of decay,
- edge knot: a knot exposed on an edge. This type of knots is potentially dangerous in structural members, because it can cause the reduction of cross-sectional height,
- traversing knot: a knot exposed on two parallel faces of a structural member,
- classification of knots depending on their shape:
  - round knot,
  - oval knot,
  - spike knot (the ratio between the maximum and the minimum dimension exceeds 4),

- classification of knots depending on their size:
  - pin knot diameter $d < 6.5$ mm,
  - small knot diameter $6.5 \text{ mm} < d < 20$ mm,
  - medium knot diameter $20 \text{ mm} < d < 40$ mm,
  - large knot diameter $d > 40$ mm.

Because the straight fibres in clear wood are distorted around the knot, the mechanical properties of the members containing the knots are always decreased. Additionally, discontinuity of fibres around a knot caused by the sawing process results in the concentration of stress in the surrounding area of the knot. Therefore, the size of the knots and their amount constitute one of the main factors in the visual grading of timber.

The negative influence of the inclusions of knots on the strength is greater in tension, than in compression. Whereas in bending this effect is different, depending on the localization of a knot, as shown in Figure 1.6. Naturally, the presence of a knot near the centreline of the cross section has little effect, due to the low value of the stress existing in this zone. It is worth mentioning that transverse compression strength and shear strength may be exceptionally increased in the presence of sound, tight knots, which act as pegs.

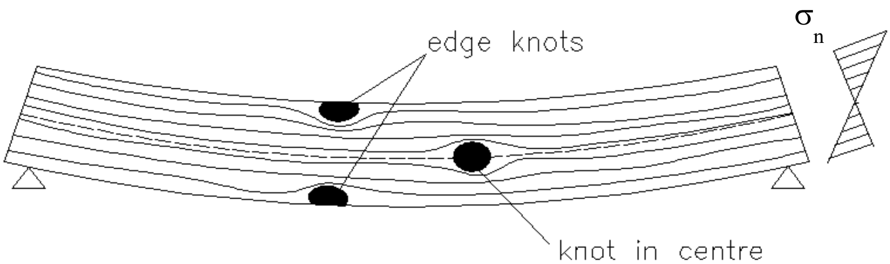


Figure 1.6. Localization of the knots in a beam and normal stress σ_n

Juvenile wood

So-called juvenile wood is produced directly around the pith of the tree trunk and covers from 5 to 20 growth rings depending on the wood species. Because of the differences in cell wall thickness and lumen diameter the specific gravity and strength properties of juvenile wood are lower than that of ordinary wood. For example tensile strength is lower by 5% to 50%.

A spiral grain occurs much more often in juvenile wood.

Inbark

Inbark is a piece of bark, which is partially or fully ingrown in the tree. It disrupts the regular fibre system and should be removed from the timber member.

Eccentric heart (pith)

An eccentric heart is a deviation of the pith from the middle of the cross-section. An irregular system of annular rings occurs, resulting – as a consequence – in the differences in mechanical properties on both sides of the pith. This defect may be dangerous in the case of compressed members, due to increased buckling effect.

Cracks

There are many kinds of cracks in wood. The main are:

- shakes: a lengthwise separation of the latewood fibres along the grain caused by bacterium belonging to the clostridium genus,
- checks: a lengthwise separation resulting from moisture loss during seasoning,
- splits: an effect of tearing apart of the wood cells, often as a result of the drying process.

The presence of all these defects limits the usefulness of timber and constitutes the basis for the visual grading of timber.

Defects caused by insects

The main type of damage made by insects is a hole in the timber. The reduction of the strength can be considerable. It is a good and safe practice to reject all timber containing insect holes, before using a timber member in a structure.

Fungi decay

Fungi attack dead and living trees, also the timber in building structures. Most wood-decay fungi destroy the wood by metabolizing the cellulose fraction of the wood, thus strongly reduces its strength. The popular blue-stain fungus is a kind of exception, because it feeds on substances within the cell cavities and only has a minor effect on the strength.

The effective way of counteracting fungi is keeping a structure dry. Most decay fungi cannot exist if the moisture of wood is below the fibre saturation point, which means a value of ab. 30%.

1.3. Shrinkage

The natural processes of drying and loss of moisture contained in wood result in the shrinkage of the wood. Shrinkage occurs only if the moisture content decreases below the fibre saturation point. This means that changes in the timber dimensions due to the shrinkage phenomena are caused by the loss of bound water only.

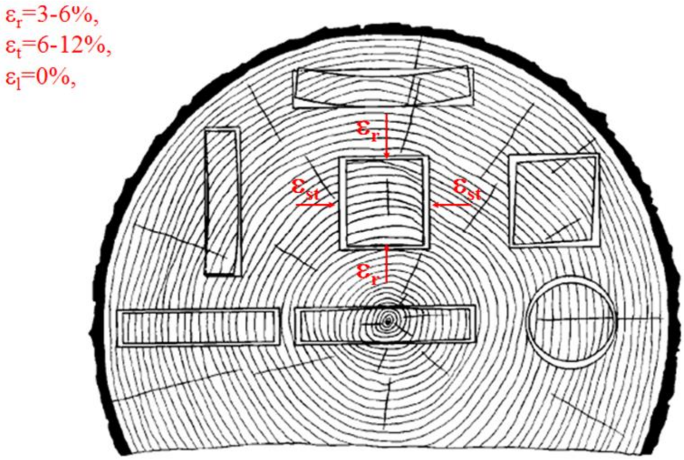


Figure 1.7. Shrinkage of wood and its effect on the cross-sectional shape, according to Ross et al (2021)

Due to the strongly anisotropic structure of wood, the shrinkage value differs depending on its direction. In the cross-sectional plane of a tree trunk, the radial and tangential shrinkage are specified, as shown in Figure 1.7.

According to the exemplary data listed in Table 1.5 tangential shrinkage is about twice as great as radial shrinkage. Generally, this rule is true for all wood species. Both, radial and tangential shrinkage values are relatively high, comparing to other structural materials, for example concrete. Fortunately, longitudinal shrinkage (along the wood fibres) equals virtually zero.

Table 1.5. Shrinkage of some species of wood

Wood specie	**Shrinkage**		**radial/tangential ratio**
	radial	**tangential**	
pine	2.1–5.4	5.6–7.7	1.4–2.7
spruce	3.7–4.3	6.8–7.8	1.8
fir	2.6–4.5	6.9–9.2	2.0–2.6
oak	4.0–4.6	7.8–10.0	1.95–2.2

The differences in tangential and radial shrinkage values result in wood warping, which is illustrated in Figure 1.7. This is one of the most unpleasant defects of timber, which sometimes makes a timber member completely useless. Therefore boards or logs with growth rings parallel to their edges are of more value than the others – they shrink but do not warp.

2. Strength properties of timber in Eurocode

There is a set of European standards devoted to the testing and calculating of strength properties (generally mechanical properties) of timber.

Standards for testing and estimating of the strength properties:

- EN 14081-1 Timber structures – Strength graded structural timber with rectangular cross-section – Part 1. General requirements.
- EN 14081-2 Timber structures – Strength graded structural timber with rectangular cross section – Part 2: Machine grading, additional requirements for initial type testing
- EN 14081-3 Timber structures – Strength graded structural timber with rectangular cross section – Part 3: Machine grading, additional requirements for factory production control
- EN 384:2010 Structural Timber – Determination of characteristic values of mechanical properties and density
- EN 408: 2010 Timber structures – Structural Timber and glued laminated timber – Determination of some physical and mechanical properties

Standards containing strength properties of timber:

- EN 338 Structural Timber: Strength Classes
- EN 14080 Timber structures – Glued laminated timber and glued solid timber – Requirements

Design values of strength are calculated according to the methods presented in the primary standard for the designing of timber structures:

- Eurocode 5: Design of timber structures. Part 1-1: General – Common rules and rules for buildings

This standard is discussed in more detail in Chapter 5 and later. Here, only information about the calculations of the timber strength is presented.

Apart of standards mentioned above, there are separate standards devoted to other wood-based materials, such as glued laminated timber, wood-based structural products and wood-based panels, as well as adhesives and mechanical fasteners.

2.1. Design value of strength

The design value of timber strength f_d is used at the Ultimate Limit State verification. It is calculated from Equation (2.1), which is fully compatible with the basic standard Eurocode 0.

$$f_d = \frac{k_{mod} \cdot f_k}{\gamma_M}, \tag{2.1}$$

where:

f_k is the characteristic value of strength (see 2.2),

k_{mod} is the modification factor, which is a conversion factor covering the effects of load duration and moisture content (see 2.3),

γ_M is the partial factor for a material property (see 2.4).

2.2. Characteristic value of strength

A characteristic value of strength is defined as a 5% fractile of a strength population. It means that 95% of tested specimens should not have the strength below the characteristic value.

The characteristic values of timber strength are given in EN 338. This standard distinguishes 12 strength classes for softwoods marked with the letter "C" – coniferous and 14 strength classes for hardwoods marked with the letter "D" – deciduous. The number following the letter "C" or "D" indicates the characteristic bending strength in N/mm^2. The bending strength is the leading kind of strength for timber. However, the last version of standard EN 338:2016 introduces also classification of timber based on tension tests, which was dedicated for glulam and cases where tension is the dominating load. These 18 strength classes are marked with the letter T. Tables 2.1 and 2.2 contain strength classification for softwood and hardwood based on edgewise bending tests.

Table 2.1. Strength classes and characteristic values of strength for softwoods, according to EN 338

strength classes	characteristic value of strength, N/mm^2					
	bending	compression		tension		shear
		parallel to grain	perpend. to grain	parallel to grain	perpend. to grain	
	$f_{m,k}$	$f_{c,0,k}$	$f_{c,90,k}$	$f_{t,0,k}$	$f_{t,90,k}$	$f_{v,k}$
C14	14.0	16.0	2.0	7,2	0.4	3.0
C16	16.0	17.0	2.2	8.5	0.4	3.2
C18	18.0	18.0	2.2	10.0	0.4	3.4
C20	20.0	19.0	2.3	11.5	0.4	3.6
C22	22.0	20.0	2.4	13.0	0.4	3.8
C24	24.0	21.0	2.5	14.5	0.4	4,0
C27	27.0	22.0	2.5	16.5	0.4	4.0
C30	30.0	24.0	2.7	19.0	0.4	4.0
C35	35.0	25.0	2.7	22.5	0.4	4.0
C40	40.0	27.0	2.8	26.0	0.4	4.0
C45	45.0	29.0	2.9	30.0	0.4	4.0
C50	50.0	30.0	3.0	33.5	0.4	4.0

For softwoods (C-classes), the tensile, compression, and shear strengths can be calculated on the basis of the bending strength from following relationships given in EN 384:

- tensile strength (parallel to grain) $f_{t,0,k}$ = -3,07 + 0.73 $f_{m,k}$,
- compression strength (parallel to grain) $f_{c,0,k} = 4{,}3(f_{m,k})^{0.5}$
- shear strength $f_{v,k} = \min(4.0;\ 1.6 + 0.1\ f_{m,k})$
- tensile strength (perpendicular to grain) $f_{t,90,k}$ = 0,4
- compression strength (perpendicular to grain) $f_{c,90,k} = 0.007\rho_k$.

Table 2.2. Strength classes and characteristic values of strength for hardwoods, according to EN 338

strength classes	characteristic value of strength, N/mm²					
	bending	compression		tension		shear
		parallel to grain	perpend. to grain	parallel to grain	perpend. to grain	
	$f_{m,k}$	$f_{c,0,k}$	$f_{c,90,k}$	$f_{t,0,k}$	$f_{t,90,k}$	$f_{v,k}$
D18	18.0	18.0	4.8	11.0	0.6	3.5
D24	24.0	21.0	4.9	14.0	0.6	3.7
D27	27.0	22.0	5.1	16.0	0.6	3.8
D30	30.0	24.0	5.3	18.0	0.6	3.9
D35	35.0	25.0	5.4	21.0	0.6	4.1
D40	40.0	27.0	5.5	24.0	0.6	4.2
D45	45.0	29.0	5.8	27.0	0.6	4.4
D50	50.0	30.0	6.2	30.0	0.6	4.5
D55	55.0	32.0	6.6	33.0	0.6	4.7
D60	60.0	33.0	10.5	36.0	0.6	4.8
D65	65.0	35.0	11.3	39.0	0.6	5.0
D70	70.0	36.0	12.0	42.0	0.6	5.0
D75	75.0	37.0	12.8	45.0	0.6	5.0
D80	80.0	38.0	13.5	48.0	0.6	5.0

Despite the fact that any strength class of timber can be applied in the calculation, in practice the availability of the material should be taken into consideration. In reality, a solid timber of strength class of more than C27 is very difficult or impossible to achieve on the market. Therefore, the designing of timber structures with the use of higher strength classes, sometimes leads to executing the structure with the use of material of downsized strength parameters or the necessity of redesigning the structure. Whereas the lowest classes C14 and C16 are commonly treated as non-constructional classes of timber.

Apart from strength, other mechanical properties of timber are also given in EN 338, such as modulus of elasticity, shear modulus and density (characteristic and mean values). All these values constitute a set of data, which is necessary in the designing process and were showed in Table 2.3.

Table 2.3. Modulus of elasticity, shear modulus and density for softwoods, according to EN 338

strength classes	stiffness properties, kN/mm^2				density, kg/m^3	
	Mean Modulus of Elast. Parrallel	**5% Modulus of Elast. Parrallel**	**Mean Modulus of Elast. Perpend.**	**Mean Shear Modulus**	**5% Density**	**Mean Density**
	$E_{m,0,mean}$	$E_{m,0,k}$	$E_{m,90,mean}$	G_{mean}	ρ_k	ρ_{mean}
C14	7.0	4.7	0.23	0.44	290	350
C16	8.0	5.4	0.27	0.50	310	370
C18	9.0	6.0	0.30	0.56	320	380
C20	9.5	6.4	0.32	0.59	330	400
C22	10.0	6.7	0.33	0.63	340	410
C24	11.0	7.4	0.37	0.69	350	420
C27	11.5	7.7	0.38	0.72	360	430
C30	12.0	8.0	0.40	0.75	380	460
C35	13.0	8.7	0.43	0.81	390	470
C40	14.0	9.4	0.47	0.88	400	480
C45	15.0	10.1	0.50	0.94	410	490
C50	16.0	10.7	0.53	1.00	430	520

2.2.1. Visual grading

Visual grading of solid timber is the most common way of specifying of the strength class. A qualified grader manually examines each timber member, essentially assessing the number and size of knots, the slope of grain and the presence of the other defects. Most European countries have their own standards and guidance for the visual grading of solid timber, which should be compatible to European Standard EN 14081-1:2005. For example, in Poland the solid timber is visually classified into one of three classes: KW, KS or KG, which correspond to the strength classes as shown in Table 2.3.

Table 2.3. Assignment of visual classes established according to PN-D-94021 to strength classes for softwood of thickness ≥ 22 mm, in Poland

wood specie	**visual classes**		
	KW	**KS**	**KG**
	strength classes		
pine	C35	C24	C20
spruce	C30	C24	C18
fir	C22	C18	C14
larch	C35	C30	C24

2.3. Modification factor

The characteristic strength of timber is established by a test that lasts about 300 seconds and is carried out on specimens with a moisture content of 12%. As was described in Chapter 1, environmental factors, such as moisture content and duration of load influence significantly the timber strength. These two factors are covered by modification factor, which is the analogue to the conversion factor, introduced in EC-0. The values of the modification factor are listed in Table 2.4.

Table 2.4 Values of modification factor k_{mod} for solid timber and the most popular wood-based structural materials, according to EC-5

material	**service class**	**load-duration class**				
		permanent	**long term**	**medium term**	**short term**	**instantaneous**
Solid timber	1	0.6	0.7	0.8	0.9	1.1
	2	0.6	0.7	0.8	0.9	1.1
	3	0.5	0.55	0.65	0.7	0.9
Glued laminated timber	1	0.6	0.7	0.8	0.9	1.1
	2	0.6	0.7	0.8	0.9	1.1
	3	0.5	0.55	0.65	0.7	0.9
LVL	1	0.6	0.7	0.8	0.9	1.1
	2	0.6	0.7	0.8	0.9	1.1
	3	0.5	0.55	0.65	0.7	0.9
Plywood	1	0.6	0.7	0.8	0.9	1.1
	2	0.6	0.7	0.8	0.9	1.1
	3	0.5	0.55	0.65	0.7	0.9
OSB/2	1	0.3	0.45	0.65	0.85	1.1
OSB/3	1	0.4	0.5	0.7	0.9	1.1
OSB/4	2	0.3	0.4	0.55	0.7	0.9

In order to determine the factor k_{mod} the structure should be assigned to the relevant service class and load-duration class.

2.3.1. Service classes

The service class takes into account the influence of the moisture contained inside the timber on its strength properties. The moisture content depends on the environmental conditions, mainly the temperature and humidity of the surrounding air, according to the data given in Table 2.5.

Table 2.5. Equilibrium moisture content of wooden material in relation to the relative humidity of the surrounding air, according to Ross R.J. (2021)

moisture content, %			
relative humidity, %	**solid wood**	**plywood**	**OSB**
10	2.5	1.2	0.8
20	4.5	2.8	1.0
30	6.2	4.6	2.0
40	7.7	5.8	3.6
50	9.2	7.0	5.2
60	11.0	8.4	6.3
70	13.1	11.1	8.9
80	16.0	15.3	13.1
90	20.5	19.4	17.2

The service classes are defined as follows:

- service class 1
  the moisture content corresponds to a temperature of 20°C and the relative humidity of the surrounding air only exceeds 65% for a few weeks per year. The average moisture content in timber does not exceed 12% (Table 2.5),
- service class 2
  the moisture content corresponds to a temperature of 20°C and the relative humidity of the surrounding air only exceeds 85% for a few weeks per year. The average moisture content in timber does not exceed 18% (Table 2.5),
- service class 3
  Environmental conditions lead to a higher moisture content than in service class 2. The average moisture content in solid timber exceeds 18%.

In some European countries, the National Annex to EC-5 contains information on the assignment of a timber structure to the relevant service class. The useful practical data is given in Table 1 in Chapter 1.

