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Streszczenie

Praca ta stanowi podsumowanie badań dotyczących harmonogramowania procesów budowlanych z zastosowaniem Metod Sprzężeń Czasowych (ang. *Time Couplings Method TCM*). W monografii opracowano nierozwiązane dotychczas problemy modelowania robót budowlanych z uwzględnieniem teorii sprzężeń czasowych. W szczególności dokonano analizy zależności technologicznych i organizacyjnych wpływających na bezkolizyjność procesów budowlanych. Przedstawiono zagadnienia synchronizacji procesów budowlanych o równoległej i szeregowej strukturze wewnętrznej. Opisano warianty oddziaływania sprzężeń czasowych na procesy budowlane oraz ich kompleksy odwzorowujące praktyczne ograniczenia technologiczne i organizacyjne. Opracowano algorytmy obliczeniowe umożliwiające harmonogramowanie robót budowlanych z uwzględnieniem minimalno-czasowej funkcji celu. W pracy przedstawiono wiele przykładów obliczeniowych ilustrujących i rozwiązujących problemy praktyczne pojawiające się w praktyce budowlanej. W ostatnie części pracy omówiono dorobek autorów dotyczący zastosowania Metod Sprzężeń Czasowych z uwzględnieniem narzędzi sztucznej inteligencji. Wskazano kierunki rozwoju badań z zastosowaniem teorii sprzężeń czasowych i doskonalenia metod obliczeniowych. Praca ta wpisuje się w nurt zagadnień typu *flow shop problem* w teorii szeregowania, ze szczególnym uwzględnieniem harmonogramowania w budownictwie.

Słowa kluczowe:
Metody sprzężeń czasowych, harmonogramowanie robót budowlanych, flow shop problem

Abstract

This work summarizes research on construction process scheduling using Time Couplings Method TCM. The monograph deals with previously unsolved problems of modeling construction works taking into account the time coupling theory. In particular, the analysis of technological and organizational dependencies influencing collision-free construction processes is presented. The issues of synchronization of construction processes of parallel and serial internal structure. Variants of time coupling influence on construction processes and their complexes mapping practical technological and organizational constraints were described. Computational algorithms were developed to enable scheduling of construction works taking into account the minimal-time objective function. The paper presents many computational examples illustrating and solving practical problems arising in construction practice. The last part of the paper discusses the authors' achievements concerning the application of Time Coupling Methods including artificial intelligence tools. The directions of research development of time coupling theory and improvement of computational methods. This work is part of the stream of flow shop problem in scheduling theory, with particular emphasis on scheduling in the construction industry.

Keywords:
Time coupling methods, construction scheduling, flow shop problem

Introduction

The complexity of realization processes in construction, where new technological solutions are introduced continuously, requires the continuous improvement of construction work organization methods. This applies to all stages of the investment process from planning, through design, to realization. Therefore increasingly sophisticated methods must to be developed and applied in practice to solve optimally organizational-technological problems. In construction, the process of planning and scheduling has typically been accomplished using network scheduling techniques. The most common is the critical path method (CPM). Challenges in scheduling method has been offered as a solution to these problems by: Arditi 1986, 2001, 2002 [13, 14], Hegazy 1993 [38], Johnston 1984 [62]. Line of balance (LOB) is a variation of linear scheduling method (LSM) and present by Arditi, Tokdemir, Suh (2001) [13], Hamerlink and Rowings (1998) [35], Rahbar nad Rowings (1992) [88], provide an algorithm for determining the controling activity path in a linear schedule. Solutions of many particular problems have been presented by: Chrzanowski and Johnson (1986) [21], Harris and Ioannou (1998) [36]. Solutions of choice of optimal scheduling variants with application of many criteria have been presented by: Zavadskas (2002) [108, 109]. It has been presented the algorithms of construction work graphic schedule which takes into account the proper sequencing problem Afanasjev (1994, 2000) [1, 2, 3, 4, 5], Mrozowicz (1997, 1999, 2001) [84, 85, 86], Hejducki (1999, 2000, 2001) [40, 41, 42].

Taking into account constraints which may occur in construction practice it has been observed that the presented interpretation of the notion of a time coupling is incomplete and thus limits the organization methods' application area. In paper [1, 2, 3, 4, 5] a proposition was made to broaden this notion and to adopt time couplings determined on the basis of the possible combinations of the earliest and latest dates of work commencement and completion. Thus a basis for a theory of time couplings was created and this made it possible to broaden the area of varying work organization methods.

Research was undertaken to identify time couplings mapping the technological and organizational relationships between construction works and to indicate directions in which organization methods should be varied. The results were presented in [6, 7]. That was the first time this problem was brought to light thereby opening up new possibilities of varying and creating new work organization methods incorporating many additional technological and organizational constraints.

Continuing the research into the identification of time couplings, practical possibilities of applying new kinds of such couplings and their combinations were investigated and the correctness of the mathematical notations has been verified. It has been found that the new kinds of time couplings constitute a significant element contributing to the development of new organizational methods. As presented in [1, 2, 3, 4], new ways of determining time characteristics for the new variants of couplings make it possible to extend the methods' application area in construction.

Several kinds of time couplings are distinguished but here only questions connected with diagonal couplings linking various works in a technological sequence according to the principle that each work on a given project in a preceding technological sequence (of kinds of works) is linked to the work on the preceding project in a given technological sequence.

An analysis carried out for the new cases of time couplings for the sixth work organization method has led to several practical conclusions.

This book has been made within the project of The International School of Flow Organization Methods, which has been founded and developed by Professor Victor A. Afanasevs' team from St. Petersburg State University of Architecture and Civil Engineering. Based on the materials of the authors presented in the work by: Hejducki Z.: *Time couplings incorporated in organizational metods of complex construction processes*, Oficyna Wydawnicza Politechniki Wrocławskiej, Wrocław 2000; Hejducki Z., Rogalska M.: *Time coupling methods: construction scheduling and time/cost optimization*, Oficyna Wydawnicza Politechniki Wrocławskiej, Wrocław 2011; Hejducki Z., Rogalska M.: *Harmonogramowanie procesów budowlanych metodami sprzężeń czasowych*, Wydawnictwo Politechniki Lubelskiej, Lublin 2017; Hejducki Z., Rogalska M.: *The application of time coupling methods in the engineering of construction projects*, *Technical Transactions 9/2017*.

Fundamental principles of St. Petersburg's school are:

- Formation of all competitive methods of organization and the choice of one which is the most suitable for concrete conditions of construction;
- Formation and optimization of flows at the stages of projecting and process of construction with detalization fixing time and place of work of each brigade;
- Optimization of flows according to the following criteria: minimum duration of whole complex of work, minimum duration of using resources and working front, minimum cost of production;
- Formation and optimization of complex flows considering previously done unit flows;
- Summing up of the experience of work organization by means of making executive calendar graphs and their analyses;
- Usage of the suggested differentiated criteria of quality of organization of construction when uniting them into integral one considering criteria of significance;

- Usage of suggested algorithms and programs of formation, optimization, evaluation and choice of the most effective variant with the help of computers.