It is worth noting that structural members of roofs are commonly classified to service class 2, despite the fact that they normally satisfy the conditions of service class 1. However, it is rather irrelevant, because there is no difference in the design value of strength for commonly used wooden structural materials, between the service classes 1 and 2. According to data listed in Table 2.4, modification factors k_{mod} have the same values.

2.3.2. Load-duration classes

Many test results prove that the more time the load acts on the timber member, the more its strength reduces. This rule is taken into account by introducing the load-duration class into the calculating process. There are five load-duration classes contained in EC-5 (Table 2.6). Each class is described by giving an accumulated duration of load and is also illustrated by a relevant example of the load.

Table. 2.6. Load-duration classes, according to EC-5

load-duration class	**order of accumulated duration of load**	**examples**
permanent	more than 10 years	self-weight
long-term	6 months – 10 years	storage
medium-term	1 week – 6 months	imposed load, snow
short-term	less than 1 week	snow, wind
instantaneous		wind, accidental loads

It tends to be quite clear how to assign the load-duration class if the single action is acting on the member. The problem occurs if another kind of action appears, acting simultaneously in combination. It seems that, because of the accumulated duration of load increases, the strength should decrease. However, the clearly formulated EC-5 rule stands that if a load combination consists of actions belonging to different load-duration classes a value of k_{mod} should be chosen that corresponds to the action with the shortest duration. For example: a timber member that is subjected to only the action of self-weight is assigned to the duration class *permanent*, and if simultaneously an action of wind is applied (of any value), according to the above mentioned rule, the load-duration class changes to *short-term*. As a result both the modification factor k_{mod} and the design strength of the timber increases by 30%.

The designers of timber structures recommend calculating each possible load combination separately, assigning the appropriate load-duration class each time, and – as a consequence – another design strength of timber the next time. However, this idea leads to time-consuming calculations because the ultimate limit state conditions should be verified separately for each load combination.

2.4. Partial factors

In order to ensure the required level of safety, EC-5 uses partial factor γ_M for material properties. According to EC-0 γ_M is defined as:

$$\gamma_M = \gamma_m \cdot \gamma_{Rd}, \qquad (2.2)$$

where

γ_m is a partial factor for the material property, which takes account of the possibility of an unfavourable deviation of material property (strength) from its characteristic value and the random part of the conversion factor (k_{mod}),

γ_{Rd} is a partial factor covering the uncertainty in the resistance model and geometric deviations of a member.

EC-5 gives only the resulting value of γ_M that is listed in Table 2.7.

Table 2.7. Recommended partial factors γ_M for material properties and resistances for fundamental combinations, according to EC-5

Material	γ_M
Solid timber	1.3
Glued laminated timber	1.25
LVL, plywood, OSB	1.2
Particleboards	1.3
Fibreboards (all types)	1.3
Connections	1.3
Punched metal plate fasteners	1.25

The recommended value of partial factor γ_M for accidental combinations equals 1.0

2.5. Size effect

The strength of the wood is related to the element of the maximum cross sectional dimension h, equal to 150 mm or 600 mm, for solid and glued laminated timber respectively. Therefore, for smaller members the strength may be multiplied by the factor

$$k_h = min\left(\left(\frac{150}{h}\right)^{0.2} ; 1,3\right) \tag{2.3}$$

for solid timber, and

$$k_h = min\left(\left(\frac{600}{h}\right)^{0.1} ; 1,1\right) \tag{2.4}$$

for glued laminated timber,

where

h is the depth (a vertical cross-sectional edge) for bending members or width (the maximum cross-sectional dimension) for tension members, in [mm]. It means that in the case of two-dimensional bending the strength may be different for each bending direction. The depth h is always the edge dimension perpendicular to the axis of bending.

3. The use of timber in buildings

Timber is one of the oldest building materials, and has been used for thousands of years. Many timber buildings are monuments to this. Today, completely wooden structures, as well as parts of them, are made using timber and wood-based materials.

3.1. Roof systems

Most suburban houses have a timber roof. From the constructional point of view roofs are divided into several groups.

3.1.1. Rafter roof

A rafter roof is the simplest structure. It consists of only a pair of rafters that are based directly on each other, creating so-called couple (Figure 3.1). From the economical point of view, the span of such roofs should not exceed 7 m, otherwise the rafters will be very long and will require timber logs of an oversized cross-section. In such roofs a horizontal force at the rafter feet occurs causing roof spreading. To avoid the spreading of the rafters a tie-beam can be added to the structure, creating a simple truss. This rafter tie often constitutes a ceiling joist in a building.

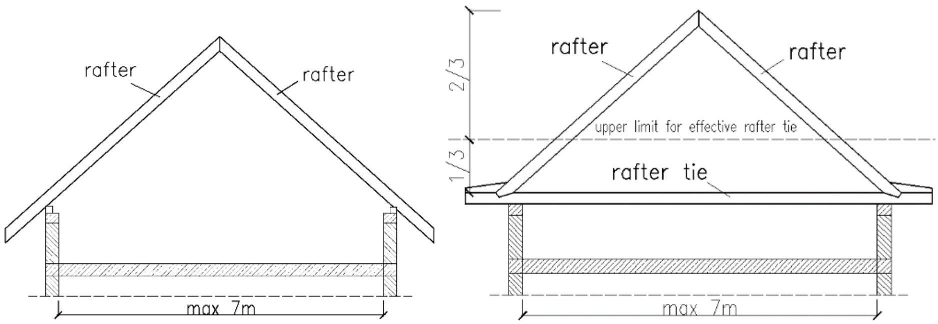


Figure 3.1. Rafter roofs. The drawings don’t show the fasteners

3.1.2. Collar roof

A rafter can be supported by a collar beam, roughly in the middle of the span as is illustrated in Figure 3.2. The precise location of the collar beam takes into account the design and functional requirements. A collar beam is installed between the pair of rafters. Because a collar beam constitutes a kind of support for rafters as it acts as the strut. Its correct stress state is compression.

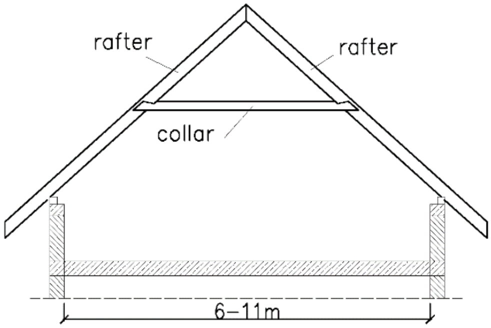


Figure 3.2. Collar roof

Sometimes, when roof spreading appears, collars can be tensioned. But it is rather an emergency state, rafters are no longer supported by the collar and their deflection increases dangerously. To avoid this situation, rafters should be firmly jointed to the walls. Of course, the typical collar beam should be distinguished from a rafter tie, which is installed in the same place, but is not designed as the rafters prop. Collar roofs are vulnerable to one-sided loads, for example wind action or snow lying at one side of the roof only, which produces disproportionate increase of the internal forces. The span of collar roofs should not exceed 9–11 m.

3.1.3. Purlin roof

A purlin is a longitudinal beam that supports all the rafters in the structure. Purlins are subjected to two-dimensional bending and require their own supports. Typically, the vertical support is a timber column (strut), and the horizontal support is a collar tie. Usually these supports are placed under every third rafter, but always the decision belongs to the designer of the structure. In order to ensure additional support for the purlin, diagonal struts bedded on columns are installed (Figure 3.3).

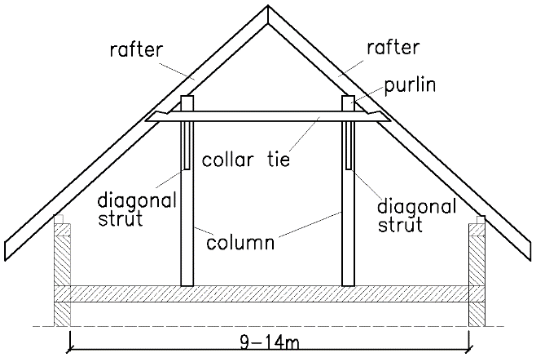


Figure 3.3. Purlin roof

Purlin roofs are not especially vulnerable to one-sided loading. Therefore the optimal span of purlin roofs can be a little greater, and equals 9–14 m. Contrary to the collar structures, the horizontal spread of the purlin roof is negligible. Instead, the load is mostly transited through the columns to the floor. It forces the design of a stronger floor structure.

3.1.4. Traditional roof trusses

Formerly, very popular roof trusses were king post trusses (Figure 3.4) and queen post trusses (Figure 3.5). A significant advantage is a lack of roof spread.

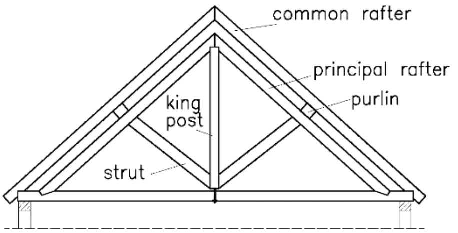


Figure 3.4. King post truss

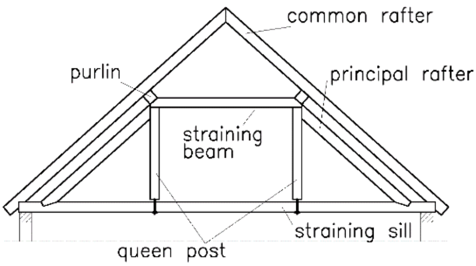


Figure 3.5. Queen post truss

Vertical members, the king post and the queen post, constitute the main parts of the trusses and they are under tension. Sometimes these members are replaced by steel rods.

3.2. Timber buildings

3.2.1. Frame buildings

Frame structures can be divided into light-frame and heavy-frame, depending on the type of members that are used. The main supporting members can be a timber posts or framed walls. Floors are constructed with the use of timber joists of standard dimensions, from 38 by 184 mm to 38 by 286 mm. Floor sheathing is made of timber boards or (more recently) wood-based materials, such as OSB boards or plywood. Frame buildings are very popular timber structures, especially in the USA and Canada.

3.2.2. Log buildings

Log buildings feature walls made of heavy timber logs, stacked one on the other. The walls are always connected in the corners creating a stiff structure. This kind of structure was often used in the past, and many historical buildings, such as churches, mills, stables etc., have a log structure. Today, log buildings are popular for small houses, but also guest houses and hotels.

4. Glued structural members (glulam)

4.1. Introduction

Structural glued-laminated timber (commonly known as “glulam”) consists of some layers of timber, joined together with the use of specially prepared glue. A characteristic for this material is that the grain of the timber is parallel to the longitudinal axis of the member, hence the structural properties of glulam are similar to sawn timber. While the board-shaped glued timber elements (cross laminated timber) are produced from layers with alternately oriented fibers. According to EN 14080 such elements are called X-lam.

The basic requirements for glulam were included in EN 14080. The standard distinguishes also other types of glued laminated timber, according to Figure 4.1.

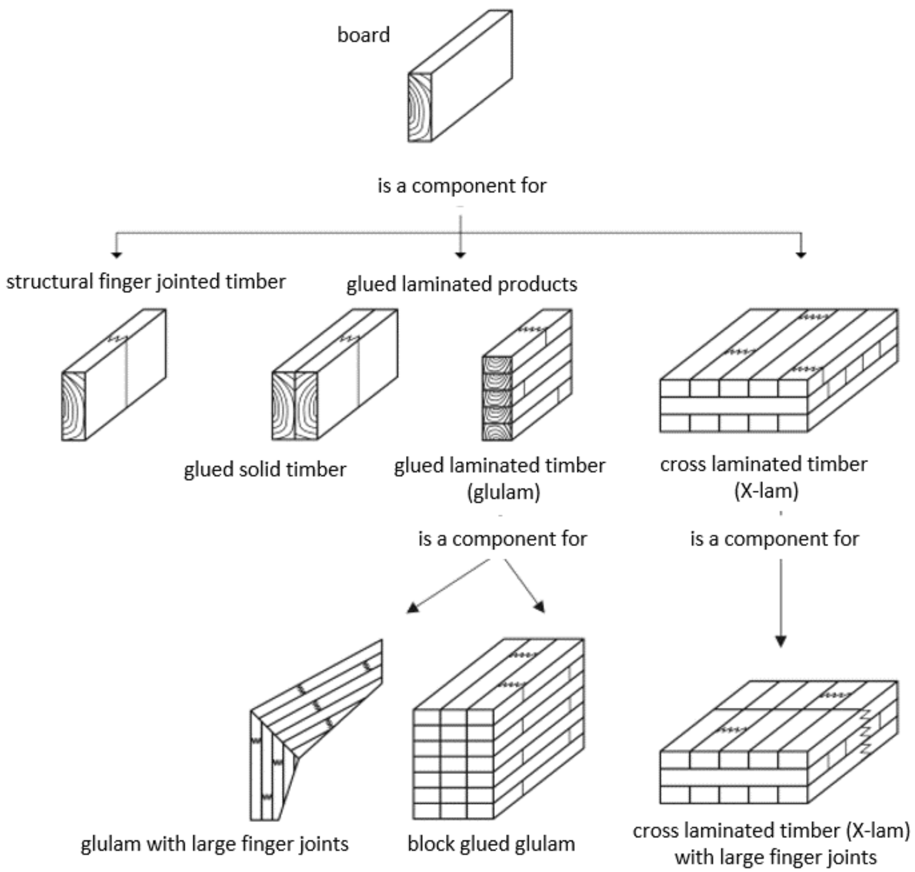


Fig. 4.1. Structural timber products, according to EN 14080 (Figure 1)

Glulam was used for the first time in the construction of an auditorium in Basel in 1893 by German engineer Otto Hetzer. Since then glulam has constantly improved and is now one of the most popular structural materials, specially used for structures with large open spaces. The production process allows members of any size to be obtained, and are limited in practice mainly by the transportation possibilities. This is one of the great advantage of glulam in comparison with sawn timber.

4.2. Manufacturing process

The manufacture of glulam items takes place in specialized plants. Manufacturing process can be divided into a few steps: preparing of timber, forming of the laminations, shaping and finishing the item.

4.2.1. Preparation of timber

According to Ross et al (2021), almost any wood species can be used for glulam timber, provided its mechanical and physical properties are appropriated. However, softwoods such as spruce or pine are usually preferred, and rarely larch or fir. In some countries hardwood such as beech is also used. Glulam consist of relatively thin pieces of timber, the thickness of which ranges between 15 mm (for sharply curved members) and 50 mm (for straight members), but the most popular thickness is 40mm. The total thickness of glulam is often a multiple of the single board thickness.

With the aim of diminishing dimensional changes of members after the manufacture process, timber should be dried to a moisture content that matches that in the structure. Because of this, the moisture content should range between 12 and 18%. However, most manufacturing plants uses timber dried to the value of 12% ± 2%, which is the average equilibrium moisture content in buildings. To achieve this level kiln drying is commonly used.

Timber is graded on the base of the typical standard procedures, using visual inspections - and for higher strength classes - mechanical methods. The limitation of knot size provides the basis for the visual evaluation. Potentially weak parts of wood containing natural defects such as cross grain, slope of grain, dead knots etc., are removed mainly using mechanical methods.

4.2.2. Forming of the lamination

After finishing the preparing process, the laminations are formed by gluing timber end-to-end to achieve the desired length. The most common joint is the so-called finger joint (Figure 4.2), because of its advantages, such as relatively small length (about 28 mm), possibility of mechanical cutting and gluing, good strength and durability achieved by large surfaces. Since the curing process is accelerated by the action of heat, the full strength of the joint is obtained within a few hours, depending on the type of the glue used.

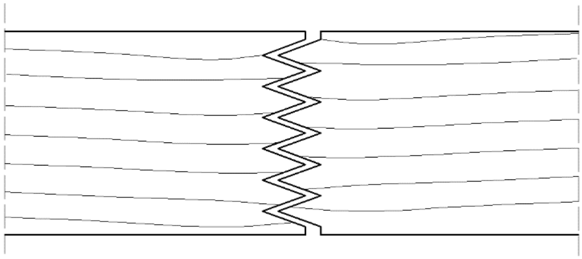


Figure 4.2. Finger joint

The wet-use types of glue are used almost exclusively, because of their improved performance compared to dry-use glues under severe conditions. Liquid one-component polyurethane adhesive, resorcinol base, phenol-resorcinol base or melamine-base adhesives are the most popular currently, all of which have strength capabilities in excess of the capacity of the wood itself. From the aesthetic point of view, the colour of the hardened glue should be considered: dark red-brown for resorcinols and a more pleasant bright white for melamines.

The lamination's length is not limited by the requirements of the gluing process, therefore this stage of glulam manufacturing does not limit the size.

4.2.3. Shaping of the members

Laminations are carefully planned to obtain plain, parallel surfaces before gluing together face-to-face. Adhesives are spread mechanically with a glue extruder, and finally the laminations are assembled into the required form in the clamping bed, and forced together with screws (Figure 4.3). Straight and arched glulam is manufactured in the same way. The important limitation for the shaping of arched members is, the radius of curvature R should not exceed the value $125 \cdot t$, where t is the lamination's thickness.

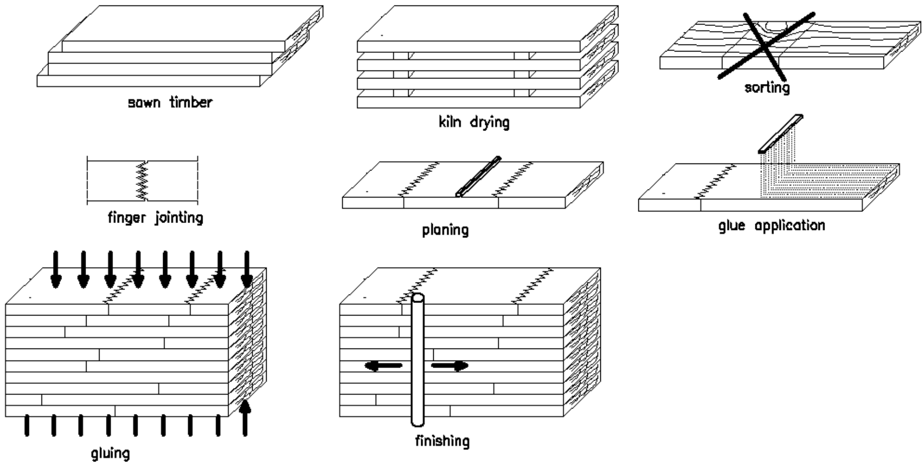


Figure 4.3. Manufacturing process of glulam

4.2.4. Finishing

After the shaping process is finished, the faces of the glulam are planned to remove adhesive residue and to align the laminations surfaces. The necessary cuts and holes are made, and connectors for joining with another members are mounted. This finishing work is called fabrication. If the glulam is to meet severe conditions with a high moisture content (20% and higher), a preservative treatment should be carried out. It means the application of creosote, oil borne or waterborne products to finished members. In the case of waterborne or light-oil preservatives, their application before gluing could give better results.