A lot of projects have been made according to “The Principles of St. Petersburg’s School of Flow Work Organisation” elaborated by Professor V.A. Afanasevs’ team. They have been later presented during international conferences, for instance in: Russia, USA, Canada, Cube, Finland, Germany, Great Britain, Singapore, Poland and many other countries.

The special essential contribution in development of above mentioned problem has been made, apart from Professor Afanasevs’ team, also by: A. Melski (USA), M. Joudi (Syria), K. Fidler, J. Pialek (Germany), T. Izmajlow (The Czech Republic), G. Ortega-Mengibura (Cuba), J. Mrozowicz (Poland) and others. Many problems have been also solved in tens of authors’ articles.

1. Basic terminology

Construction work organization methods are divided and classified roughly by considering the following characteristics:

- The degree of concurrency in the performance of jobs of different kinds on different projects;
- The degree of parallelism in the performance of jobs of one kind;
- The degree of rhythmicity of jobs;
- Couplings between the particular jobs;
- Dates of completion of uncritical jobs;
- The rate of the processes.

A graphic representation of the classification of construction work organization methods is shown in fig. 1.1.

The degree of concurrency in the performance of jobs of different kinds and the degree of parallelism in the performance of jobs of one kind are the basic criteria used to define the general character of work organization. The two characteristics are interrelated and they should be considered jointly. The notion of the concurrency in the performance of jobs of different kinds and the notion of the parallelism in the performance of jobs of one kind reflect the fact that the jobs are performed simultaneously. However, there are some differences between these notions. The concurrent performance of jobs of different kinds is always advantageous whereas there must be a reason for the adoption of the concurrent performance of jobs of one kind on different projects since this usually entails serious technological-organizational problems.

If jobs of different kinds are performed nonconcurrently or jobs of one kind are performed nonparallely, they are performed in succession. In this method of work execution only one job is performed at a time and the jobs form of a chain of jobs.

If, however, jobs of different kinds are performed concurrently and there is no parallelism in the performance of jobs of one kind, then we are dealing with flow methods[1]. The lead time for a complex of building structures in this case can be divided into the following stages: formation (deployment), full operation and winding down.

If jobs of one kind are carried out on different projects, this is referred to as a parallel performance method. Since each group of jobs performed in parallel can occur independently of any other group of jobs performed in parallel, there are three versions of this method:

[1] In the Russian-language literature on the subject such methods are commonly referred to as uniform work methods [1, 2, 3, 4, 5, 6, 7, 9, 10].

- With no concurrency in the performance of jobs of different kinds – a parallel-consecutive version;
- With concurrency in the performance of a part of jobs of one kind – a parallel-flow version;
- With concurrency in the performance of jobs of different kinds – a parallelconsecutive-flow version.

The degree of rhythmicity of jobs is an important consideration in the classification of organization methods. Depending on the rhythmicity with which jobs of one kind or different kinds are performed, i.e. depending on their lead time ratios, the following are distinguished:

- Rhythmic realization (all the jobs are performed at the same rhythm), fig. 1.2a;
- Heterorhythmic realization (jobs of one kind are performed at the same rhythm but jobs of different kinds are performed at different rhythms), fig. 1.2b;
- Unrhythmic realization (jobs of all kinds are performed at different rhythms), fig. 1.2.

The proper handling of construction-organization problems in the realization of building structures consists in taking into account couplings[2] between jobs. In this new approach, couplings are the basis on which different work organization methods are devised. The following kinds of time couplings are distinguished:

- Couplings between realization means (resources);
- Couplings between projects (lots);
- Diagonal and reverse diagonal couplings.

Couplings between realization means link jobs of one kind, defining a degree of their continuity within each kind of jobs or a degree of continuity in the use of resources. If gangs after completing work on one project (lot) move without a standstill to the next project (lot) within a given kind of jobs, the couplings between the realization means are equal to zero. These couplings are highly important in building practice since they reflect the degree of use of gangs. Frequent standstills in the work of gangs affect adversely their productivity or the use of the equipment.

Couplings between projects link successive jobs of different kinds within each partial complex[3], defining a degree of continuity in the occupation of partial projects. If as the preceding job has been completed on a project, the next job

[2] Couplings between jobs are described in [1, 2, 8, 84, 85].

[3] A complex of jobs carried out during the realization of building structures can be divided into jobs of one kind (earth work, concreting work, assembly) or technological processes (partial flows) and partial complexes (also called partial projects) which include jobs of different kinds carried out in succession on particular projects. In the matrix, kinds of jobs are put in the columns and partial complexes – in the rows (the WF arrangement).

begins (no downtimes), the couplings are equal to zero. Couplings between projects are instrumental in the reduction of the total time in which jobs are realized. The time is minimized if the putting of realized building structures into service in the shortest time is the main criterion.

Diagonal couplings[4] are more complex. They link jobs of different kinds in adjoining kinds of jobs so that each job on a project in the preceding kind of jobs combines with a job on the preceding partial project in the next kind of jobs. Diagonal couplings reflect a degree of continuity in the succession of jobs in the consecutive execution method. In the case of flow methods, they reflect a degree of simultaneity in the commencement or completion of jobs linked by such couplings. If jobs linked by diagonal couplings begin or end simultaneously, the diagonal coupling value for the earliest or latest dates equals zero.

Reverse diagonal couplings link jobs on a partial project in a particular kind of jobs with jobs in the next partial project in the preceding kind of jobs.

The following construction work organization methods, which incorporate the above couplings, are considered in this book:

1. Methods with zero couplings between means of realization (with the continuous use of realization means);
2. Methods with zero couplings between projects (with the continuous occupation of projects);
3. Methods which take into account couplings between both realization means and projects;
4. Methods which take into account couplings between both realization means and projects and diagonal couplings;
5. Methods which take into account couplings between both realization means and projects and reverse diagonal couplings;
6. Methods which take into account both diagonal couplings and inversely diagonal couplings.

As the classic organizational methods incorporating time couplings were applied in construction practice it became apparent that the interpretation of the notion time coupling could be further extended. Therefore it was accepted that time couplings should be determined from possible combinations of the earliest and latest work commencement dates.

Through combinations of time couplings many new variants (related by the basic assumptions) of the above methods can be created.

In this book, sixty eight new variants created by combining time couplings according to the assumptions made for the six organizational methods are presented.

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[4] In matrixes these couplings are perpendicular to the main diagonal. They link jobs having an equal sum of subscripts (e.g. t14, t23, t32, t41) and hence they are also called grade couplings. In papers by W.A. Afanasev diagonal couplings are referred to as rank couplings (cf [2, 8]).