4.3. Strength properties of glulam

The strength properties of glulam usually exceed those of sawn timber, which can be the result of many factors. First, during the preparation process many defects in the timber are removed and do not diminish the strength of the glulam. Secondly, potentially weak parts of the wood such knots, cross grain etc., are dispersed throughout the cross-section of the member. It provides more safety under load, and means that from a designer’s point of view, glulam can be regarded as a homogenous material. It creates a more advantageous strength distribution with comparison to sawn timber.

If a glulam section is manufactured using laminates of the same strength, it is called single-graded or homogeneous. Its strength class symbol contains the letter “h”, for example GL32h. If the strength of the laminates is different the class symbol contains the letter “c”. For glulam with an asymmetrical layup mark “ca” is added, for example GL32ca, and for resawn glulam the class name has additional letter “s”, for example GL32cs.

Table 4.1. Strength classes and characteristic values of strength for glued laminated timber, according to EN 14080

strength classes	characteristic value of strength, N/mm^2					
	bending	compression		tension		shear
		parallel to grain	perpend. to grain	parallel to grain	perpend. to grain	
	$f_{m,k}$	$f_{c,0,k}$	$f_{c,90,k}$	$f_{t,0,k}$	$f_{t,90,k}$	$f_{v,k}$
GL20h	20	20	2,5	16	0,5	3,5
GL20c	20	18,5	2,5	15	0,5	3,5
GL22h	22	22	2,5	17,6	0,5	3,5
GL22c	22	20	2,5	16	0,5	3,5
GL24h	24	24	2,5	19,2	0,5	3,5
GL24c	24	21,5	2,5	17	0,5	3,5
GL26h	26	26	2,5	20,8	0,5	3,5
GL26c	26	23,5	2,5	19	0,5	3,5
GL28h	28	28	2,5	22,3	0,5	3,5
GL28c	28	24	2,5	19,5	0,5	3,5
GL30h	30	30	2,5	24	0,5	3,5
GL30c	30	24,5	2,5	19,5	0,5	3,5
GL32h	32	32	2,5	25,6	0,5	3,5
GL32c	32	24,5	2,5	19,5	0,5	3,5

4.4. The use of glulam

Thanks to the large cross-sectional dimensions that can be achieved, glulam is used for structures with a long span. In practice glulam beam use is very common. The double tapered (Figure 4.4) and pitched cambered (Figure 4.5) beams are very effective for the roof girders with a span up to 24 m. Exceeding this value usually results in the non-economical cross-section of the beam. However, some manufacturers technical guides give the maximum span of 40 m. During the designing of the above mentioned beams, as well as curved beams, a tensile stress perpendicular to the grain should be taken into account, which occurs due to the action of the bending moment. This stress has the greatest value in the apex zone of the beam, and tries to delaminate it. A typical counteraction is the placing of steel rods in the apex zone with the aim of taking the tensile stress.

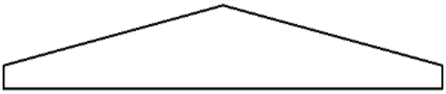


Figure 4.4. Double tapered beam

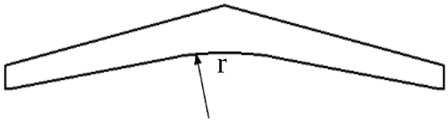


Figure 4.5. Pitched cambered beam

Glulam enables much larger distances to be spanned. Some typical glulam structures are listed below:

- double tapered, pitched cambered beams span: 10–40 m,

Figure 4.6. Pitched cambered beam of 18 m span in a school gym in Krasnystaw city, Poland. Source: Author's archive

- portal frames span: 15–100 m,
- arches span: 20–60 m,
- trusses, arched trusses span up to 100 m
  The most famous application: Olympic Stadium in Norway, Vikingship, the arched trusses with a the span to 96.4 m
- domes span up to 180m
  Application example: Tacoma Dome, a span of 161.5 m.

5. Calculations of timber elements according to EC-5

In the European Union, timber structures are designed on the basis of the rules that were described in the collection of the *Eurocode 5* standards: *Design of timber structures*. There are three parts of the Eurocode 5 standard, which are devoted to timber structures:

- Part 1-1: General – Common rules and rules for buildings,
- Part 1-2: General – Structural fire design,
- Part 2: Bridges

The overall rules contained in the Eurocodes also are used for timber. It means that the process of designing structural timber elements is based on the limit state method, in conjunction with the partial factor method, and should follow the requirements for the safety, serviceability and durability of the structure established in the basic part – *Eurocode: Basis of design*. These requirements are presented in the form of mathematical criteria:

$$E_d \leq R_d \tag{5.1}$$

for ultimate limit states of resistance (STR/GEO) and

$$E_d \leq C_d \tag{5.2}$$

for serviceability limit states,
where:

E_d is the design value of the effects of actions given in *Eurocode 1: Actions on structures,*

R_d is the design value of resistance,

C_d is the limiting design value of the serviceability criterion

Both: R_d and C_d are calculated according to the relevant part of the Eurocode. In the case of timber structures it is EC-5. The most important rules for these calculations are described below.

5.1. Ultimate Limit States

The Ultimate Limit States (ULS) deal with any form of failure of the structure or its parts. Verification of the ULS conditions ensures that the structure is safe, in other words that the probability of its failure is acceptably low. The calculations of the load capacity of timber elements R_d are described in EC-5-1 for solid timber, glued laminated timber and wood-based structural products. The relevant rules concern tension, compression, bending and torsion acting on the member as a single action or in conjunction with other factors.

The member is assumed as a constant cross-section element and its principal axes are marked as shown in Figure 5.1. The x-axis is the longitudinal direction of the member, the y- and z- axes are the cross-sectional axes, the strong and the weak ones, respectively.

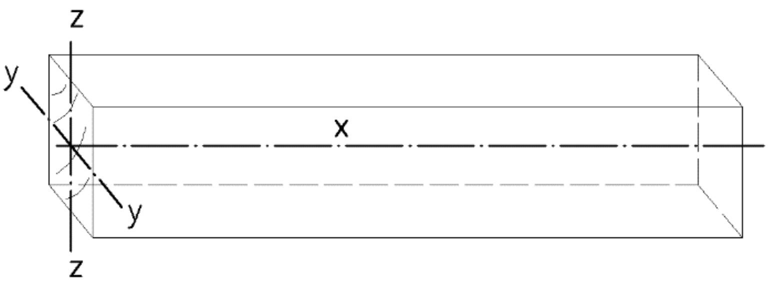


Figure 5.1. Principal member axes

5.1.1. Tension parallel to the grain

The uniform tension field throughout the entire cross section (Figure 5.2) occurs in the tension elements of the trusses such as ties and braces, and traditional roof systems such as the king post– and queen post elements.

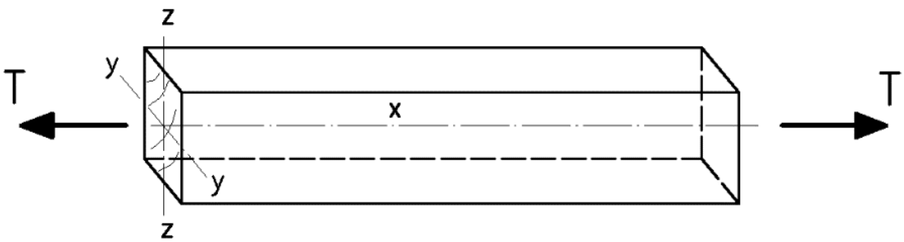


Figure 5.2. Axial tension

The following expression shall be satisfied for the tension elements

$$\frac{\sigma_{t,0,d}}{f_{t,o,d}} \leq 1.0\,, \tag{5.3}$$

where:

$\sigma_{t,0,d}$ is the design tensile stress along the grain,

$f_{t,0,d}$ is the design tensile strength along the grain.

A uniform tensile stress is obtained by

$$\sigma_{t,0,d} = \frac{T}{A}\,, \tag{5.4}$$

where:

T is the designed value of axial tensile load

A is the cross-sectional area.

Designed tensile strength is calculated taking into account the influence of the moisture content and duration of load, according to the rules described in

Chapter 2. For timber with a small cross section EC-5 also mentions the size effect.

5.1.2. Tension perpendicular to the grain

Tension in the direction perpendicular to the grain occurs relatively rarely. A typical example is the apex zone of double tapered, curved or pitched cambered beams, where tensile stress perpendicular to the grain stems from the action of bending, and should satisfy the expression

$$\sigma_{t,90,d} \leq k_{dis} \cdot k_{vol} \cdot f_{t,90,d} \,, \tag{5.5}$$

where:

$\sigma_{t,90,d}$ is the design tensile stress perpendicular to the grain,

k_{dis} is the factor depending on the effect of the stress distribution in the apex zone and equals 1.4 for double tapered and curve beams and 1.7 for pitched cambered beams,

k_{vol} is a volume factor, as follows:

$$k_{vol} = 1{,}0 \tag{5.6}$$

for solid timber, and

$$k_{vol} = \left(\frac{0{,}01m^3}{V}\right)^{0{,}2}, \tag{5.7}$$

for GL and LVL with all veneers parallel to the beam axis,

where:

V is the volume of apex zone in m^3, determined on the basis of the drawing from the standard EN 1995-1-1.

5.1.3. Compression parallel to the grain

Timber columns, posts, collar beams and some truss members are typical examples of compressed elements as shown in Figure 5.3.

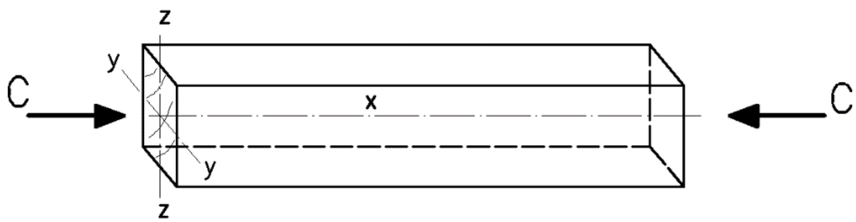


Figure 5.3. Axial compression

Compression of a structural element significantly differs from tension because a member under an axial compression load may exhibit the characteristics of deformation and buckling. The buckling effect may significantly reduce the load capacity of the compressed member as illustrated in Figure 5.4.

Figure 5.4. Buckling failure of a test column. Source: Author's archive

The influence of buckling increases with the increase of slenderness ratio λ, defined as

$$\lambda = \frac{l_e}{i}, \tag{5.8}$$

where

l_e is the effective length of the compressed element,

i is the radius of gyration of the cross section.

The above mentioned values are calculated both about the y and z axes. As a result, a slenderness ratio is also calculated about the y and z axes, and marked λ_y and λ_z respectively. When buckling occurs about the y axis, it means that the bending of the element appears in the z-x plane. Similarly, buckling about the z axis means bending in the y-x plane.

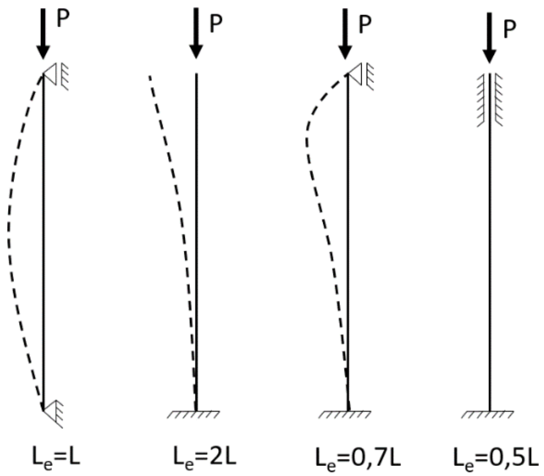


Figure 5.5. Effective length of the compressive element – example values

The effective length l_e is the distance along the element between adjacent points of contra-flexure. This calculation includes the support conditions at the ends of the element, following the instructions given in Figure 5.5. Strictly speaking, the effective length is difficult to adopt with full accuracy. In practical designing, the intuition of the designer is frequently used here.

Using the Euler buckling load equation the relative slenderness ratio is calculated from the equations given in EC-5 for the y and z directions respectively:

$$\lambda_{rel,y} = \frac{\lambda_y}{\pi}\sqrt{\frac{f_{c,0,k}}{E_{0,05}}}, \tag{5.9}$$

$$\lambda_{rel,z} = \frac{\lambda_z}{\pi}\sqrt{\frac{f_{c,0,k}}{E_{0,05}}}, \tag{5.10}$$

where
$f_{c,o,k}$ and $E_{0,05}$ are the material properties described in chapter 2.

The influence of the slenderness ratio on the load capacity of the element is shown in Figure 5.6. Since the strength reduction is very high for large slenderness ratios, it is limited to the value of 150.

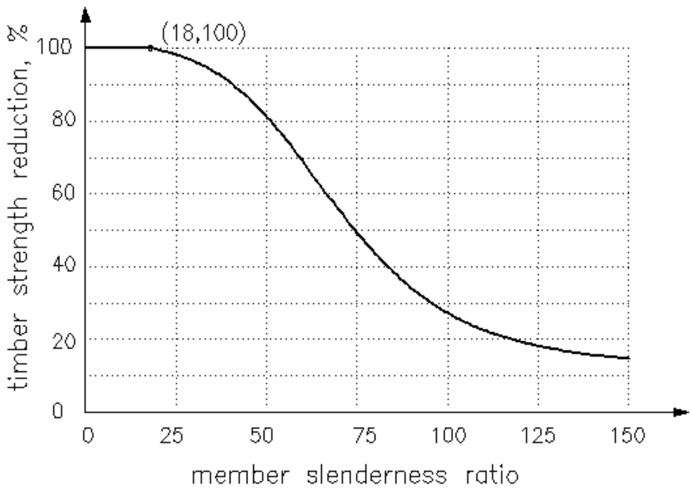


Figure 5.6. The influence of a slenderness ratio on strength. The typical plot

When an element is stocky and the slenderness ratio is below the value of 18, buckling does not occur. It approximately corresponds to the situation, where the relative slenderness ratio does not exceed the value of 0.3. This value appears in Equation (5.11) and Equation (5.12). These equations are transitional steps to obtain the buckling strength reduction factors $k_{c,y}$ and $k_{c,z}$, as follows:

$$k_y = 0{,}5\left(1 + \beta_c\left(\lambda_{rel,y} - 0{,}3\right) + \lambda_{rel,y}^2\right), \tag{5.11}$$

$$k_z = 0{,}5\left(1 + \beta_c\left(\lambda_{rel,z} - 0{,}3\right) + \lambda_{rel,z}^2\right), \tag{5.12}$$

where

β_c takes into account the initial geometric imperfection of the element and equals 0.2 or 0.1 for solid timber and glued laminated timber respectively.

The buckling strength reduction factors, also called instability factors in EC-5 are calculated from the expressions:

$$k_{c,y} = \frac{1}{k_y + \sqrt{k_y^2 - \lambda_{rel,y}^2}}, \tag{5.13}$$

$$k_{c,z} = \frac{1}{k_z + \sqrt{k_z^2 - \lambda_{rel,z}^2}}, \tag{5.14}$$

and finally

$$k_c = min\left(k_{c,y}, k_{c,z}\right). \tag{5.15}$$

Taking into account the influence of buckling for compressed elements, the ultimate limit state is verified from the formula

$$\sigma_{c,0,d} \leq k_c \cdot f_{c,0,d}. \tag{5.16}$$

5.1.4. Compression perpendicular to the grain

A compression perpendicular to the grain is commonly described as a bearing. For example, it occurs when the ends of a beam bear on a supporting member or when another member transfers its load to a beam and causes the bearing stresses.

The bearing stresses should not exceed the bearing strength, satisfying the expression

$$\sigma_{c,90,d} \leq k_{c,90} \cdot f_{c,90,d} \, , \tag{5.17}$$

where:

$\sigma_{c,90,d}$ is the design compression stress perpendicular to the grain,

$k_{c,90}$ is the factor taking into account the load configuration, the possibility of splitting and the degree of compressive deformation. Usually $k_{c,90} = 1.0$, but also higher values can be adopted (for detailed information s. p. 6.1.5 in EC-5).

The compression stress is caused by a load acting on a bearing area and is calculated from the expression

$$\sigma_{c,90,d} = \frac{F_{c,90,d}}{A_{ef}}, \tag{5.18}$$

where

$F_{c,90,d}$ is the design compressive load perpendicular to the grain, mainly the reaction transferred to the support,

A_{ef} is the effective contact area (bearing area).

The effective bearing area in calculation can be greater than only the geometrical contact area because the unloaded adjacent area (if it exists) positively influences the bearing capacity. Therefore EC-5 allows the contact area to be enlarged by 30 mm at each side, but under several conditions (s. p. 6.1.5 in EC-5).

5.1.5. Bending

Bending stresses occur when a structural member is subjected to a load applied perpendicularly to the longitudinal axis of the member. A bending moment causes tension and compression in the relevant member zones (Figure 5.6). The typical timber members that are subjected to bending are floor beams, purlins, rafters etc.

The bending stress σ_m in any point in the cross-sectional area can be calculated from the general formula

$$\sigma_m = \frac{M}{I/a},\tag{5.19}$$

where

M is the bending moment,

I is the second moment of the area of the cross-section

a is the distance from the neutral axis to the point where σ_m is calculated (Figure 5.7).

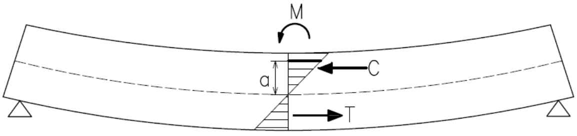


Figure 5.7. Bending stress in the cross-section of a beam

The effect of excessive stresses in tensile zone is showed in Figure 5.8.