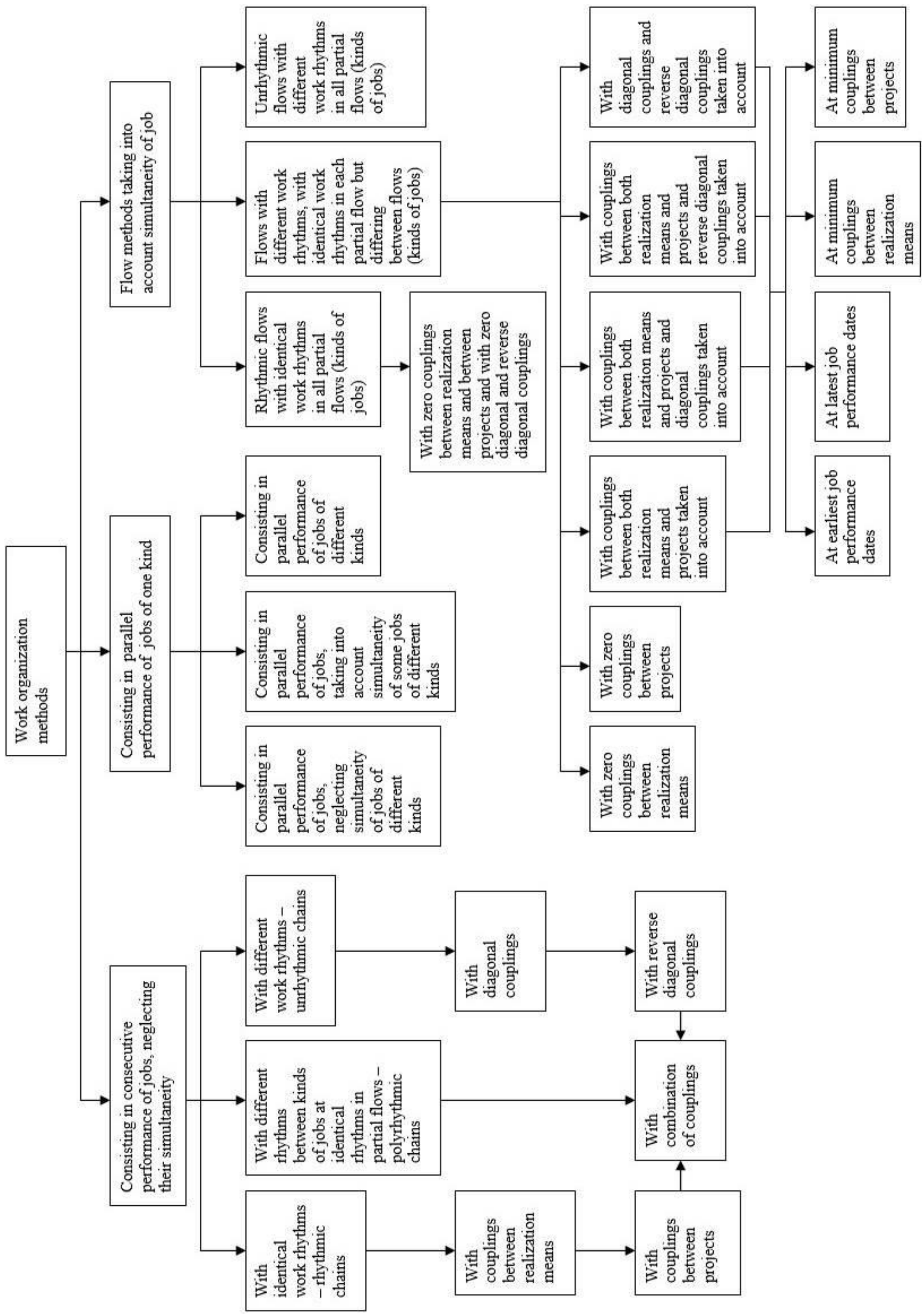


Figure 1.1. The graphic representation of the classification of construction work organization methods

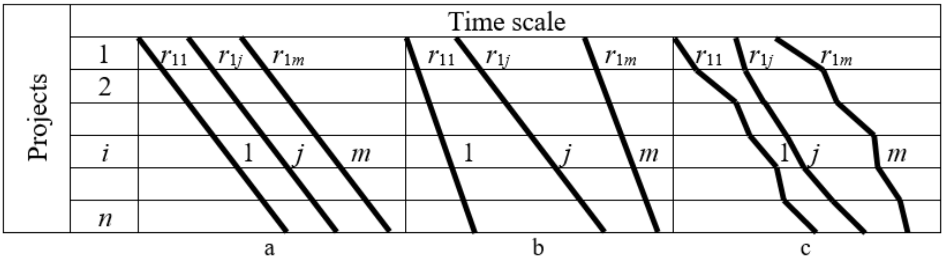


Figure 1.2. Cyclograms representing realization of complexes of building structures:
a) rhythmic realization, b) heterorhythmic realization, c) unrhythmic realization

The organization of jobs, consisting in the determination of connections between the jobs in time and space, requires realization models which take into account such couplings. When constructing and solving models, all possible work organization methods should be considered. A selected variant should be best suited to the particular conditions and guarantee the highest technical-economic realization indices. To find an optimum solution of the models, the latter must be in a suitable form. There are different ways of representing models in work organization. The most common are graphic and tabular representations.

One form of tabular representation is a matrix. This representation has many advantages: one can quickly determine the character of the involved processes and the characteristics of the considered work organization methods. Also it makes for the clarity of calculations aimed at determining the significant features of the models to be solved. Through a design of the matrix the values of all the important characteristics can be presented in a concise way. A suitable matrix notation gives a clear picture of the couplings between particular jobs.

Several forms of mathematical notation are used in models of construction processes. The simplest notation is similar to that commonly used in mathematics. A more complex form have the matrices used for calculating characteristics and those used in solving scheduling problems[5]. Matrix elements are: labour consumption values, work duration values and costs or other quantities which characterize processes.

Matrices can be constructed in two arrangements (tab. 1.1., 1.2). In one arrangement, the rows of a matrix correspond to projects and the columns to technological processes or kinds of work. Partial projects or partial complexes are written in the rows, whereas partial flows are put in the columns of the matrix. This arrangement is denoted as WF (projects are put in rows along the Y-axis).

[5] In practice, these are problems of determining an optimum sequence in which jobs are to be carried out on the particular projects (lots) (cf [1, 2, 23]) or a sequence of performing operations on machines.

The other arrangement is obtained through the transposition of the WF system and is denoted as WK – work kinds (technological processes, partial flows) are put in the rows along the Y-axis.