Figure 5.8. Failure of wooden fibres caused by tensile stress. Source: Author's archive

The maximum bending stress in the cross-section is located on the compressed or stretched edge and can be calculated inserting into Equation (5.20) $a = 0.5h$, as follows

$$\sigma_m = \frac{M}{I/0{,}5h} = \frac{M}{W},\tag{5.20}$$

where

h is the depth of the beam,

W is the section modulus.

Some of the members are subjected to the bending moment acting about both the weak and the strong axes. In this case it is important to correctly identify the directions. A bending moment denoted by the index y is called a bending moment about the y axis (M_y), and is caused by a load acting in the direction of the z axis. Analogically, a bending moment about the z axis M_z is caused by the load acting in the direction of the y axis, according to the so called screw rule. The bending stresses $\sigma_{m,y}$ and $\sigma_{m,z}$ generated by the action of the bending moments M_y and M_z can be calculated from developed Equation (5.21), as follows

$$\sigma_{m,y} = \frac{M_y}{W_y}, \tag{5.21}$$

$$\sigma_{m,z} = \frac{M_z}{W_z}, \tag{5.22}$$

where

$W_y = \frac{b \cdot h^2}{6}$ and $W_z = \frac{h \cdot b^2}{6}$ are calculated for rectangular cross-sections. Designation of the symbols used above in accordance with the Figure 5.9.

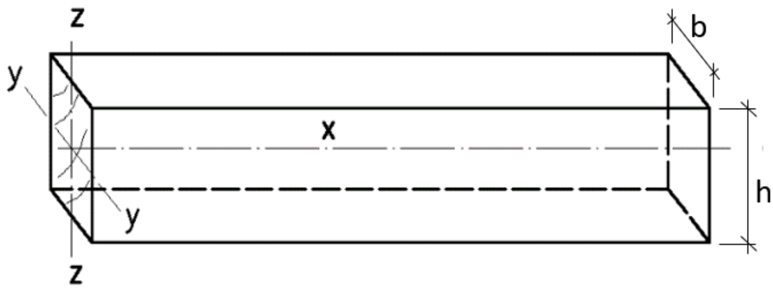


Fig. 5.9. Designations of axes and dimensions of the cross-section

The ultimate limit state condition for bending should be verified using the two following equations

$$\frac{\sigma_{m,y,d}}{f_{m.,y,d}} + k_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1, \tag{5.23}$$

$$k_m \frac{\sigma_{m,y,d}}{f_{m.,y,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1, \tag{5.24}$$

where

$\sigma_{m,y,d}$ and $\sigma_{m,z,d}$ are the design bending stresses about the y and z axes, respectively,

$f_{m,y,d}$ and $f_{m,z,d}$ are the design bending strengths about the y axis and z axis respectively (these values can be different in case of using the size factor k_h),

k_m is a factor of the value 0.7 or 1.0 for rectangular cross-section and other cross-sections, respectively.

lateral stability

Bending of the beam about its strong y axis may lead to the excessive lateral displacements of the compression face of the beam in the direction of the z axis. The excessive lateral displacement causes a dangerous increase of the bending stresses and as a consequence – a failure of the member.

Figure 5.10. The illustration of the lateral buckling effect of the beam, showed by B. Pelletier and G. Doudak 2019

The important remark made in EC-5 requires limitations for initial deviation from straightness of beams (and also other elements such for example columns) where lateral instability can occur to 1/500 times the length for glulam and LVL member and 1/300 times for solid timber. The following calculation rules are only valid under this assumption. A lateral torsional instability phenomenon occurs in the case of insufficient restraining of the compression zone of the beam. A key parameter is the relative slenderness ratio $\lambda_{rel,m}$, which is defined in EC-5 as

$$\lambda_{rel,m} = \sqrt{\frac{f_{m.k}}{\sigma_{m,crit}}}\,, \tag{5.25}$$

where

$\sigma_{m,crit}$ is the critical bending stress, calculated from Equation (5.26), according to the classical theory of stability:

$$\sigma_{m,crit} = \frac{\pi\sqrt{E_{0.05} \cdot I_z \cdot G_{0.05} \cdot I_{tor}}}{l_{ef} \cdot W_y}, \tag{5.26}$$

where

$E_{0.05}$ is the characteristic value of modulus of elasticity,
$G_{0.05}$ is the characteristic value of shear modulus,
I_z is the second moment of the area about the z axis,
I_{tor} is the torsional moment of inertia,
W_y is the section modulus about the y axis,
l_{ef} is the effective length of the beam, according to data included in Table 5.1.

Table 5.1. The ratio of the effective length l_{ef} to the design span of the beam (enhanced version of Table 6.1 from EC-5), by Kermani and Porteous, (2013)

Beam type	**Loading type (applied in the centre of gravity)**	**L_{ef} / L**
Simply supported	Constant moment	1.0
	Uniformly distributed load	0.9
	Concentrated load at mid span	0.8
	Point loads at quarter and three quarter points	0.96
	Moment M at one end M/2 in opposite direction at the other end	0.76
	Moment M at one end and zero moment at the other end	0.53
Fully fixed at both ends	Uniformly distributed load	0.78
	Concentrated load at mid span	0.64
Simply supported and restrained from lateral torsional movement at mid span	Concentrated load at mid span	0.28
Cantilever	Uniformly distributed load	0.5
	Concentrated load at the free end	0.8

The assumptions to Table 5.1:

- beam end conditions for case 1 and 2: restrained in position laterally, restrained torsionally, free to rotate in plan,

- cantilever end conditions: fixed end is restrained laterally in position, restrained torsionally, prevented from rotating in plan, free to move laterally, free to rotate at the free end,
- if the load is applied at the compression edge of the beam, l_{ef} should be increased by $2h$,
- if the load is applied at the tension edge of the beam, l_{ef} should be decreased by $0.5h$.

If lateral displacements of the beam are blocked at positions along its length by any bracing members, the effective length is based on the distance between them.

In the case of the most common practical designs rectangular cross-section of the beam, the critical bending stress can be calculated using the following equation

$$\sigma_{m,crit} = \frac{0.78 \cdot b^2}{l_{ef} \cdot h} \cdot E_{0.05} \,. \tag{5.27}$$

The relative slenderness ratio $\lambda_{rel,m}$ is used to define the influence of the beam lateral instability on its load capacity by the reduction of timber bending strength. The lateral buckling reduction factor k_{crit} is calculated according to the following relations

$$k_{crit} = 1 \quad for \;\; \lambda_{rel,m} \leq 0.75 \tag{5.28}$$

$$k_{crit} = 1.56 - 0.75 \cdot \lambda_{rel,m} \quad for \quad 0.75 < \lambda_{rel,m} \leq 1.4 \tag{5.29}$$

$$k_{crit} = \frac{1}{\lambda_{rel,m}^2} \quad for \quad 1.4 < \lambda_{rel,m} \tag{5.30}$$

If the slenderness ratio $\lambda_{rel,m} \leq 0.75$ and the factor k_{crit} equals 1.0 the beam is fully (or adequately) secured from lateral displacement, and its bending strength is not reduced. When $\lambda_{rel,m} > 0.75$ the following condition for bending about the y axis must be satisfied:

$$\frac{\sigma_{m,y,d}}{k_{crit} \cdot f_{m.,y,d}} \leq 1 \,. \tag{5.31}$$

Equation (5.31) constitutes the ultimate limit state condition, which proves that lateral instability of the beam will not occur.

5.1.6. Shear

Shear stresses are generated in laterally loaded members and usually occur together with bending stresses. Due to the strongly anisotropic structure of the wood, the shear crack usually occurring in the beam's supporting zone runs parallel to the grain, is illustrated in Figure 5.11.

Figure 5.11. Longitudinal fracture caused by shear. Source: Author's archive

The shear stress at a given level in the cross-section of a beam is calculated from the equation

$$\tau = \frac{V \cdot S}{I \cdot b}, \tag{5.32}$$

where

V is the shear force at the cross-section,

S is the first moment of the area created above the given level about the neutral axis,

I is the second moment of cross-sectional area about the neutral axis,

b is the width of the cross-section.

For the most common rectangular cross-section of a beam the maximum shear stress arises at the middle of the cross-section and can be calculated as

$$\tau = 1.5 \frac{V}{b \cdot h}. \tag{5.33}$$

A small difference is introduced for timber members. Instead of the width b EC-5 gives the effective width b_{ef}, which takes into account the influence of longitudinal cracks:

$$b_{ef} = k_{cr} \cdot b, \tag{5.34}$$

where

k_{cr} = 0.67 for solid timber and glued laminated timber,

k_{cr} = 1.0 for other wood-based products.
The shear stress at the notched support should be calculated using the effective depth h_{ef} instead of h. Also for notched beams the shear strength is reduced by factor k_v. For details see EC-5. p. 6.5.2.

For shear with a stress component parallel to the grain, as well as for shear with both components perpendicular to the grain, the ultimate limit state condition is formulated as follows

$$\frac{\tau_d}{f_{v,d}} \leq 1.0\,. \tag{5.35}$$

5.1.7. Combined stresses

bending and axial tension

EC-5 assumes that the failure tensile strength will be achieved in the extreme fibre at the tension zone of the member. The ultimate state condition is a simple assembly of conditions expressed by Equation (5.3), Equation (5.23) and Equation (5.24)

$$\frac{\sigma_{t,0,d}}{f_{t,o,d}} + \frac{\sigma_{m,y,d}}{f_{m.,y,d}} + k_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1\,. \tag{5.36}$$

$$\frac{\sigma_{t,0,d}}{f_{t,o,d}} + k_m \frac{\sigma_{m,y,d}}{f_{m.,y,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1\,. \tag{5.37}$$

In practical designing, lateral torsional instability should also be considered, using the information given in 5.1.5.

bending and axial compression

EC-5 gives different rules for verifying the ultimate limit state condition depending on whether a member is classified as column or a beam. This classification is rather clear usually. However, there are situations when it is not so certain. Therefore, the general way of calculating if members are subjected to combined bending and compression, considering the effects of lateral instability, is described briefly below. This problem is presented in more detail in Kermani and Porteous (2013).

Three design cases are determined due to the lateral stability and buckling criteria. The ultimate limit state conditions are given for each case, according to EC-5.

1. $\lambda_{rel,m} \leq 0.75$ and ($\lambda_{rel,y} \leq 0.3$ and $\lambda_{rel,z} \leq 0.3$)
no lateral torsional instability and no buckling will occur

There is no reduction in strength because a member is thick. EC-5 uses the plastic theory approach, which enables the bending part of the combined stress to be enhanced (the compression stress to strength ratio is squared), which is illustrated in Figure 5.12.

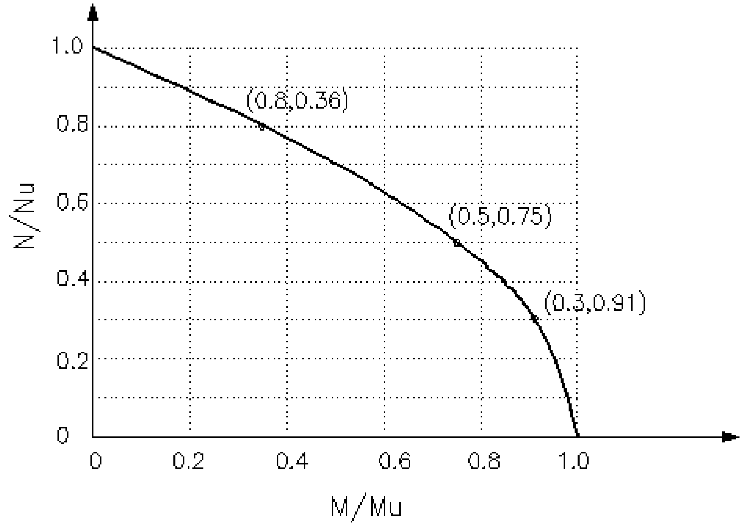


Figure 5.12. Interaction between compression and bending

The following expressions shall be satisfied:

$$\left(\frac{\sigma_{c,0,d}}{f_{c,o,d}}\right)^2 + \frac{\sigma_{m,y,d}}{f_{m.,y,d}} + k_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1\,, \tag{5.38}$$

$$\left(\frac{\sigma_{c,0,d}}{f_{c,o,d}}\right)^2 + k_m \frac{\sigma_{m,y,d}}{f_{m.,y,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1\,. \tag{5.39}$$

2. $\lambda_{rel,m} \leq$ 0.75 and ($\lambda_{rel,y}$ > 0.3 and/or $\lambda_{rel,z}$ > 0.3)
no lateral torsional instability will occur, but buckling is possible

Due to the necessity of taking into account the buckling effects, the elastic theory is used. Here the compression stress to strength ratio is no longer squared. In turn the buckling strength reduction factors occur. The ultimate limit state condition is verified from the expressions:

$$\frac{\sigma_{c,0,d}}{k_{c,y} \cdot f_{c,o,d}} + \frac{\sigma_{m,y,d}}{f_{m.,y,d}} + k_m \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1\,, \tag{5.40}$$

$$\frac{\sigma_{c,0,d}}{k_{c,z} \cdot f_{c,o,d}} + k_m \frac{\sigma_{m,y,d}}{f_{m.,y,d}} + \frac{\sigma_{m,z,d}}{f_{m,z,d}} \leq 1. \tag{5.41}$$

3. $\lambda_{rel,m} > 0.75$

lateral torsional instability is possible

This case applies to members subjected to axial compression and bending about the strong y axis only. EC-5 gives no conditions for the situation where bending occurs about both the y axis and the z axis at the same time, and $\lambda_{rel,m}$ exceeds the value of 0.75.

The ultimate limit state condition is based on the plastic behaviour similar to that described above (Equation 5.37 and Equation 5.38), and is formulated as follows:

$$\frac{\sigma_{c,0,d}}{k_{c,z} \cdot f_{c,o,d}} + \left(\frac{\sigma_{m,y,d}}{k_{crit} \cdot f_{m.,y,d}}\right)^2 \leq 1\,. \tag{5.42}$$

5.2. Serviceability Limit State (SLS)

The serviceability limit state for timber members consider the limitation of its deformations only. Limiting of deformation of a structural member aims to ensure:

- safe and appropriate use of finishing materials such as claddings, plasters and plasterboards etc.,
- safe and appropriate use of non-structural building equipment such as installations, windows, doors etc., which are associated with the structural member,
- no vibrations that cause discomfort to users
- functionality of a structure,
- good appearance of a structure.

Generally speaking, serviceability limit states are established to ensure a trouble-free structure. Exceeding the SLS does not lead to the failure of the structure. However, it does not mean that SLS is not important. A properly designed structure has to satisfy both the SLS and ULS conditions. However, EC-0 assumes three types of SLS that can be agreed with the client:

- no exceedance of the SLS is permitted. SLS conditions should be satisfied using the characteristic values of actions in the load combination,
- the frequency and duration of exceeding the SLS is agreed with a client, but the exceedance is removable when the action causing it is removed. SLS conditions are calculated using the frequent value of actions with the relevant combination factor ψ_1,
- the long-term (removable) exceedance is agreed with a client. According to EC-5 SLS conditions are calculated using the quasi-permanent values of actions with the relevant quasi-permanent combination factor ψ_2.

According to the rules presented in EC-5, SLS conditions are calculated with the use of characteristic values of actions. However it is not a principle rule.

An initial response of a timber member to applied load is called the instantaneous deformation $u_{inst.}$. Deformations of a timber member increase over time because of the creep phenomenon. Fortunately, this increase (known as creep deformation u_{creep}) stabilizes after some period of time and the deformation reaches the final value u_{fin}, as shown in the example of the simple supported beam in Figure 5.13.

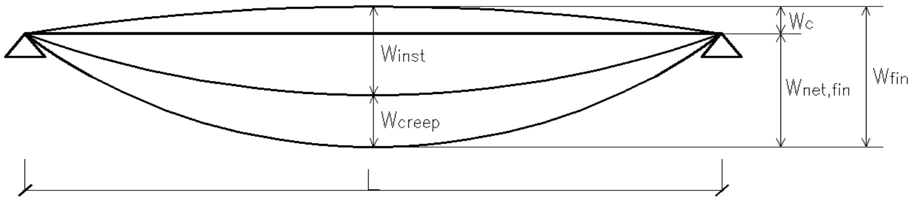


Figure 5.13. Deflection of a beam, according to EC-5

EC-5 gives no particular rules in order to calculate the instantaneous deflection of the beam, assuming that it is rather common knowledge. However, for example, the Polish National Annex to EC-5 contains useful expressions, which help to calculate the instantaneous deflection for some practical cases.

- beams of solid section

$$u_{inst} = \alpha \cdot \frac{5}{384} \cdot \frac{q_k \cdot L_{ef}^4}{E_{0,mean} \cdot I} \left[1 + 19.2 \left(\frac{h}{L_{ef}} \right)^2 \right], \tag{5.43}$$

where

q is the characteristic value of uniform distributed load,

L_{ef} is an effective length of span,

h is the depth of the beam

$E_{0,mean}$ is a mean value of modulus of elasticity,

I is the second moment of area about a relevant axis,

α is a factor that takes into account a static scheme of the beam, according to the data listed in Table 5.2;

The expression in square brackets may be omitted if $L_{ef}/h > 20$.

- double tapered beams

$$u_{inst} = \alpha \cdot \frac{5}{384} \cdot \frac{q_k \cdot L_{ef}^4}{E_{0,mean} \cdot I} \frac{1 + 19.2\left(\frac{h}{L_{ef}}\right)^2}{\left\lfloor 0.15 + 0.85\frac{h_p}{h_{ap}} \right\rfloor}, \tag{5.44}$$

where

h_p is the depth of a beam at the support,

h_{ap} is the depth of a beam at the apex;

- solid double T beams and box beams

$$u_{inst} = \alpha \cdot \frac{5}{384} \cdot \frac{q_k \cdot L_{ef}^4}{E_{0,mean} \cdot I} \left[1 + \eta_1 \left(\frac{h}{L_{ef}}\right)^2\right], \tag{5.45}$$

where

η_1 is the factor taking into account the influence of a shear on the deflection of the beam, according to the data given in Table 5.3.

- double tapered double T beams and double tapered box beams

$$u_{inst} = \alpha \cdot \frac{5}{384} \cdot \frac{q_k \cdot L_{ef}^4}{E_{0,mean} \cdot I} \frac{1 + \eta_1\left(\frac{h}{L_{ef}}\right)^2}{\left\lfloor 0.4 + 0.6\frac{h_p}{h_{ap}} \right\rfloor}. \tag{5.46}$$

Table 5.2. The factor α taking into account a static scheme of beams, according to Polish NA to EC-5

Structural system	**Load**	
	Permanent	**Variable**
Simply supported beam	1.0	
End span of continuous beam	0.65	0.9
Interior span of continuous beam	0.25	0.75

Table 5.3. Values of η_1 factor for I-beam and box beam, according to Polish NA to EC-5

Solid timber				
Width of web to width of flange ratio	1.0	0.5	0.33	0.25
η_1	19.2	30.0	40.0	51.0

Creep depends on many factors, such as:

- level of stress in the timber member,
- temperature of timber,
- moisture content in the timber member,
- duration of the load.

The first factor can be ignored in calculations, because the level of stresses at the serviceability limit state is relatively constant and its influence on creep is stable. The influence of temperature changes at normal conditions is not very high, whereas moisture content and load duration have to be considered as follows:
For permanent action *G*

$$u_{creep,G} = k_{def} \cdot u_{inst,G}, \tag{5.47}$$

And for main variable action Q_1

$$u_{creep,Q1_1} = \psi_{2,1} \cdot k_{def} \cdot u_{inst,Q1}. \tag{5.48}$$

For accompanying variable action Q_i (i>1) $u_{creep,Qi}$ is calculated similarly to Equation (5.48), taking into account the combination factor ψ_0 additionally, see Equation (5.51).
The deformation factor k_{def} in Equation (5.47) and Equation (5.48) determines the creep deformation of the timber member under permanent load. EC-5 gives values of the deformation factor due to the service class (determined by the moisture content), as was shown in Table 5.4.

Table 5.4 Values of deformation factor k_{def} for solid timber and the most popular wood-based structural materials, according to EC-5

material	**service class**		
	1	**2**	**3**
Solid timber	0.6	0.8	2.0
Glued laminated timber	0.6	0.8	2.0
LVL	0.6	0.8	2.0
Plywood (depending on the type, they may not be present in class 2 or 3)	0.8	1.0	2.5
OSB/2 OSB/3, OSB/4	2.25 1.5	- 2.25	- -

Assuming that a pre-camber u_c is not used, the final deflection is given by the expressions:

for permanent action *G*

$$u_{fin,G} = u_{inst,G} + u_{creep,G} = u_{inst,G} \cdot (1 + k_{def}), \tag{5.49}$$

for main variable action Q_1

$$u_{fin,Q1} = u_{inst,Q1} + u_{creep,Q1} = u_{inst,Q1} \cdot (1 + \psi_{2,1} \cdot k_{def}), \tag{5.50}$$

And for accompanying variable action Q_i (i>1)

$$u_{fin,Qi} = u_{inst,Qi} + u_{creep,Qi} = u_{inst,Qi} \cdot (\psi_{0,i} + \psi_{2,i} \cdot k_{def}). \tag{5.51}$$

The final deflection is a simple superposition of fragmentary deflections, as follows:

$$u_{fin} = u_{fin,G} + u_{fin,Q1} + u_{fin,Qi}. \tag{5.52}$$

The net final deflection below the straight line between the supports is marked $u_{net,fin}$. Applying so-called pre-camber u_c aims to diminish the resulting deflection. Typical value of pre-camber for beams is about 1,5 times dead load deflection, according to Brejer D.E, Fridley K. J, Cobeen K. E., 2020. The net final deflection is

$$u_{net,fin} = u_{fin} - u_c. \tag{5.53}$$

In order to satisfy the SLS requirements, deformations of the structural member should not exceed the maximum value. EC-5 recommends limiting not only the final deflection but also the instantaneous one, according to the data included in Table 5.5.

Table 5.5. Examples of limiting values for deflections of beams, according to EC-5

	w_{inst}	$w_{net,fin}$	w_{fin}
beam on two supports	L/300 to L/500	L/250 to L/350	L/150 to L/300
cantilever beams	L/150 to L/250	L/125 to L/175	L/75 to L/150

6. Connections

Connections provide the integrity of a structure. The safety of a structure is greatly influenced by the correct design and the execution of the connections of the members. There are many types of connections and fasteners that create them. Recently, the most popular are mechanical connections with the use of metal fasteners, which can be divided into two main groups:

- connections with metal dowel-type fasteners,
- connections with bearing-type connectors.

6.1. Connections with metal dowel-type fasteners

Dowel-type fasteners transfer the load from one timber member to another by their dowel-like part. Typical examples of such fasteners are shown in Figure 6.1.

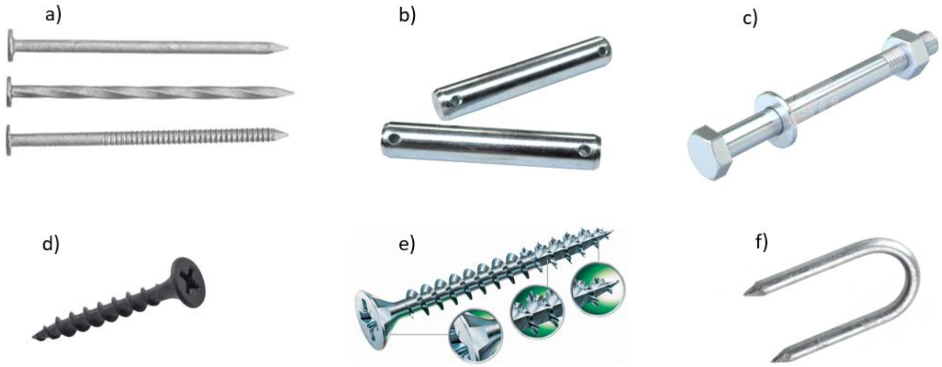


Figure 6.1. Dowel-type fasteners for timber structures:
a) nails, b) dowels, c) bolts d,e) screws, f) staples

The fasteners can be loaded in a perpendicular or parallel direction to the dowel axis. The perpendicular loading is called lateral, and the parallel loading is called withdrawal, as illustrated in Figure 6.2. Fundamentally, the dowel-type fasteners are designated for lateral type of loading. Some dowel-type fasteners such as nails (especially smooth nails) or staples should not be subjected to withdrawal loads if it is not necessary.

The dowel-type fasteners should not be placed in a timber member in the direction parallel to the grain, because of their poor load capacity in such a position, especially under long-term loadings.

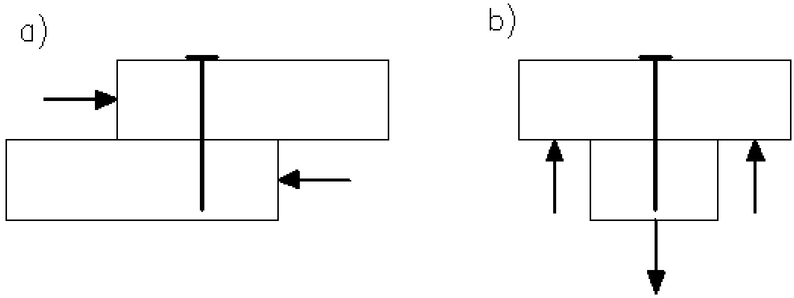


Figure 6.2. Lateral loading (a) and withdrawal loading (b)

6.1.1. Lateral load-carrying capacity

Depending on the geometry of the joint, there are single shear, double shear and multi shear connections. The basis for this distinction is a number of shear planes per fastener, according to the sketch in Figure 6.3. The necessary condition is that the fastener is placed into the last member in the connection respecting the ideal penetration length.

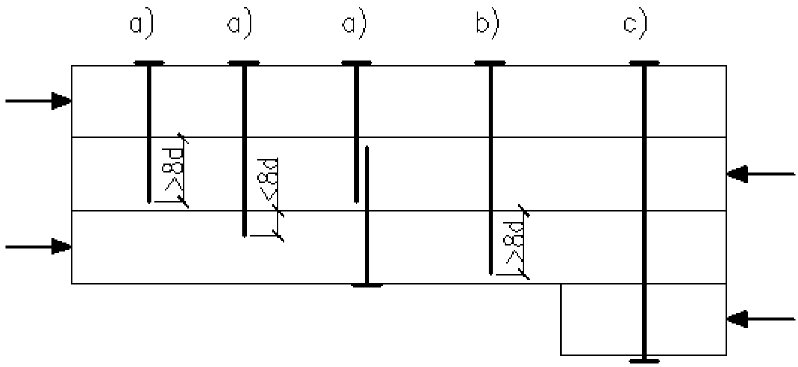


Figure 6.3. Single (a), double (b), and multi shear connections (c)

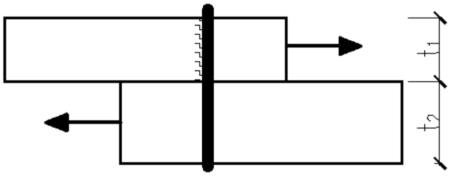
The terms "single shear" or "double shear" is somewhat misleading because it suggests a shear-type mechanism of failure. In fact a dowel in a connection is subjected to a combination of shear and flexure. The failure mechanism resembles rather the flexure-type in timber-to-timber connections. Also, in the adjacent timber fibres bearing (embedment) stresses arise.

EC-5 adopts the modified Johansen's equations for calculating the load capacity of connections, which takes into account the possibility of one of several failure modes arising, which are illustrated below. However, for a given connection it is difficult or quite impossible to predict which failure mode will be decisive. Therefore each possible failure mode should be considered and calculated. The lowest calculated value is assumed as the load-carrying capacity of the connection.

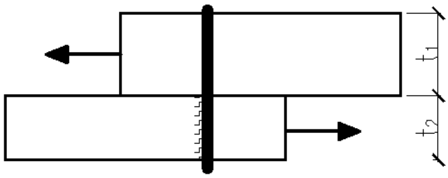
Below, the particular failure modes are shown with corresponding equations. The equations enable the characteristic load–carrying capacity per fastener per shear plane $F_{v,Rk}$ to be calculated.

Timber to timber (also panel to timber), single shear connection:

mode (a) $F_{v,Rk,1} = f_{h,1,k} \cdot t_1 \cdot d,$ (6.1)

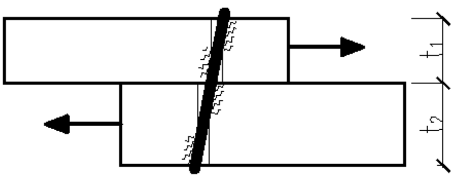


mode (b) $F_{v,Rk,2} = f_{h,2,k} \cdot t_2 \cdot d,$ (6.2)



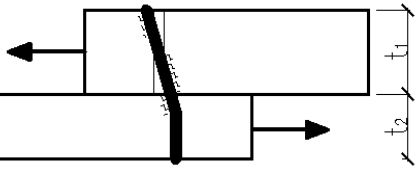
mode (c)

$$F_{v,Rk,3} = \frac{f_{h,1,k} \cdot t_1 \cdot d}{1+\beta}\left[\sqrt{\beta + 2\beta^2\left[1 + \frac{t_2}{t_1} + \left(\frac{t_2}{t_1}\right)^2\right] + \beta^3\left(\frac{t_2}{t_1}\right)^2} - \beta\left(1 + \frac{t_2}{t_1}\right)\right] + \frac{F_{ax,Rk}}{4}, \quad (6.3)$$



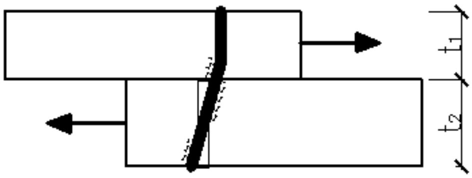
mode (d)

$$F_{v,Rk,4} = 1.05\frac{f_{h,1,k}\cdot t_1\cdot d}{2+\beta}\left[\sqrt{2\beta(1+\beta)+\frac{4\beta(2+\beta)M_{y,Rk}}{f_{h,1,k}\cdot t_1^2\cdot d}}-\beta\right] + \frac{F_{ax,Rk}}{4}, \tag{6.4}$$



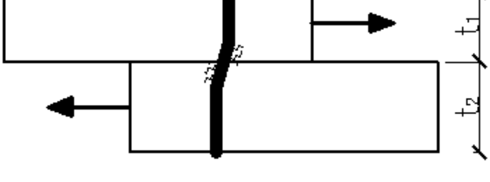
mode (e)

$$F_{v,Rk,5} = 1.05\frac{f_{h,1,k}\cdot t_2\cdot d}{1+2\beta}\left[\sqrt{2\beta^2(1+\beta)+\frac{4\beta(1+2\beta)M_{y,Rk}}{f_{h,1,k}\cdot t_2^2\cdot d}}-\beta\right] + \frac{F_{ax,Rk}}{4}, \tag{6.5}$$



mode (f)

$$F_{v,Rk,6} = 1.15\sqrt{\frac{2\beta}{1+\beta}}\cdot\sqrt{2M_{y,Rk}\cdot f_{h,1,k}\cdot d}+\frac{F_{ax,Rk}}{4}. \tag{6.6}$$

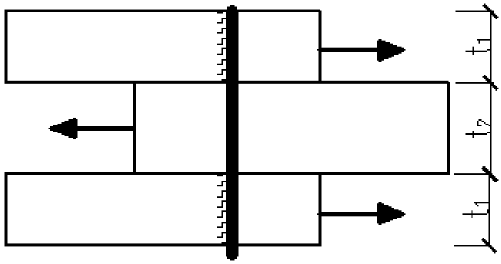


Finally:

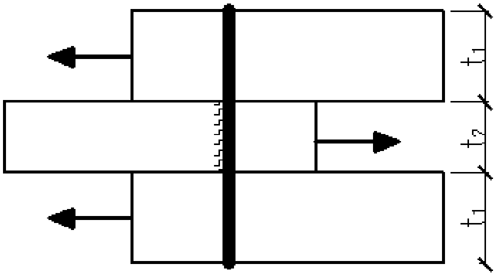
$$F_{v,Rk} = min(F_{v,Rk,1} \ldots F_{V,Rk,6}) \tag{6.7}$$

Timber to timber (panel to timber), double shear connection:

mode (g) $f_{v,Rk,1} = f_{h,1,k} \cdot t_1 \cdot d,$ (6.8)

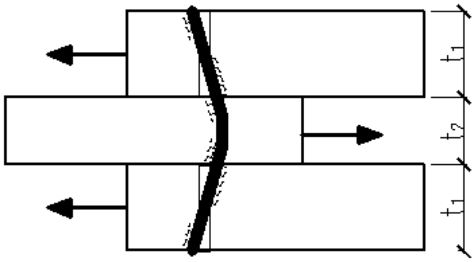


mode (h) $f_{v,Rk,2} = 0.5 f_{h,2,k} \cdot t_2 \cdot d,$ (6.9)



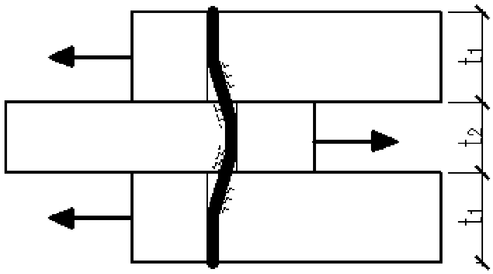
mode (i)

$$f_{v,Rk,3} = 1.05\frac{f_{h,1,k} \cdot t_1 \cdot d}{2+\beta}\left[\sqrt{2\beta(1+\beta) + \frac{4\beta(2+\beta)M_{y,Rk}}{f_{h,1,k} \cdot t_1^2 \cdot d}} - \beta\right] + \frac{F_{ax,Rk}}{4}, \tag{6.10}$$



mode (j)

$$f_{v,Rk,4} = 1.15\sqrt{\frac{2\beta}{1+\beta}} \cdot \sqrt{2M_{y,Rk} \cdot f_{h,1,k} \cdot d} + \frac{F_{ax,Rk}}{4}. \tag{6.11}$$



Finally:

$$F_{v,Rk} = min(F_{v,Rk,1}...F_{V,Rk,4}), \tag{6.12}$$

where

d is the diameter of the fastener,

t_1 is the headside thickness of the first timber member (Equations 6.1 - 6.6) or minimum of the headside timber thickness and pointside penetration (Equations 6.8 – 6.11) in connection,

t_2 is the penetration length (Equations 6.1 - 6.6) or the thickness (Equations 6.8 – 6.11) of the central timber member in connection, as was illustrated in Figure 6.4

$f_{h,k,1}$ is the characteristic embedment strength in the first timber member in the connection,

$f_{h,k,2}$ is the characteristic embedment strength in the second timber member in the connection,

β is the ratio between the embedment strength of the timber members, as follows:

$$\beta = \frac{f_{h,2,k}}{f_{h,1,k}}, \tag{6.13}$$

$M_{y,Rk}$ is the characteristic fastener yield moment, given in EC-5 or in manufacturer's specifications for different types of fasteners

$F_{ax,Rk}$ is the characteristic axial withdrawal capacity of the fastener, according to the relevant calculation procedure given in EC-5.

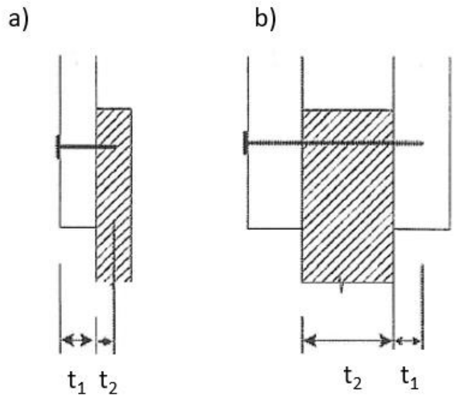


Figure 6.4. Definitions of thickness t_1 and t_2 for single (a) and double (b) shear connections, according to EC-5

Embedment strength

The embedment strength is experimentally derived from the tests, which rely on determining the maximum possible force, by pressing a dowel into the timber, as shown in Figure 6.5.