Table 1.1. Matrix in WF arrangement

couplings between projects →

couplings between realization means ↓

	ACTIVITY j								
	WF	1	2					m-1	m
PROJECT i	1	t_{11}	t_{12}					$t_{1,m-1}$	$t_{1,m}$
	2	t_{21}	t_{22}					$t_{2,m-1}$	$t_{2,m}$
			diagonal couplings						
					t_{11}				
						reverse diagonal			
							couplings		
	n-1	$t_{n-1,1}$	$t_{n-1,2}$						$t_{n-1,m}$
	n	$t_{n,1}$	$t_{n,2}$						t_{nm}

Table 1.2. Matrix in WK arrangement

couplings between realization means →

couplings between projects ↓

	PROJECT i								
	WK	1	2					n-1	n
ACTIVITY j	1	t_{11}	t_{12}					$t_{1,n-1}$	$t_{1,n}$
	2	t_{21}	t_{22}					$t_{2,n-1}$	$t_{2,n}$
			diagonal couplings						
					t_{11}				
						reverse diagonal			
	m-1	$t_{m-1,1}$	$t_{m-1,2}$			couplings			$t_{m-1,n}$
	m	$t_{m,1}$	$t_{m,2}$						t_{mn}

In a matrix in the WF arrangement, couplings between realization means follow a vertical direction, couplings between projects follow a horizontal direction and diagonal couplings and reverse diagonal couplings link jobs (matrix elements) having the same total of subscripts (tab. 1.1, 1.2).

If the realization of a complex of building structures is to be optimized, scheduling problems (determining the sequence of operations) must be solved. A change in the order in which jobs are performed on the particular projects has a significant effect on the technical-economic indices, particularly on the lead time for a complex of building structures[6]. Therefore it is essential to determine the most advantageous (rational) sequence, but this is a highly laborious task requiring the testing of all the possible variants (n!). For example, there are 120 possible variants for five projects (lots) but already for ten projects (a frequent case in building practice) there are 3628800 variants. Thus a controlled survey (directional search) is needed to find quickly an optimum sequence. This problem is solved by applying a modified branch-and-bound method.

The branch-and-bound method makes it possible to choose a solution with an extreme value of a feature from a set of possible solutions differing in the value of the feature. The selection is made in consecutive computational stages following a strategy of division into increasingly smaller partial subsets. Through the group estimation of objective functions in the subsets and the elimination of subsets including solutions with inappropriate objective function values from further calculations one quickly reaches the sought extreme solution. The division of an initial set into subsets is illustrated by a tree diagram[7] (fig. 1.3) whose nodes represent subsets of possible solutions and whose segments represent the order in which the nodes are divided.

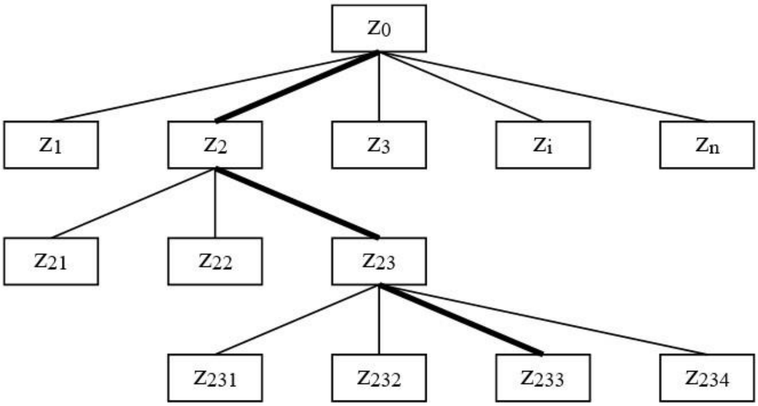


Figure 1.3. General diagram showing how set of solutions is divided

One of the division strategies is the choice of a variant with the lowest value of the object function's lower bound. In construction process organization methods which take into account time couplings between jobs, the minimization of the lead time for a complex of jobs is usually the objective function (the criterion). When searching for the most advantageous sequence in which to perform

[6] In the author's experience through a change in the sequence of jobs on projects one can reduce the lead time for building structures by as much as 15%.

[7] A tree of variants (solutions) is also called a "dendrite" or a "porphyrian".

jobs on the particular projects, the lead time alone is not always a reliable indicator since also the technological relationships between the particular kinds of jobs must be taken into consideration. Therefore an auxiliary index, substituting for the above criterion at the intermediate stages of the search in the subsets represented by the tree-diagram nodes, should be introduced. This index is called the lowest possible minimum and denoted as LPM.

The LPM index is calculated from a relation given in [84, 85, 86] examine the relation, we shall find that the shortest lead time for a complex of jobs depends on the time in which the longest jobs of one kind are carried out and on the shortest times in which the preceding and following jobs are performed:

$$T_{\min} = \min_i \sum_{j=1}^{j_0 - 1} t_{ij} + \sum_{i=1}^{n} t_{ij} + \min_i \sum_{j=j_0+1}^{m} t_{ij} \ , \qquad (1.1)$$

where:

T_{min} – the shortest lead time for a complex of jobs,
t – the lead time for the j-th job on the i-th project,
j_0 – the number of the longest job,
m – a number of jobs,
n – a number of projects.

In the algorithms, the main operations performed to determine the optimum sequence for an unrhythmic complex of building structures are:

- division into subsets (branches in the tree) – an operation common for all algorithms,
- the calculation of LPM – an operation specific to particular work organization methods.

The organizational methods presented in this work can be represented graphically in the form of: Gantt charts, cyclograms and networks of relationships. Gant charts are often modified to suit the represented methods and to take into account couplings between jobs and division into partial projects. Depending on the represented method, cyclograms are plotted in the WF or WK arrangement (it is possible to include some couplings). Networks of relationships can be presented (against a time scale or without it) for the earliest and latest work completion dates.

2. Construction work organization methods taking into account time couplings

2.1. Description of work organization method taking into account time couplings

2.1.1. Work organization method with zero couplings between realization means

This method ensures the continuity of work of gangs, machines and equipment. Specialist gangs move without breaks from one plot to another keeping up work continuity in the partial flows. This is a common organizational constraint.

A flow is modelled according to the above method in two stages: first a rational sequence in which jobs are to be performed on lots (structures) is determined and then time characteristics needed for graphic models (the Gantt chart, the cyclic line diagram and the network diagram) are calculated.

A sequencing problem can be solved by means of the modified S.M. Johnson algorithm. Together with the branch and bound method it allows one to determine a rational sequence of jobs on lots (objects) with uninterrupted continuity of work in partial flows and minimized realization time.

Although this method enables the planning of the continuous work of gangs, it often results in downtimes of work fronts. Downtimes can be eliminated only if rhythmic flows are modelled.

In a flow designed in this way, the time of deployment of partial flows (the time which passes between the entrance of successive work gangs into a building site) increases as the number of work lots increases. Research has shown that this may considerably prolong the task lead time. The latter may become longer than the time of realization of a work complex planned using the method that takes into account couplings between both realization means and work fronts.

2.1.2. Work organization method with zero couplings between work fronts

The main feature of this method is the possibility of planning continuous work on work fronts. The minimum work complex realization time can be obtained if such a sequence of jobs is determined at which the sum of first construction process realization times and times of winding down the remaining processes is the smallest. If we look at graphical representations (cyclograms) of the flow we can see that the middle block of construction processes has no effect on the work complex lead time. This fact was exploited in an algorithm searching for a sequence of objects to be realized which would ensure the shortest lead time. The larger the number of jobs “covered” by the first process, the smaller the sum of winding down times. A review of possible combinations involving the use of the tree diagram and an auxiliary index referred to as the lowest possible minimum (LPM) will lead to a rational sequence of performing jobs on work fronts. Then the basic time characteristics of the flow modelled by means of this method are determined.

Continuous work on lots is possible if breaks in the work of gangs are permitted. The gangs can pass from one lot to another immediately after the preceding gang leaves the lot. This condition can be fulfilled if a given work gang has finished the job on the preceding (in a fixed order) lot and waits for work. This constraint affects the work complex realization time which is usually longer than the time determined taking into account the couplings between the realization means and the work fronts.

2.1.3. Work organization method taking into account couplings between both realization means and work fronts

This method ensures the shortest lead time for a work complex. Since both time couplings are taken into account at the same time a critical path appears in the model. By adding up the lead times of the jobs along the critical path the work complex lead time (T) is obtained.

If the sequence of jobs on work fronts (structures) is changed, the order of jobs along the critical path also changes and so does the work complex lead time.

The minimum lead time for a complex of structures can be achieved if such an order of their realization is established that the sum of the lead times of the jobs that precede and close a leading partial flow or work complex will be the smallest, assuming that the critical path leads through the leading partial flow or complex.

The sequencing problem is solved by means of the branch and bound method which allows one to search the solution space.

After a rational order of jobs on lots (structures) is established the time characteristics of the flow are determined taking into account two kinds of couplings: between realization means and between work fronts.

In this method breaks in the work of gangs and work front idle time are allowed in order to arrive at such a critical path which ensures the shortest lead time for the investment task. The work front idle time or the work gang's time of waiting for employment are reduced substantially through the immediate availability of work gangs for employment. But the achievement of the shortest lead time for a complex of structures through this method poses difficulties in the planning of uniform and continuous use of realization means.

2.1.4. Work organization methods taking into account couplings between both realization means and work fronts, diagonal couplings and reverse diagonal couplings

The algorithm used for searching for the sequence of building structures to be realized by these methods is similar to the one described above. The difference is in the way the flow is modelled. For each of the methods, flow time characteristics are determined taking into account appropriate couplings.

Tests have shown that work complex lead times in this case are on the whole longer or equal to the ones obtained by the above method.

2.1.5.Work organization methods taking into account diagonal couplings and reverse diagonal couplings

In this method, jobs having the same rank form diagonals whose duration is equal to that of critical jobs. A change in the order of jobs on the particular fronts alters the set of critical jobs and as a result changes the work complex lead time. The minimum job lead time can be achieved by putting jobs in such an order that jobs of longest duration are located on one diagonal or on as few diagonals as possible.

Basic time characteristics are determined taking into account appropriate diagonal and reverse diagonal couplings.

One of the factors which determine the effectiveness with which construction works are carried out is their proper sequence taking account of technological and organizational constraints. The problem belongs to the deterministic theory of scheduling tasks and it has permutational character and n! possible variants. Because of the large number of its possible solutions, the problem is considered to be highly complex computationally. In practice, one can establish a rational sequence in which works are to be carried out by changing the order of works on the particular work fronts. However, it would be difficult, or even impossible, to check all the possible realization variants because of the computational complexity of the combinatorial problems.

Therefore review division and limitation algorithms, enabling the focused surveying of the space of solutions, are used. Because of the relatively high computing speed they are more effective, but less accurate, than a comprehensive review of realization variants would be. They allow one to find a suboptimal solution.

To organize building works, methods of division and limitation allowing one to establish a rational sequence of the works are employed. They take into account the specificity of construction processes through an index called “Limit Possible Minimum – LPM” which characterizes nodes in a tree diagram constructed according to the rules of the division and limitation method.

2.1.6. Elimination and selection rules

A set of division and limitation algorithms which is used to solve the permutation problem can be characterized by a set of parameters. Among them the selection rule – allowing us to determine the search tree's next vertex at which division is to take place – and the elimination rules – enabling the reactivation of the tree's generated nodes – should be singled out.

The matrix notation of input data is used in construction work organization methods:

$$T = [t_{ij}],\ i = 1,\dots,m,\ \ j = 1,\dots,m\ , \tag{2.1}$$

where:

t_{ij} – is a time of carrying out the j-th work on the i-th front.

The notation facilitates the division of works into subsets comprising matrix rows and the formation of tree diagram levels with a dwindling number of nodes. The first level will include n nodes and the last level – two nodes.

The elimination and selection rules allow us to move, via calculation procedures, up the successive levels of the tree diagram and to discard some of the generated vortices. To do it we use an algorithm for determining the nodes' numerical characteristics by means of LPM (Limit Possible Minimum). Since the construction work organization methods have different properties, the tree's nodes are described in different ways according to the peculiarities of the methods. Algorithms for determining LPM for a construction work organization method which takes into account couplings between realization means, couplings between work fronts and their combinations are presented below.

The so-called tree of variants is used to analyze the different possible sequences of work fronts.

In the first stage we must consider n variants. The j-th (j = 1, …, n) variant consists in this that the i-th variant is realized as the first and the order of the other fronts has not been established. For each of the variants we calculate the so-called limit possible time minimum (LPTM) which has the property of being less than time T_1 calculated for any sequence of the fronts whose order has not been fixed. Thus we know for sure that by expanding further, by establishing the order of the next fronts, the variant with calculated LPM1 we shall not obtain time T_1 less than LPM1. At the end of stage I we check for which variant the calculated LPM1 is the least.

In the second stage we expand next the variant for which LPM1 is currently the least. The expansion of the variant with a certain number of rows s whose sequence has been fixed consists in the forming of new variants (n-s) using one row with a fixed position from the rows whose positions have not been established.

The whole procedure of searching for a sequence of work fronts is completed when the least of the LPM1s calculated so far is realized only by variants whose all fronts have been determined, i.e. at the very bottom of the variant tree.

Since LPM1 calculated for the above final variants is equal to T_1 and LPM1 cannot be less after the expansion of a given variant, the above procedure will yield all the possible sequences of work fronts with shortest time T of the realization of the whole work complex.

2.2. Algorithms for establishing sequence in construction work organization method with zero couplings between realization means

We apply a formula for the duration of the realization of a whole work complex for the continuous use of realization means:

$$T = \sum_{j=2}^{m} t_j^r + \sum_{i=1}^{n} t_{i,m} \quad , \tag{2.2}$$

where t_j^r is the duration of the expansion of the next j-th partial stream, i.e. the difference between the time of commencement of the j-th partial stream and that of the j-1th partial stream.