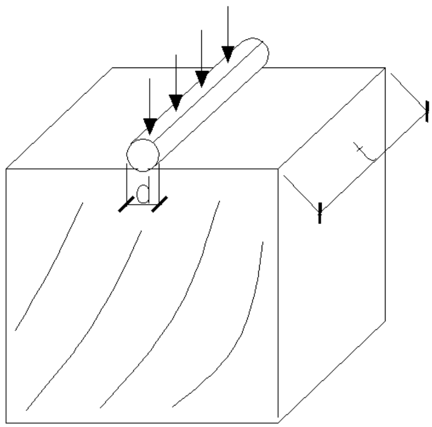


Figure 6.5. The test schema for determining the embedment strength of a timber member

Unlike the other material properties, the embedment strength of timber is not given in EN-338, because it is rather a system property than a material property. The test results show that embedment strength depends not only on material resistance – the density of the wood in this case, but also on other factors, for example, the diameter of the dowel. EC-5 includes equations that enable the embedment strength for different fasteners to be calculated. The following are the most frequently used equations, according to EC-5:

- **nails of diameter $d \leq 8$ mm**
  for timber and LVL connections:
  without pre-drilling holes
  $$f_{h,k} = 0.082\rho_k \cdot d^{-0.3}, \tag{6.14}$$
  with pre-drilling holes
  $$f_{h,k} = 0.082(1 - 0.01d)\rho_k, \tag{6.15}$$
  for plywood connections (head diameter of at least $2d$):
  $$f_{h,k} = 0.11\rho_k \cdot d^{-0.3}, \tag{6.16}$$
  for hardboard connections (head diameter of at least 2d):
  $$f_{h,k} = 30t^{0.6} \cdot d^{-0.3}, \tag{6.17}$$
  for particleboard and OSB connections (head diameter of at least $2d$):
  $$f_{h,k} = 65t^{0.1} \cdot d^{-0.7}, \tag{6.18}$$
- **nails of diameter d > 8 mm, bolts $d \leq$ 30mm and dowels**
  for timber and LVL connections:
  load parallel to the grain
  $$f_{h,0,k} = 0.082(1 - 0.01d)\rho_k, \tag{6.19}$$
  load at angle α to the grain
  $$f_{h,a,k} = \frac{f_{h,0,k}}{k_{90}\, sin\alpha^2 + cos\alpha^2}, \tag{6.20}$$
  for plywood-to-timber connections:
  $$f_{h,k} = 0.11(1 - 0.01d)\rho_k, \tag{6.21}$$
  for particleboard-to-timber and OSB-to-timber connections:
  $$f_{h,k} = 50t^{0.2} \cdot d^{-0.6}, \tag{6.22}$$

where
ρ_k is a timber characteristic density, according to EN 338 in kg/m^3,
d is a diameter of the fastener dowel in mm,
t is the minimum of panel thickness or penetration length of the nail in mm,
k_{90} is the embedment strength parallel to the grain to the embedment strength perpendicular to the grain ratio, as follows:

$$k_{90} = 1{,}35 + 0{,}015d \text{ for softwoods,} \tag{6.23}$$

$$k_{90} = 1{,}3 + 0{,}015d \text{ for LVL,} \tag{6.24}$$

$$k_{90} = 0{,}9 + 0{,}015d \text{ for hardwoods.} \tag{6.25}$$

Characteristic fastener yield moment

A fastener yield moment $M_{y,Rk}$ constitutes the parameter of the steel fastener, which gives information about the bending capacity of the dowel-type fasteners in the connection. Such data can be found in the technical specification of each product. Whereas EC-5 uses the experimentally derived equations for different types of fasteners, as follows:

- smooth round nails produced form a wire of tensile strength $f_u \geq 600$ N/mm^2

$$M_{y,Rk} = 0.3 f_u \cdot d^{2.6}, \tag{6.26}$$

- square and grooved nails produced form a wire of tensile strength $f_u \geq 600$N/mm^2

$$M_{y,Rk} = 0.45 f_u \cdot d^{2.6}, \tag{6.27}$$

- staples produced form a wire of tensile strength $f_u \geq 800$ N/mm^2

$$M_{y,Rk} = 240 \cdot d^{2.6}, \tag{6.28}$$

- bolts and dowels

$$M_{y,Rk} = 0.3 f_u \cdot d^{2.6}. \tag{6.29}$$

The unit of the calculated fastener yield moment is N·mm.

The design load–carrying capacity per fastener per shear plane $F_{v,Rd}$ which is used to verify a ULS condition is calculated taking into account the influence of moisture content and load duration, with the use of modification factor k_{mod}

$$F_{v,Rd} = \frac{F_{v,Rk} \cdot k_{mod}}{\gamma_M}, \tag{6.30}$$

where modification factor k_{mod} and partial factor γ_M were detail described in 2.3 and 2.4.

6.1.2. Spacings

Driving a fastener into wood is associated with a risk of the fibres splitting, which is a kind of brittle-type failure, illustrated in Figure 6.6.

Figure 6.6. Splitting of fibres in the timber connection. Source: Author's archive

When the splitting of timber occurs, the parts of the connection cannot fully cooperate, and the connection will not achieve its full load capacity. To avoid this negative effect, EC-5 gives detailed requirements for minimum spacings between fasteners, and minimum distances between fasteners and edges/ends of the timber member.

The required minimum spacings and distances are presented in the form of tables in EC-5.

6.2. Connections with bearing-type connectors

Bearing-type connectors are designated for connections subjected to high loads. The bearing-type connectors comprise:

- rings and split-rings (Figure 6.7a)
- single-sided and double-sided tooth plates (Figure 6.7b),
- discs

The connectors are embedded into the timber members, with the use of a mechanically press (tooth plates) or they are installed into formerly prepared slots (rings). The lateral load is transferred between members in the connection by a bearing.

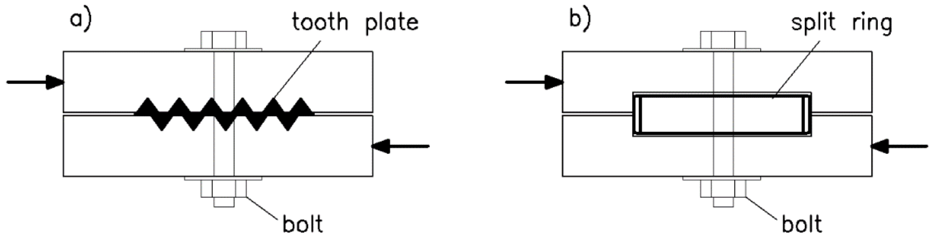


Figure 6.7. a) tooth plate, b) split ring in the connection

To avoid splitting the connection bolts are used that hold the members together in the connection.

7. Timber composite beams

7.1. General information

Solid timber beams were often used in building as structural members of floors. Formerly, to achieve high load capacity of the floor a big cross-section of the beam was used. Recently however, because of economic reasons, there is a tendency to design and produce timber beams of a rather small cross-sectional area. When the high load capacity (or big span) of the beam is required, it is advantageous to use a timber composite beam, made of several components connected with use of glue or mechanical fasteners. Figure 7.1 illustrates the common types of timber composite structures

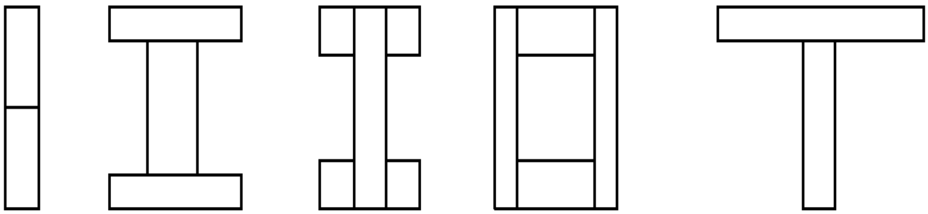


Figure 7.1. Examples of timber composite structure profiles

There are many types of glued composite beams, often manufactured from solid timber and wood-based products, such as OSB, plywood or particleboards. Such beams, characterized by their high strength to weight ratio, are also relatively easy to fabricate.

In turn, beams made with the use of mechanical fasteners, are rather used on-site, during strengthening of existing structures or when practical reasons are decisive. Mechanical fasteners transit the shearing force from one member to another in the connection providing a composite action of the assembly. The stiffness of the connection is crucial, according to the illustration in Figure 7.2.

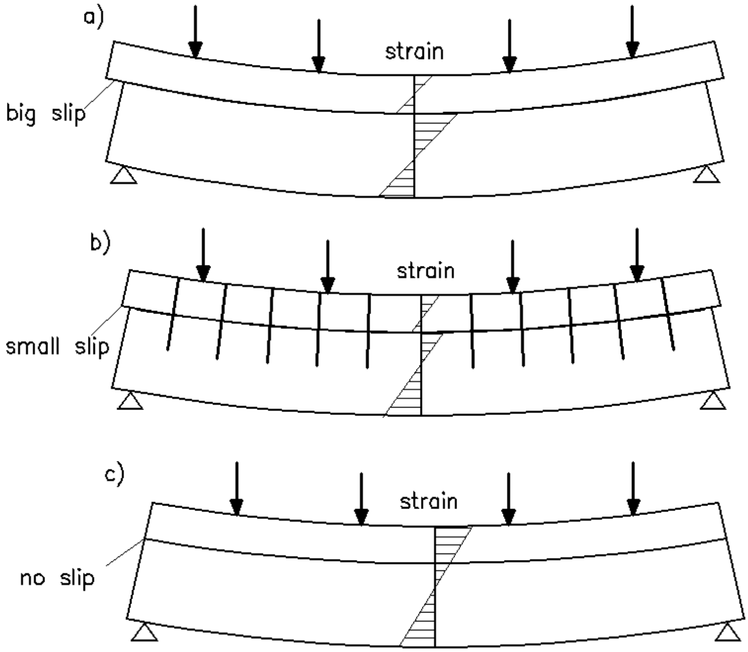


Figure 7.2. Deformations of the beam depending on the level of composite action a) no connection, b) partial connection c) full level of connection

For glued composite sections slippage between timber members is negligible, therefore such an assembly can be treated as one uniform piece. However, in mechanically jointed composites slippage always has to be considered. Figure 7.3 shows deformation of a composite double-T beam tested in the laboratory.

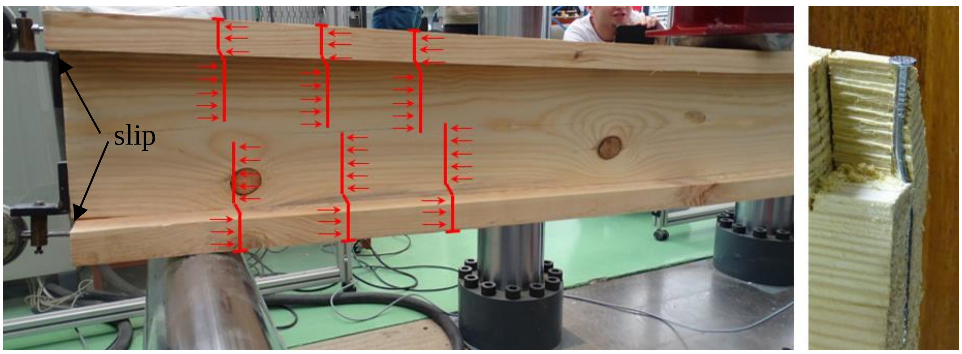


Figure 7.3. Composite double-T beam deformation and the fastener deformation. Source: Author's archive

The slippage visible on the left side of the beam depends on the connection stiffness, specially the fastener's diameter, length and spacing. Failure Mode (f) was observed for this particular case.

7.2. Calculations of mechanically jointed beams, according to EC-5

EC-5 includes the informative Annex B that is fully devoted to the method of calculating of composite beams with the use of mechanical dowel-type fasteners. Crucial to the understanding of the behaviour of such composite beams, is the phenomena of diminishing of the assembly bending stiffness because of the slip in the joint. This slip determines both the load-carrying capacity, and the deflection of the beam and its calculations is represented by slip coefficient γ_i.

The calculations can be performed for assembly profiles:

- T cross-section,
- double T cross-section,
- box section.

7.2.1. Effective bending stiffness

The effective bending stiffness of the composite beam $(EI)_{ef}$, takes account of the slip in the joint and is calculated from the expression:

$$(EI)_{ef} = \sum_{i=1}^{3} \left(E_i I_i + \gamma_i E_i A_i a_i^2\right), \qquad (7.1)$$

where

E_i is the mean value of the modulus of elasticity of the profile "i" in the assembly,

I_i is the second moment of area of the profile "i" in the assembly,

A_i is the cross-sectional area of the profile "i" in the assembly,

a_i is the distance between the centre of gravity of the profile "i" and the centre of gravity of the assembly,

γ_i is the slip coefficient for a profile "i", according to Equation 7.2.

The amount of profiles in the assembly is limited to 3.

Usually the profiles in the assembly are numbered from top to bottom, so the profile on the top has an index "1".

7.2.2. Slip coefficient

The key factor - slip coefficient γ_i is calculating from the formula:

for i = 2 $\gamma_2 = 1$

for i = 1, 3: $$\gamma_i = \frac{1}{1 + \frac{\pi^2 \cdot E_{k,i} \cdot A_i \cdot s_i}{K_i \cdot l_{ef}^2}}, \qquad (7.2)$$

where

$E_{k,i}$ is the characteristic value of the modulus of elasticity of the profile "i" in the assembly,

s_i is the spacing between the fasteners in the joint. Index "1" means the upper joint, index "3" means the lower joint. The spacing is determined summarizing all the fasteners in the connection per unit length.

l_{ef} is the effective length of the beam, as follows:
l_{ef} = 1 for beam on 2 supports,
l_{ef} = $0.8 \cdot l$ for continuous beam,
$l_{ef} = 2 \cdot l_c$ for cantilevered beam, where l_c is the cantilever length,

K_i is the slip modulus of a profile "i" in the assembly, according to the data given in Table 7.1.

7.2.3. Slip modulus

The slip modulus K is the ratio of lateral load acting on the fastener to its slip, and enables the deformation in the connection (slip) to be calculated. It is worth noting that EC-5 does not limit the slip in the joint, leaving the decision to the designer. While serviceability limit state conditions are calculated the value of K_{ser} is taken into account.

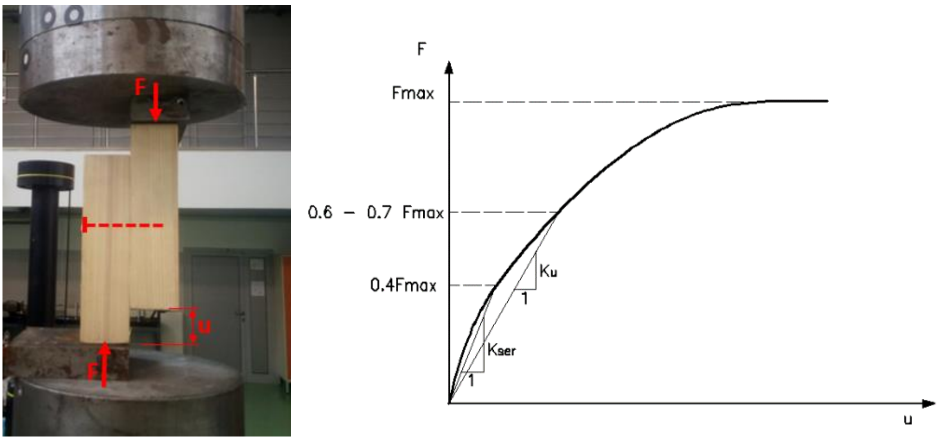


Figure 7.4. Measurement device and relationship between deformation u and load F acting in a nailed connection. Source: Author's archive

According to the relationship shown in Figure 7.4., K_{ser} is determined as a secant modulus of the load-displacement curve at a load level of about 40% of maximum load that the fastener can resist, assuming that the load is instantaneous.

In turn, instantaneous slip modulus in the ultimate limit state K_u is obtained at the load level of 60 – 70% of the maximum load. The relationship between both factors is

$$K_u = {}^2\!/_3\, K_{ser} \,. \tag{7.3}$$

The procedure described above is still discussed and developed (for example Sandhaas, Munch-Andersen, Dietsch, 2018).

On the basis of test results EC-5 gives the values of K_{ser} for most common types of fasteners in timber-to-timber and wood-based panel-to-timber connections, as shown in Table 7.1.

Table 7.1. Slip modulus K_{ser} per shear plane per fastener, according to EC-5

fastener type	**K_{ser}, N/mm**
dowels, bolts, screws, nails with pre-drilling	$\frac{\rho_m^{1.5} \cdot d}{23}$
nails without pre-drilling	$\frac{\rho_m^{1.5} \cdot d^{0.8}}{30}$
staples	$\frac{\rho_m^{1.5} \cdot d^{0.8}}{80}$
split-ring connectors type A, shear-plate connectors type B	$\frac{\rho_m \cdot d_c}{2}$
toothed-plate connector types C1 – C9	$\frac{1.5\rho_m \cdot d_c}{4}$
toothed-plate connector types C10 – C11	$\frac{\rho_m \cdot d_c}{2}$

The equations in Table 7.1. include:

d, d_c - diameter of the fastener, mm

ρ_m - mean density of timber, kg/m^3. If the joint consists of members of different densities $\rho_{m,1}$ and $\rho_{m,2}$ then

$$\rho_m = \sqrt{\rho_{m,1} \cdot \rho_{m,2}} \,. \tag{7.4}$$

As a consequence, slip coefficient γ_i should be calculated both in SLS and ULS, as follows:

for serviceability limit state conditions:

$$\gamma_{ser,i} = \frac{1}{1 + \frac{\pi^2 \cdot E_{k,i} \cdot A_i \cdot s_i}{K_{ser,i} \cdot l_{ef}^2}}, \tag{7.5}$$

for ultimate limit state conditions:

$$\gamma_{u,i} = \frac{1}{1 + \frac{\pi^2 \cdot E_{k,i} \cdot A_i \cdot s_i}{K_{u,i} \cdot l_{ef}^2}}. \tag{7.6}$$

Both values have to range between “0” and “1”. The value of “0” means that there is no connection at all, and the value of “1” means that the members in connection are fully jointed. In reality, when mechanical fasteners are used, the value of γ_i usually ranges between 0.3 and 0.6.

7.2.4. Normal stresses

Normal stresses, perpendicular to the cross-sectional plane, arise due to the bending of the beam. The distribution of normal stresses in the cross-sectional plane of the composite beam is shown in Figure 7.4.