The second summand, i.e. $\sum_{i=1}^{n} t_{i,m}$ does not depend on the sequence in which works are carried out on the particular fronts. Thus a search for shortest duration T of the realization of a whole complex of works can be limited to the minimum-time optimization of the first summand, i.e. $T_1 = \sum_{j=2}^{m} t_j^r$.

The time of the expansion of t_j^r depends on the mutual synchronization of two adjacent partial streams and so it depends on the sequence in which works are carried out on the particular work fronts. It can be expressed as follows:

$$t_j^r = \max_{1 \le k \le n} \left[\sum_{i=1}^{k} t_{i,j-1} - \sum_{i=1}^{k-1} t_{i,j} \right]. \tag{2.3}$$

If one solution is sufficient, one needs to test the variant tree only until LPM1 is currently minimum for any fully expanded variant.

To represent fully the way in which the right sequence of carrying out works is established it is necessary to define the calculation procedure for LPM1. For this purpose one should use modified Johnson's algorithm which allows one to establish the proper sequence of work fronts for two partial streams.

2.2.1. Johnson's algorithm which takes into account technological and organizational relationships between construction works

The input data is a two-column matrix: $\mathbf{A} = [a_{i,k}]$, $i = 1, \ldots, n$, $k = 1, 2$, corresponding to two adjacent partial streams. Then we obtain a three-column matrix: $\mathbf{B} = [b_{i,k}]$, $i = 1, \ldots, n$, $k = 1, 2, 3$.

The first two columns of matrix **B** consist of the same rows as matrix **A** does, but in the order established by Johnson's algorithm. The shifted rows' initial numbers bearing information about which work front they come from are in the third column.

Modified Johnson's algorithm taking account of relationships specific for building works is presented below.

Table 2.1. Modified Johnson's algorithm

Instructions	**Commentary**
a) Make substitutions: sg:1; Sd: n.	sg (ds) – number of currently highest (lowest) free row.
b) Find such indices i_0, k_0 that $a_{i,0}, k_0 = \max_{\substack{1 \le i \le n \\ k=1,2}}$	Searching for lowest value in table A.
c) If $k0 = 1$, make substitutions: $b_{sg,1} = a_{i,0,1}$; $b_{sg,2}: = a_{i,0,2}$; $b_{sg,3}: = i_0$; sg : = sg + 1.	Shifting upwards row with minimum element on left side.
If $k_0 = 2$, make substitutions: $b_{sd,1} : = a_{i0,1}$; $b_{sd,2} : = a_{i0,2}$; $b_{sd,3} : = i_0$; sd : = sd - 1.	Shifting downwards row with minimum element on right side.
d) Let M be large number – larger than all elements of matrix **A**, e.g. $M = 10^{30}$. Make substitutions: $a_{i0,1} : = M$; $a_{i0,2} : = M$.	
e) If sg < sd, go back to point b) and if sg = ds, end algorithm.	Testing if all rows of matrix **A** have been exhausted.

Prior to the testing of the variant tree one should calculate successively auxiliary matrices $\mathbf{B}^j$, $j = 1,\ldots,(m-1)$. Matrix $\mathbf{B}^j$ is created by applying Johnson's algorithm to data matrix **A** consisting of columns having numbers j, $(j+1)$, i.e. $\mathbf{A} = [t_{i,j}]$, $i = 1,\ldots, n$, $k = 1,\ j+1$.

Now we calculate LPM1 for a case when the sequence of fronts (rows) bearing numbers $(u_1,\ldots, u_s)$, $s \le n$ has been established. Let us denote set $(u_1,\ldots, u_s)$, consisting of the numbers of the rows whose sequence has been established, by D.

Table 2.2. Modified Johnson's algorithm

Instructions	Commentary
a) For each $j = 1,\ldots,(m-1)$ create matrix $\mathbf{C}^j = [C^j_{i,k}]$, $i = 1, \ldots, n$; $k = 1, 2$ as follows:	Matrix $\mathbf{C}^j$ shows relationship between adjacent columns.
i) For all indices $i = 1,\ldots s$; $k = 1, 2$ substitute $C^j_{i,\mathrm{k}} := t_{ui,\, j+1-1}$	First s rows have already been determined.
ii) Substitute: $p := s+1$; iii) For all in turn $i = 1,\ldots n$ do if $b^j_{i,3}$ ε D, substitute: $p := p+1$; $C^j_{p,1} = b^j_{i,k}$ for $k = 1, 2$	Introduction of remaining rows in order established by auxiliary matrix $\mathbf{B}^j$.
b) For all $j = 1,\ldots,(m-1)$ calculate $t^r_j = \max_{1\le k\le n}[\sum_{i=1}^{k} c_{i,1} - \sum_{i=1}^{k-1} c_{1,2}]$	Period of expansion of matrix $\mathbf{C}^j$'s second column relative to first one is calculated.
c) $\mathrm{LPM1} = \sum_{j=1}^{m-1} t^r_j$	

2.2.2. Practical example

In this paper a set of algorithms for scheduling construction work has been presented. As an illustration of the algorithm for work planning by the work organization method with zero couplings between realization means, an investment problem, consisting in the realization of four structures (A-B-C-D) on which seven technological processes: I – earth work, II – foundation work, III – masonry work, IV – concreting work, V – roofing work, VI – plaster work and VII – finishing work are to be carried out, was formulated. Work durations, constituting elements of a work lead time matrix, were determined. A detailed calculation procedure based on the algorithm described above was worked out.

SEQUENCING WORKS IN WORK COMPLEX BY METHOD WITH ZERO COUPLINGS BETWEEN REALIZATION MEANS – METHOD I

Work lead times for particular structures – initial matrix

	Earth work	Foundation work	Masonry work	Concreting work	Roofing work	Plaster work	Finishing work
	I	II	III	IV	V	VI	VII
Object A	4	2	26	23	1	8	32
Object B	6	2	17	5	5	10	37
Object C	3	4	29	22	3	19	39
Object D	3	4	20	13	11	13	34

$\Sigma = 142$

Deployment times for particular works

I	II
4	2
6	2
3	4
3	4

$tr_F = 9$

II	III
2	26
2	17
4	29
4	20

$tr_M = 2$

III	IV
26	23
17	5
29	22
20	13

$tr_B = 44$

IV	V
23	12
5	5
22	3
13	11

$tr_D = 43$

V	VI
12	8
5	10
3	19
11	13

$tr_T = 12$

VI	VII
8	32
10	37
19	39
13	34

$tr_Z = 8$

Lead time for structure

$\mathbf{T} = t_{rZ} + t_{rF} + t_{rM} + t_{rB} + t_{rD} + t_{rT} + \Sigma t_{im9w0}$

$\mathbf{T}_S = 9 + 2 + 44 + 43 + 12 + 8 + 142 = 260$ days

Matrices reconstructed according to Johnson algorithm

I'	II'
3	4
3	4
4	2
6	2

II'	III'
2	17
2	26
4	20
4	29

III'	IV'
29	22
26	23
20	13
17	5

IV'	V'
23	12
13	11
5	5
22	3

V'	VI'
3	19
5	10
11	13

VI'	VII'
8	32
10	37
13	34

Matrices with successively determined rows

	1	2		3		4		5		6		7
A	4	2		26		23		12		8		32
	3	4	2	17	29	22	13	11	3	19	10	37
	3	4	4	20	20	13	5	5	5	10	13	34
	6	2	4	29	17	5	22	3	11	13	19	39
											Σ =	142

$GMM^I_A = 6 + 2 + 34 + 35 + 12 + 8 + 142 =$ **239**

	1	2		3		4		5		6		7
B	4	2		17		5		5		10		37
	3	4	2	26	29	22	23	12	3	19	8	32
	3	4	4	20	26	23	13	11	11	13	13	34
	4	2	4	29	20	13	22	3	12	8	19	39
											Σ =	142

$GMM^I_B = 7 + 2 + 45 + 35 + 5 + 10 + 142 =$ **246**

	1	2		3		4		5		6		7
C	3	4		29		22		3		19		39
	3	4	2	17	26	23	23	12	5	10	8	32
	4	2	2	26	20	13	13	11	11	13	10	37
	6	2	4	20	17	5	5	5	12	8	13	34
											Σ =	142

GMM^{I}_{C} = 6 + 4 + 34 + 43 + 3 + 19 + 142 = **251**

	1	2		3		4		5		6		7
D	3	4		20		13		11		13		34
	3	4	2	17	29	22	23	12	3	19	8	32
	4	2	2	26	26	23	5	5	5	10	10	37
	6	2	4	29	17	5	22	3	12	8	19	39
											Σ =	142

GMM^{I}_{D} = 6 + 4 + 40 + 35 + 11 + 13 + 142 = **251**

Matrices constructed at intermediate stages

	1	2		3		4		5		6		7
A	4	2		26		23		12		8		32
B	6	2		17		5		5		10		37
	3	4	4	20	29	22	13	11	3	19	13	34
	6	2	4	29	20	13	22	3	11	13	19	39
											Σ =	142

GMM^{II}_{AB} = 9 + 2 + 44 + 35 + 12 + 8 + 142 = **252**

	1	2		3		4		5		6		7
A	4	2		26		23		12		8		32
C	3	4		29		22		3		19		39
	3	4	2	17	20	13	13	11	11	13	10	37
	6	2	4	20	17	5	5	5	10	12	13	34
											Σ =	142

GMM^{II}_{AC} = 6 + 2 + 34 + 43 + 12 + 8 + 142 = **247**

	1	2		3		4		5		6		7
A	4	2		26		23		12		8		32
D	3	4		20		13		11		13		34
	3	4	2	17	29	22	5	5	3	19	10	37
	6	2	4	29	17	5	22	3	5	10	19	39
											Σ =	142