The normal stresses should be determined in the middle and at the edges of all profiles in the assembly. The stresses in the middle of the flange can be interpreted as an effect of the cooperation of the cross-sectional profiles: the upper flange is connected to the web, therefore the compressive stress arises in the flange. Analogically, the lower flange is subjected to tensile stress. The values of these stresses are strongly dependent on the assembly extent in the joint, which is determined by the slip coefficient γ_i. The stresses in the middle of profile “i” can be calculated from the equation:

$$\sigma_i = \frac{\gamma_{u,i} \cdot E_{0,mean,i} \cdot a_i}{(EI)_{ef}} M_{ED}, \tag{7.7}$$

where

$\gamma_{u,i}$ is the slip coefficient in ULS for a profile “i”, according to Equation 7.6,

$E_{0.mean,i}$ is the mean value of modulus of elasticity parallel to the grain of a profile “i“ in the assembly,

M_{ED} is the design bending moment acting on the beam.

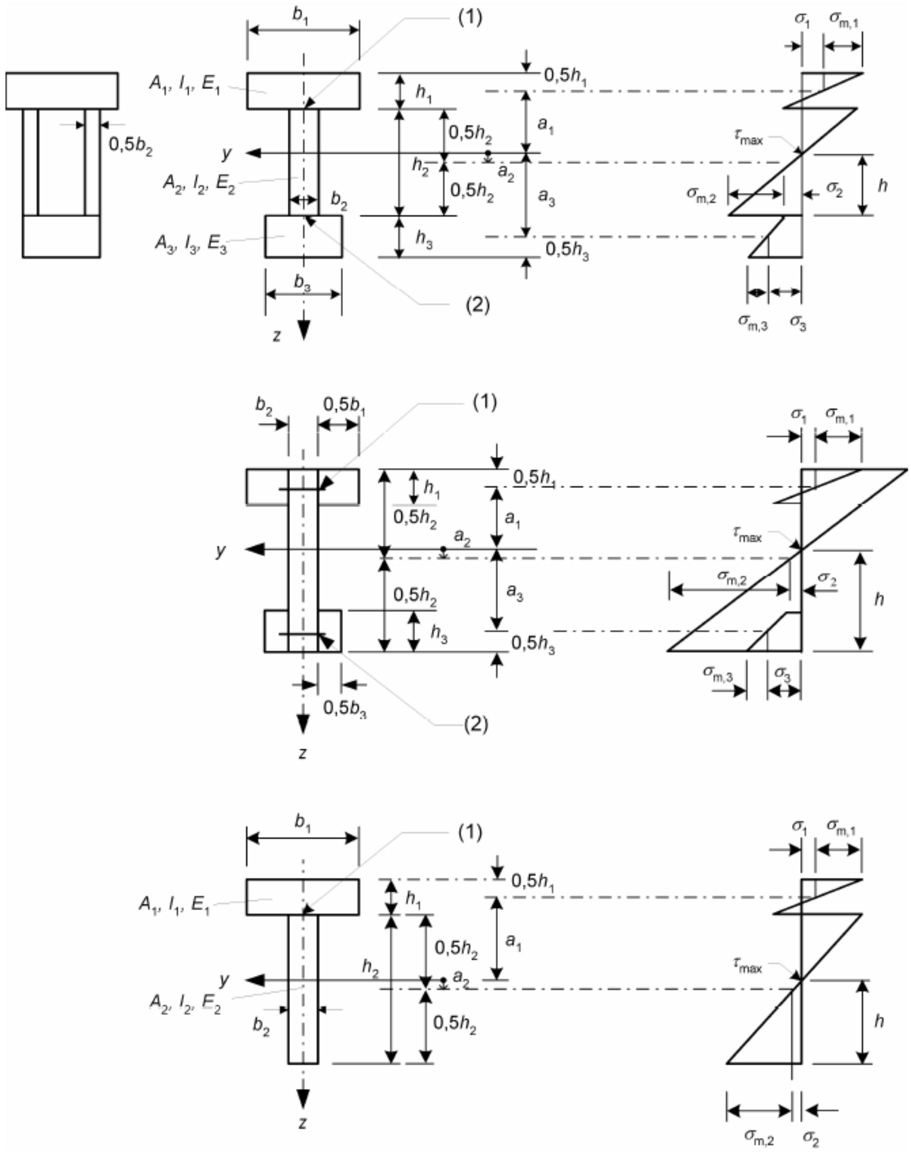


Figure 7.4. The distribution of bending stress for different types of cross-sections. EC-5, Annex B

Both flanges are subjected to the acting of the bending moment, so the stress in the cross-section of the flange is the superposition of σ_i and $\sigma_{m,i}$. The maximum value is reached at the edges, and can be calculated as:

$$\sigma_{k,i} = \sigma_i \pm \sigma_{m,i} \quad , \tag{7.8}$$

$$\sigma_{m,i} = \frac{E_{0,mean,i} \cdot 0.5h_i}{(EI)_{ef}} M_{ED} \,. \tag{7.9}$$

Equation (7.9) also concerns the central profile in the assembly – the web.

The ultimate limit state due to the bending is verified using the following six expressions:

- bending stresses in the middle of profiles

$$\frac{\sigma_1}{f_{c,0,d}} \leq 1 \quad \text{for upper flange} \tag{7.10}$$

$$\frac{\sigma_2}{f_{c,0,d}} \leq 1 \quad or \, \frac{\sigma_2}{f_{t,0,d}} \leq 1 \quad \text{respectively for web} \tag{7.11}$$

$$\frac{\sigma_3}{f_{t,0,d}} \leq 1 \quad \text{for lower flange} \tag{7.12}$$

- bending stresses at the edge of profiles

$$\frac{\sigma_{k,1}}{f_{m,d}} \leq 1 \quad \text{for upper flange} \tag{7.13}$$

$$\frac{\sigma_{k,2}}{f_{m,d}} \leq 1 \quad \text{for web} \tag{7.14}$$

$$\frac{\sigma_{k,3}}{f_{m,d}} \leq 1 \quad \text{for lower flange} \tag{7.15}$$

7.2.5. Shear stress

The maximum value of shear stress occurs at the point where normal stress equals zero. If the composite member is symmetric, the maximum shear stress in the web can be calculated as:

$$\tau_{2,max} = \frac{\gamma_{u,3} \cdot E_{0,mean,3} \cdot A_3 \cdot a_3 + 0.5E_{0,mean,2} \cdot b_2 \cdot h^2}{b_2.(EI)_{ef}} V_{ED} \,. \tag{7.16}$$

Equation (7.16) can be derived directly from Equation (5.32), taking into account a slip effect in the joint. The width of the web b_2 should be reduced to $b_{2,eff}$, according to Equation 5.34.

For cross-section symmetrical about horizontal axis dimension $h = 0{,}5h_2$ (see Figure 7.4) and equation (7.16) transforms into (7.17):

$$\tau_{2,max} = \frac{\gamma_{u,3} \cdot E_{0,mean,3} \cdot A_3 \cdot a_3 + 0.125E_{0,mean,2} \cdot b_2 \cdot h_2^2}{b_2.(EI)_{ef}} V_{ED} \tag{7.17}$$

The ultimate limit state due to the shear is verified using the following expression (for web):

$$\frac{\tau_{2,max}}{f_{v,d}} \leq 1\ . \tag{7.18}$$

7.2.6. Fastener load

The load acting on a single fastener should be calculated as:

$$F_i = \frac{\gamma_{u,i} \cdot E_{0,mean,i} \cdot A_i \cdot a_i \cdot s_i}{(EI)_{ef}} V_{Ed}\ , \tag{7.19}$$

where

i = 1 for the upper joint and i = 3 for the lower joint.

The load on a single fastener cannot exceed the design load-carrying capacity per shear plane and per fastener, which constitutes the ultimate limit state condition for the connection:

$$\frac{F_i}{F_{vR,d}} \leq 1\ . \tag{7.20}$$

8. Working examples

An example concerns the simply supported timber floor beam in residential building. Two variants of cross-section are considered:

- solid rectangular,
- double T composite, with nailed joints

Assumptions:

- clear span L_s = 5.5 m
- uniform load is acting on the floor
  - permanent load, characteristic value g_k = 0.55 kN/m2
  - variable load, characteristic value q_k = 2.8 kN/m2
- timber strength class C24

8.1. Beam with solid rectangular cross-section

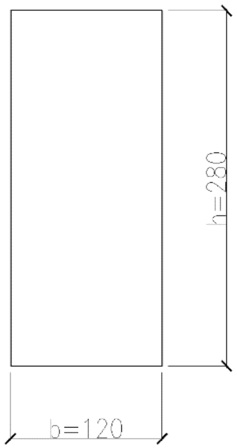


Figure 8.1. Dimensions of the rectangular cross-section

Cross-sectional parameters

Cross-sectional area $A = 0.12 \cdot 0.28 = 0.0336\ \mathrm{m}^2$

Section modulus about axis y $W_y = \frac{0.12 \cdot 0.28^2}{6} = 0.00157\mathrm{m}^3$

Second moment of area about axis y $I_y = \frac{0.12 \cdot 0.28^3}{12} = 0.00022\mathrm{m}^4$

Material properties

Characteristic values

Characteristic bending strength $f_{m,k}$ = 24 N/mm^2,

Characteristic compression strength parallel to the grain $f_{c,0,k}$ =21 N/mm^2,

Characteristic tensile strength parallel to the grain $f_{t,0,k}$ =14.5 N/mm^2,
Characteristic shear strength $f_{v,k}$ = 4.0 N/mm^2,
Characteristic value of modulus of elasticity E_k=7400 N/mm^2,
Mean value of modulus of elasticity $E_{0,mean}$=11000N/mm^2,
Characteristic density ρ_k = 350 kg/m^3,
Mean density ρ_m = 420 kg/m^3.

Design values

Modification factor for:

- serviceability class: *1*
- load-duration class: *medium-term* k_{mod} = 0.8

Partial factor for solid timber γ_M = 1.3

Design bending strength $f_{m,d} = \frac{24 \cdot 0.8}{1.3} = 14.8$N/mm^2

Design compression strength parallel to the grain $f_{c,0,d} = \frac{21 \cdot 0.8}{1.3} = 12.9$N/mm^2

Design tensile strength parallel to the grain $f_{t,0,d} = \frac{14.5 \cdot 0.8}{1.3} = 8.9$N/mm^2

Design shear strength $f_{v,d} = \frac{4 \cdot 0.8}{1.3} = 2.5$N/mm^2

Actions

Self-weight of the beam $g_{w,k}$ = 0.12 m·0.28 m·6.0 kN/m^3 = 0.2 kN/m
Distance between beams a = 0.6 m
Characteristic permanent load per single beam g_k^1 = 0.55·0.6 + 0.2 = 0.53 kN/m
Characteristic variable load per single beam q_k^1 = 2.8 · 0.6 = 1.68 kN/m
Design permanent load per single beam g_d^1 = 0.53 · 1.35 = 0.72 kN/m
Design variable load per single beam q_d^1 = 1.68 · 1.5 = 2.52 kN/m
Design total load per single beam p_d^1 = 0.72 + 2.52 = 3.24 kN/m

Bending

Effective span of the beam L_{ef} = 1.05 · 5.5 m = 5.78 m

Bending moment about axis y $M_{Ed,y} = \frac{3.24 \cdot 5.78^2}{8} = 13.5$kNm

Bending stress $\sigma_{m,y,d} = \frac{13.5}{0.00157} = 8610$kN/m^2

Beam is assumed to be laterally restrained by timber boards of the floor, therefore k_{crit}=1.0

Ultimate Limit State condition: $\frac{\sigma_{m,y,d}}{f_{m,d}} = \frac{8610}{14770} = 0.58 < 1.0$

The member satisfies the ULS condition for bending

Shear

Shear force $V_{Ed} = \frac{3.24 \cdot 5.78}{2} = 9.35$kN

Shear stress $\tau_{max} = 1.5\,\frac{9.35}{(0.67 \cdot 0.12) \cdot 0.28} = 620$kN/m^2

Ultimate Limit State condition:

$$\frac{\tau_{max}}{f_{v,d}} = \frac{620}{2500} = 0.25 < 1.0$$

<u>The member satisfies the ULS condition for shear</u>

Deflection

Instantaneous deflection for permanent load

$$u_{inst,g} = \frac{5}{384} \cdot \frac{0.53 \cdot 5.78^4}{11.4 \cdot 10^6 \cdot 0.00022}\left[1 + 19.2\left(\frac{0.28}{5.78}\right)^2\right] = 3.2mm$$

Instantaneous deflection for variable load

$$u_{inst,g} = \frac{5}{384} \cdot \frac{1.68 \cdot 5.78^4}{11.4 \cdot 10^6 \cdot 0.00022}\left[1 + 19.2\left(\frac{0.28}{5.78}\right)^2\right] = 10.6mm$$

Deformation factor for serviceability class 1
and load-duration class: *medium-term* $k_{def} = 0.6$
Combination factor for category of use A $\psi_2 = 0.6$
Final deflection for permanent load $u_{fin,g} = 3.2 \cdot (1 + 0.6) = 5.4$mm
Final deflection for variable load $u_{fin,q} = 10.6 \cdot (1 + 0.6 \cdot 0.6) = 12.5$mm
Final total deflection $u_{fin,q} = 5.4 + 12.5 = 17.9$mm
Limiting value for deflection of the beam $w_{fin} = \frac{5500}{300} = 18.3$mm
Ultimate Limit State condition:

$$\frac{u_{fin}}{w_{fin}} = \frac{17.9}{18.3} = 0.97 < 1$$

<u>The member satisfies the SLS condition</u>.

8.2. Beam with composite double T cross-section, variant A

Cross-section of the beam is assumed to be symmetric. The timber strength class of all profiles is the same.

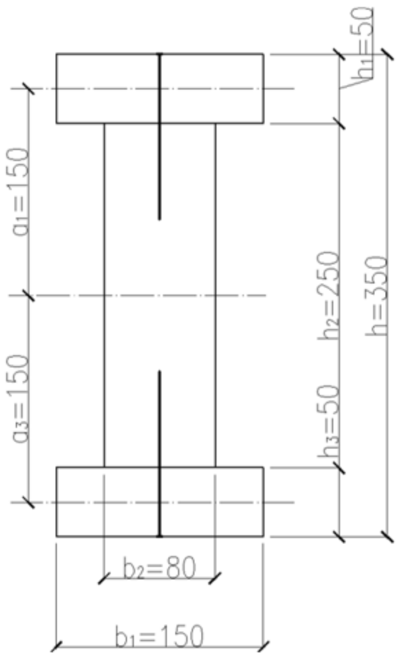


Figure 8.2. Dimensions of the beam

Cross-sectional parameters

Flange area $A_1 = 0.05 \cdot 0.15 = 0.0075$ m^2

Web area $A_2 = 0.08 \cdot 0.25 = 0.02$ m^2

Cross-sectional area $A = 2 \cdot 0.0075 + 0.02 = 0.035$ m^2

Second moment of area for flange $I_y = \frac{0.15 \cdot 0.05^3}{12} = 1.56 \cdot 10^{-6}$m^4

Second moment of area for web $I_y = \frac{0.08 \cdot 0.25^3}{12} = 1.04 \cdot 10^{-4}$m^4

$a_1 = 0.15$ m

Material properties

Characteristic and design values of material properties are the same as were assumed for rectangular cross-section.

Actions

The values of permanent and variable actions equal to values given in 8.1.3, because of virtually the same self-weight of the beam.

Fasteners

Diameter and length

Smooth nails will be used of diameter d and length l_n

$t_{min} = 55\,mm$

$d_{min} = \frac{t_{min}}{11} = \frac{55}{11} = 5.0mm$

$d_{max} = \frac{t_{min}}{6} = \frac{55}{6} = 9.2mm$

$l_{n,min} = h_1 + 1\text{ mm} + 8d + 1.5d = 50 + 1 + 8\cdot 5.6 + 1.5\cdot 5.6 = 104.2\text{ mm}$

The smooth nails **120 x 5.6** were adopted (d = 5.6 mm, l_n = 120 mm, length of the point 1,5d)

Spacing

minimum spacing along the beam $s_{min} = (5+7\cos 0^o)\cdot 5.6 = 67.2$ mm

maximum spacing along the beam $s_{max} = \min(4\cdot 67.2\ ;\ 40\cdot 5.6) = 224$ mm

It is adopted 1 row of nails with spacing ***s*** **= 70 mm**

Distance from edge of the web a=40 mm > $a_{4,c} = 5\cdot 5.6 = 28$ mm → OK

Pre-drilling

d = 5.6 mm < 6 mm → pre-drilling not required

$t_{min} = 50mm > max\left\{7\cdot 5.6 = 39.2mm; (13\cdot 5.6 - 30)\cdot \frac{350}{400} = 37.5mm\right\}$

→ pre-drilling not required

Nails will be driven direct, **without pre-drilling**

Slip effects

Slip modulus in SLS $K_{ser} = \frac{\rho_m^{1.5}\cdot d^{0.8}}{30} = \frac{420^{1.5}\cdot 5.6^{0.8}}{30} = 1138.4\text{N/mm}$

Slip modulus in ULS $K_u = {}^2\!/_3\, 1138.4 = 758.9\text{N/mm}$

Slip coefficient in SLS

$$\gamma_{ser} = \frac{1}{1 + \frac{\pi^2\cdot 7.4\cdot 10^9\cdot 7500\cdot 70}{1138.4\cdot 5780^2}} = 0.50$$

Slip coefficient in ULS

$$\gamma_u = \frac{1}{1 + \frac{\pi^2\cdot 7.4\cdot 10^9\cdot 7500\cdot 70}{758.9\cdot 5780^2}} = 0.40$$

Effective second moment of area in SLS

$I_{ef,ser} = 2\cdot 1.56\cdot 10^{-6} + 1.04\cdot 10^{-4} + 2\cdot 0.5\cdot 0.0075\cdot 0.15^2 =$

$= 2.75\cdot 10^{-4}\text{m}^4$

Effective second moment of area in ULS

$I_{ef,u} = 2 \cdot 1.56 \cdot 10^{-6} + 1.04 \cdot 10^{-4} + 2 \cdot 0.4 \cdot 0.0075 \cdot 0.15^2 =$
$= 2.41 \cdot 10^{-4}$m^4

Normal stresses

Effective span of the beam $L_{ef} = 1.05 \cdot 5.5$ m = 5.78 m

Bending moment about axis y $M_{Ed,y} = \frac{3.24 \cdot 5.78^2}{8} = 13.5$kNm

Normal stresses

in the centre of the upper flange $\sigma_1 = \frac{0.4 \cdot 0.15 \cdot 13.5}{2.41 \cdot 10^{-4}} = 3350$kN/m^2

in the centre of the web $\sigma_2 = 0$

in the centre of the lower flange $\sigma_3 = 3350$ kN/m^2

at the edge of the upper flange $\sigma_{e,1} = 3350 + \frac{0.5 \cdot 0.05 \cdot 13.5}{2.41 \cdot 10^{-4}} = 4750$ kN/m^2

at the edge of the web $\sigma_{e,2} = \frac{0.5 \cdot 0.25 \cdot 13.5}{2.41 \cdot 10^{-4}} = 7010$ kN/m^2

at the edge of the lower flange $\sigma_{e,3} = 4750$ kN/m^2

The beam is assumed to be laterally restrained by the timber boards of the floor, therefore k_{crit}=1.0

Ultimate Limit State conditions:

$$\frac{\sigma_1}{f_{c,0,d}} = \frac{3350}{12900} = 0.26 < 1.0$$
$$\frac{\sigma_3}{f_{t,0,d}} = \frac{3350}{8900} = 0.39 < 1.0$$
$$\frac{\sigma_{e,1}}{f_{m,d}} = \frac{4750}{14800} = 0.32 < 1.0$$
$$\frac{\sigma_{e,2}}{f_{m,d}} = \frac{7010}{14800} = 0.47 < 1.0$$
$$\frac{\sigma_{e,3}}{f_{m,d}} = \frac{4750}{14800} = 0.32 < 1.0$$

<u>The member satisfies the ULS condition for bending .</u>

Shear

Shear force $V_{Ed} = \frac{3.24 \cdot 5.78}{2} = 9.35$kN

Shear stress $\tau_{2,max} = \frac{0.4 \cdot 0.0075 \cdot 0.15 + 0.125 \cdot (0.67 \cdot 0.08) \cdot 0.25^2}{(0.67 \cdot 0.08) \cdot 2.41 \cdot 10^{-4}} 9.35 = 630$kN/m^2

Ultimate Limit State condition:

$$\frac{\tau_{max}}{f_{v,d}} = \frac{630}{2500} = 0.26 < 1.0$$

<u>The member satisfies the ULS condition for shear.</u>

Joints

thickness of first member in connection t_1 = 50 mm

penetration length of nail in the web t_2 = 120-50-1-1.5·5.6 = 60.6 mm characteristic embedment strength

$$f_{h,k} = 0.082 \cdot 350 \cdot 0.56^{-0.3} = 17.12N/mm^2$$

fastener yield moment $M_{y,Rk} = 0.3 \cdot 600 \cdot 5.6^{2.6} = 15869Nmm$

axial withdrawal capacity of the fastener is neglected.

Characteristic load-carrying capacity per fastener is calculated using Equations 6.1 – 6.6 for single shear connection:

$F_{v,Rk,1}$ = 4790 N (Equation 6.1)

$F_{v,Rk,2}$ = 5810 N (Equation 6.2)

$F_{v,Rk,3}$ = 2210 N (Equation 6.3)

$F_{v,Rk,4}$ = 2300 N (Equation 6.4)

$F_{v,Rk,5}$ = 2000 N (Equation 6.5)

$F_{v,Rk,6}$ = 2010 N (Equation 6.6)

$F_{v,Rk}$ = min ($F_{vR,k,1}$ ... $F_{vR,k,6}$) = 2000 N

Design load-carrying capacity per fastener

$$F_{v,Rd} = \frac{F_{v,Rk} \cdot k_{mod}}{\gamma_M} = \frac{2000 \cdot 0.8}{1.3} = 1230N$$

Load acting on a single fastener $F_i = \frac{0.4 \cdot 0.0075 \cdot 0.15 \cdot 0.07}{2.41 \cdot 10^{-4}} 9.38 = 1220N$

Ultimate Limit State condition:

$$\frac{F_i}{F_{vR,d}} = \frac{1220}{1230} = 0.99 < 1.0$$

<u>The ULS condition for the load carrying capacity of the connection is satisfied.</u>

Deflection

η_1 = 29.3

Instantaneous deflection for permanent load

$$u_{inst,g} = \frac{5}{384} \cdot \frac{0.53 \cdot 5.78^4}{11.0 \cdot 10^6 \cdot 0.000275} \left[1 + 29.3 \left(\frac{0.35}{5.78}\right)^2\right] = 2.6\text{mm}$$

Instantaneous deflection for variable load

$$u_{inst,q} = \frac{5}{384} \cdot \frac{1.68 \cdot 5.78^4}{11.0 \cdot 10^6 \cdot 0.000275} \left[1 + 29.3 \left(\frac{0.35}{5.78}\right)^2\right] = 9.0\text{mm}$$

Deformation factor for serviceability class 1
and load-duration class: *medium-term* k_{def} = 0.6

Combination factor for category of use A ψ_2 = 0.6

Final deflection for permanent load $u_{fin,g} = 2.6 \cdot (1 + 0.6) = 4.6\text{mm}$

Final deflection for variable load $u_{fin,q} = 9.0 \cdot (1 + 0.6 \cdot 0.6) = 12.2\text{mm}$

Final total deflection $u_{fin,q} = 4.6 + 12.2 = 16.9\text{mm}$

Limiting value for deflection of the beam $w_{fin} = \frac{5500}{300} = 18.3\text{mm}$

Ultimate Limit State condition:

$$\frac{u_{fin}}{w_{fin}} = \frac{16.9}{18.3} = 0.92 < 1$$

<u>The member satisfies the SLS condition.</u>

8.3. Beam with composite double T cross-section, variant B

Variant B concerns the beam with strongly nailed joints, to achieve a low slip in the connections.

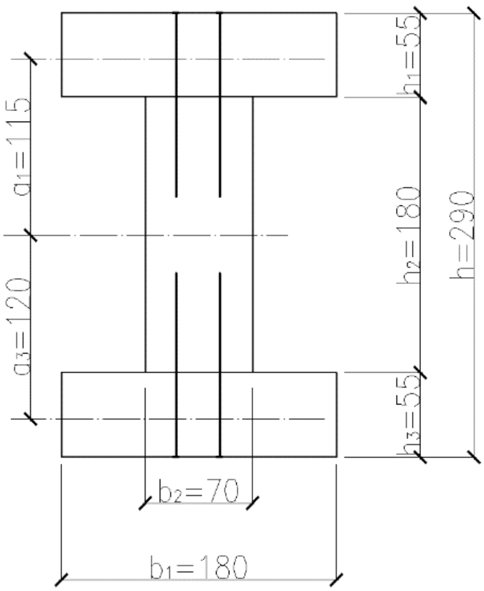


Figure 8.3. Dimensions of the cross-section

Cross-sectional parameters

Flange area	$A_1 = 0.055 \cdot 0.18 = 0.0099$ m^2
Web area	$A_2 = 0.07 \cdot 0.18 = 0.0126$ m^2
Cross-sectional area	$A = 2 \cdot 0.0099 + 0.0126 = 0.032$ m^2
Second moment of area for flange	$I_y = \frac{0.18 \cdot 0.055^3}{12} = 2.5 \cdot 10^{-6}$m^4
Second moment of area for web	$I_y = \frac{0.07 \cdot 0.18^3}{12} = 3.4 \cdot 10^{-5}$m^4
	$a_1 = 0.12$ m

Material properties

Characteristic and design values of material properties are the same as were assumed for rectangular cross-section.

Actions

The values of permanent and variable actions equal to values given in 8.1.3, because of virtually the same self-weight of the beam.

Fasteners

Diameter and length

Smooth nails will be used of diameter d and length l_n

$t_{min} = 55\,mm$

$d_{min} = \frac{t_{min}}{11} = \frac{55}{11} = 5.0mm$

$d_{max} = \frac{t_{min}}{6} = \frac{55}{6} = 9.2mm$

$l_{n,min} = h_1 + 1\text{ mm} + 8d + 1.5d = 55 + 1 + 8\cdot5.6 + 1.5\cdot5.6 = 109.2\text{ mm}$

The smooth nails **120 x 5.6** were adopted (d = 5.6 mm, l_n = 120 mm, length of the point 1,5d)

Pre-drilling

d = 5.6 mm < 6 mm → pre-drilling not required

$t_{min} = 55mm > max\left\{7 \cdot 5.6 = 39.2mm; (13 \cdot 5.6 - 30) \cdot \frac{350}{400} = 37.5mm\right\}$

→ pre-drilling not required

To diminish a slip in connection nails will be driven **with pre-drilling**

Spacing

minimum spacing along the beam $s_{min} = (4+1\cos0^o)\cdot5.6 = 39.2$ mm

maximum spacing along the beam $s_{max} = \min(4\cdot39.2\ ;\ 40\cdot5.6) = 157$ mm

2 rows of nails are adopted with spacing ***s*** **= 60 mm**

Distance from edge of the web $a = ⅓\cdot70$ mm=23 mm > 3·5.6 = 16.8 mm → OK

Slip effects

Slip modulus in SLS $K_{ser} = \frac{\rho_m^{1.5}\cdot d}{23} = \frac{420^{1.5}\cdot5.6}{23} = 2095.7\text{N/mm}$

Slip modulus in ULS $K_u = {}^2/_3\, 2095.7 = 1297.1\text{N/mm}$

Slip coefficient in SLS

$$\gamma_{ser} = \frac{1}{1 + \frac{\pi^2 \cdot 7.4 \cdot 10^9 \cdot 9900 \cdot 30}{2095.7 \cdot 5780^2}} = 0,76$$

Slip coefficient in ULS

$$\gamma_u = \frac{1}{1 + \frac{\pi^2 \cdot 7.4 \cdot 10^9 \cdot 9900 \cdot 30}{1397.1 \cdot 5780^2}} = 0,68$$

Effective second moment of area in SLS

$I_{ef,ser} = 2 \cdot 2.5 \cdot 10^{-6} + 3.4 \cdot 10^{-5} + 2 \cdot 0.76 \cdot 0.0099 \cdot 0.12^2 =$

$= 2.48 \cdot 10^{-4}\text{m}^4$

Effective second moment of area in ULS
$I_{ef,u} = 2 \cdot 2.5 \cdot 10^{-6} + 3.4 \cdot 10^{-5} + 2 \cdot 0.68 \cdot 0.0099 \cdot 0.12^2 = 2.26 \cdot 10^{-4}$m^4

Normal stresses

Effective span of the beam $L_{ef} = 1.05 \cdot 5.5$ m = 5.78 m

Bending moment about axis y $M_{Ed,y} = \frac{3.24 \cdot 5.78^2}{8} = 13.5$kNm

Normal stresses

in the centre of the upper flange $\sigma_1 = \frac{0.68 \cdot 0.12 \cdot 13.5}{2.26 \cdot 10^{-4}} = 4780$kN/m^2

in the centre of the web $\sigma_2 = 0$

in the centre of the lower flange $\sigma_3 = 4780$ kN/m^2

at the edge of the upper flange $\sigma_{e,1} = 4780 + \frac{0.5 \cdot 0.055 \cdot 13.5}{2.26 \cdot 10^{-4}} = 6420$kN/m^2

at the edge of the web $\sigma_{e,2} = \frac{0.5 \cdot 0.18 \cdot 13.5}{2.26 \cdot 10^{-4}} = 5370$ kN/m^2

at the edge of the lower flange $\sigma_{e,3} = 6420$ kN/m^2

The beam is assumed to be laterally restrained by the timber boards of the floor, therefore k_{crit}=1.0

Ultimate Limit State conditions:

$$\frac{\sigma_1}{f_{c,0,d}} = \frac{4780}{12900} = 0.37 < 1.0$$
$$\frac{\sigma_3}{f_{t,0,d}} = \frac{4780}{8900} = 0.56 < 1.0$$
$$\frac{\sigma_{e,1}}{f_{m,d}} = \frac{6420}{14800} = 0.43 < 1.0$$
$$\frac{\sigma_{e,2}}{f_{m,d}} = \frac{5370}{14800} = 0.36 < 1.0$$
$$\frac{\sigma_{e,3}}{f_{m,d}} = \frac{6420}{14800} = 0.43 < 1.0$$

<u>The member satisfies the ULS condition for bending.</u>

Shear

Shear force $V_{Ed} = \frac{3.24 \cdot 5.78}{2} = 9.35$kN

Shear stress $\tau_{2,max} = \frac{0.68 \cdot 0.0099 \cdot 0.12 + 0.125 \cdot (0.67 \cdot 0.07) \cdot 0.18^2}{(0.67 \cdot 0.07) \cdot 2.26 \cdot 10^{-4}} 9.35 = 870$kN/m^2

Ultimate Limit State condition:

$$\frac{\tau_{max}}{f_{v,d}} = \frac{870}{2500} = 0.35 < 1.0$$

<u>The member satisfies the ULS condition for shear.</u>

Joints

thickness of first member in connection t_1 = 55 mm
penetration length of nail in the web t_2 = 120-55-1-1.5·5.6 = 55.6 mm
characteristic embedment strength

$$f_{h,k} = 0.082 \cdot (1 - 0.01 \cdot 5.6) \cdot 350 = 27.09 N/mm^2$$

fastener yield moment $M_{y,Rk} = 0.3 \cdot 600 \cdot 5.6^{2.6} = 15869 Nmm$

axial withdrawal capacity of the fastener is neglected.

Characteristic load-carrying capacity per fastener is calculated using Equations 6.1 – 6.6 for single shear connection:

$F_{v,Rk,1}$ = 8340 N (Equation 6.1)
$F_{v,Rk,2}$ = 8440 N (Equation 6.2)
$F_{v,Rk,3}$ = 3480 N (Equation 6.3)
$F_{v,Rk,4}$ = 3240 N (Equation 6.4)
$F_{v,Rk,5}$ = 3220 N (Equation 6.5)
$F_{v,Rk,6}$ = 2520 N (Equation 6.6)
$F_{v,Rk}$ = min ($F_{vR,k,1}$ … $F_{vR,k,6}$) = 2520 N

Design load-carrying capacity per fastener

$$F_{v,Rd} = \frac{F_{v,Rk} \cdot k_{mod}}{\gamma_M} = \frac{2520 \cdot 0.8}{1.3} = 1550N$$

Load acting on a single fastener $F_i = \frac{0.68 \cdot 0.0099 \cdot 0.12 \cdot 0.03}{2.26 \cdot 10^{-4}} 9.38 = 980N$

Ultimate Limit State condition:

$$\frac{F_i}{F_{vR,d}} = \frac{980}{1550} = 0.63 < 1.0$$

The ULS condition for the load carrying capacity of the connection is satisfied.

Deflection

η_1 = 29.3

Instantaneous deflection for permanent load

$$u_{inst,g} = \frac{5}{384} \cdot \frac{0.53 \cdot 5.78^4}{11.0 \cdot 10^6 \cdot 0.000248}\left[1 + 29.3\left(\frac{0.29}{5.78}\right)^2\right] = 2.8\text{mm}$$

Instantaneous deflection for variable load

$$u_{inst,g} = \frac{5}{384} \cdot \frac{1.68 \cdot 5.78^4}{11.0 \cdot 10^6 \cdot 0.000248}\left[1 + 29.3\left(\frac{0.29}{5.78}\right)^2\right] = 9.8\text{mm}$$

Deformation factor for serviceability class 1
and load-duration class: *medium-term* k_{def} = 0.6

Combination factor for category of use A ψ_2 = 0.6

Final deflection for permanent load $u_{fin,g} = 2.8 \cdot (1 + 0.6) = 4.9\text{mm}$

Final deflection for variable load $u_{fin,q} = 9.8 \cdot (1 + 0.6 \cdot 0.6) = 13.4\text{mm}$

Final total deflection $u_{fin,q} = 4.9 + 13.4 = 18.3\text{mm}$

Limiting value for deflection of the beam $w_{fin} = \frac{5500}{300} = 18.3\text{mm}$

Ultimate Limit State condition:

$$\frac{u_{fin}}{w_{fin}} = \frac{18.3}{18.3} = 1.0$$

The member satisfies the SLS condition.

All the calculated cross-sections were shown in Figure 8.4.

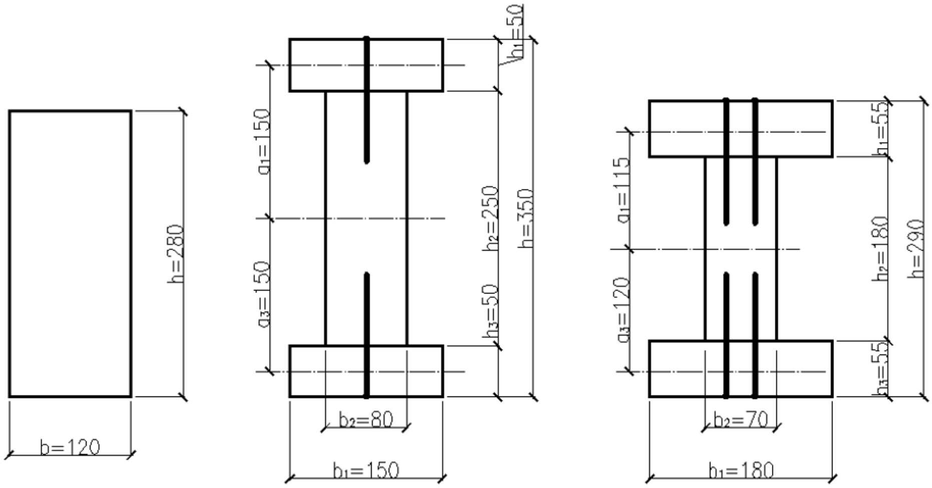


Figure 8.4. Comparison of cross-sections of the calculated beams

The above calculation examples show that with the same initial assumptions, the cross-section of wooden elements can be shaped in different ways. Especially complex cross-sections offer a lot of design possibilities. Depending on the requirements, sections of different overall height, material consumption and cost can be obtained. Figure 8.5 shows an example of a composite beam from the author's design practice.

Figure 8.5. Composite double-T beams used as floor beams in residential building. Source: Author’s archive

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