GMM^{II}_{AD} = 6 + 2 + 34 + 35 + 15 + 8 + 142 = **246**

Determining sequence of rows

	1	2	3	4	5	6	7
A	4	2	26	23	12	8	32
C	3	4	29	22	3	19	39
B	6	2	17	5	5	10	37
D	3	4	20	13	11	13	34
						Σ =	142

GMM^{III}_{ACBD} = 8 + 2 + 42 + 43 + 12 + 8 + 142 = **257**

	1	2	3	4	5	6	7
A	4	2	26	23	12	8	32
C	3	4	29	22	3	19	39
D	3	4	20	13	11	13	34
B	6	2	17	5	5	10	37
						Σ =	142

GMM^{III}_{ACDB} = 6 + 2 + 34 + 43 + 12 + 8 + 142 = **244**

	1	2	3	4	5	6	7
A	4	2	26	23	12	8	32
D	3	4	20	13	11	13	34
B	6	2	17	5	5	10	37
C	3	4	29	22	3	19	39
						Σ =	142

GMM^{III}_{ADBC} = 8 + 2 + 51 + 35 + 15 + 8 + 142 = **261**

	1	2	3	4	5	6	7
A	4	2	26	23	12	8	32
D	3	4	20	13	11	13	34
C	3	4	29	22	3	19	39
B	6	2	17	5	5	10	37
						Σ =	142

GMM^{III}_{ADCB} = 6 + 2 + 39 + 37 + 15 + 8 + 142 = **249**

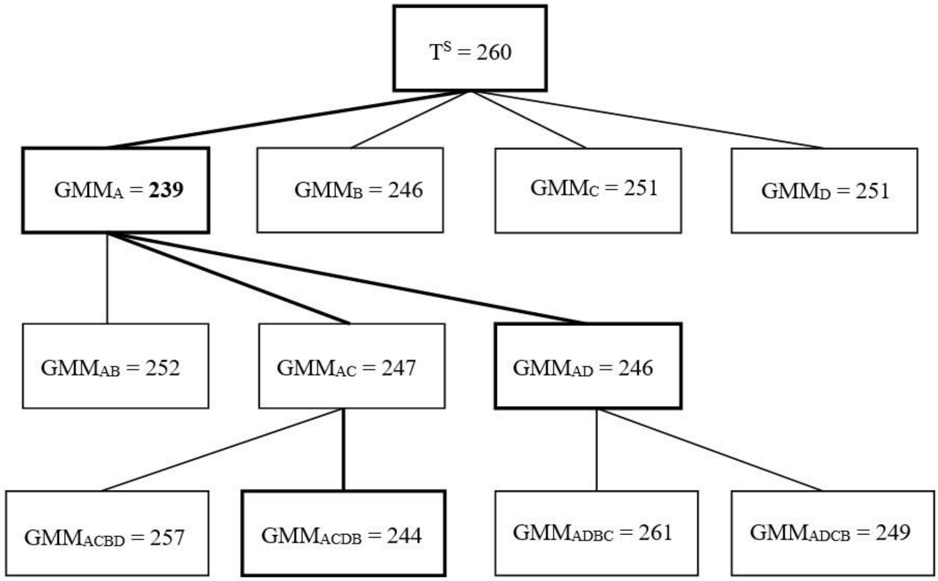


Figure 2.1. Variant tree

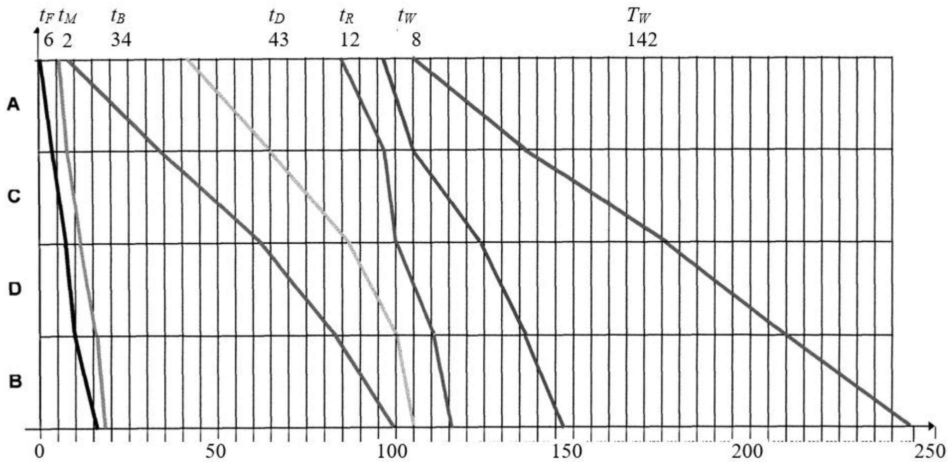


Figure 2.2. Cyclogram of realization of structures for method with zero couplings between realization means; Legend: I – Earth work, II – Foundation work, III – Masonry work, IV – Concreting work, V – Roofing work, VI – Plaster work, VII – Finishing work

2.2.3. Recapitulation

As a result of the calculations a new realization sequence: A-D-C-B was determined. This realization sequence ensures a reduction in the task lead time from 260 units to 244 units, i.e. 6.15%.

The algorithm for scheduling work by the work organization method with zero couplings between realization means was applied to a practical realization problem. A computer system named ORGANIZATOR, which incorporates the presented algorithms and enables their use in construction practice, has been developed.

2.3. Algorithm for establishing order in construction work organization method with zero couplings between work fronts

The total duration of the realization of a work complex for work organization where works on the particular fronts are carried out continuously can be represented as follows:

$$T = \sum_{j=1}^{m-1} t_{1,j} + \sum_{i=1}^{n} t_{i,m} + \sum_{r=1}^{m-1} \Delta_r \,, \qquad (2.4)$$

where Δ_r is a positive difference between the “hidden” part of front $(r + 1)$ and the “hiding” part of front r, expressed by the following formula:

$$\Delta_r = \max(\overline{\Delta_r}, 0)\,, \qquad (2.5)$$

$$\overline{\Delta_r} = \max_{1 \le k \le n} \left[\sum_{n=k}^{m-1} t_{r+1,j} - \sum_{j=k+1}^{m} t_{r,j} \right]. \qquad (2.6)$$

Numbers Δ_r are interruptions (downtimes) caused by the fact that it was impossible for each of the previous gangs to work simultaneously on two fronts r and $(r+1)$.

Similarly as in the previous method, a variant tree is used to test different possible sequences of work fronts.

The numerical value of LPM, which is less than realization time T for the rows whose sequence has already been established, should be determined for any order of the rows whose sequence has not been established.

For this purpose let us define two matrices **P** and **R**. Matrix **P** = $P_i\ J_{k = 1,\ldots,\ n}$ contains the following numbers (in a descending order):

$$\left\langle \sum_{j=2}^{m} t_{i,j} \right\rangle, \quad i = 1, \ldots, n. \qquad (2.7)$$

The above are sums of all, except the first one, the elements in a given row. Similarly the matrix: $Q = [q_{i,j}]$, $i = 1,\ldots, n$, $j = 1, 2$, contains in its first column the following numbers (in a descending order):

$$\left\langle \sum_{j=1}^{m-1} t_{i,j} \right\rangle, \quad i = 1, ..., n, \tag{2.8}$$

which are sums of all, except the last one, the elements in a given row. Whereas the second column holds the numbers of rows (fronts) prior to ordering.

The sequence of rows bearing numbers ($u_1, \ldots, u_s$) has been established. Let n_k denote the number of the k-th row of matrix **Q**, containing in the second place a number which does not belong to set ($u_1, \ldots, u_s$), i.e. a row whose position has not been established ($q_{nk,2}$ ε < $u_1, \ldots, u_s$ >). Let us define: $d_k = p_k - q_{nk,1}$, for $k = 1, \ldots, (n\text{-}s)$.

Numbers d_k are hypothetical quantities Δ_r for the rows whose sequence has not been established. When the sequence of all the rows has been established, possible interruptions (downtimes), i.e. Δ_k, will be longer.

If when calculating LPM we replace Δ_r in the formula for time T of the realization of a work complex by appropriate d_k, we shall obtain LPM not higher than the value of time T. For any order of rows whose sequence has not been established d_k allows us to find the optimum solution by means of the variant tree.

$$\text{LPM} = \sum_{j=1}^{m-1} t_{u1,j} + \sum_{i=1}^{n} t_{i,m} + \sum_{r=1}^{s-1} \Delta_r + \sum_{k=1}^{n-s} d_d \,, \tag{2.9}$$

where:

$$\Delta_r = \max(\overline{\Delta_r}, 0)\,, \tag{2.10}$$

$$\overline{\Delta_r} = \max\left[\sum_{j=k}^{m-1} t_{ur+1,j} - \sum_{j=k+1}^{m} t_{ur,j} \right]. \tag{2.11}$$

Similarly as in the previous method we search the variant tree by expanding the variant whose LPM is currently the least and the search ends when the minimum is realized by only the variants whose sequence of all rows has been established.

2.3.1. Practical example

STAGE I. INITIAL MATRIX

	1	2	3	4	5	6	7	Σ	Σ-7	Σ-1
I	15	7	26	22	5	8	26	109	83	94
II	19	29	0	100	8	30	132	318	186	299
III	10	16	75	35	12	32	68	248	180	238
IV	6	10	42	17	15	12	34	136	102	130
V	11	12	48	40	7	20	43	181	138	170

STAGE II. INTERMEDIATE MATRICES WITH SUCCESSIVELY DETERMINED ROWS I, II, III, IV, V INSTEAD OF FIRST ONE

	1	2	3	4	5	6	7				
I	15	7	26	22	5	8	26	109			
	109	+	132	+	68	+	34	+	43	=	386
	186	299	0								
	180	238	0								
	138	170	0								
	102	130	0								
			0								

GMM I = 386 + 0 = 386

	1	2	3	4	5	6	7				
II	19	29	0	100	8	30	132	318			
	318	+	26	+	68	+	34	+	43	=	489
	180	299	0								
	138	238	0								
	102	170	0								
	83	130	0								
			0								

GMM II = 489 + 0 = 489

	1	2	3	4	5	6	7				
III	10	16	75	35	12	32	68	248			
	248	+	26	+	132	+	34	+	43	=	483
	186	299	0								
	138	238	0								
	102	170	0								
	83	130	0								
			0								

GMMIII = 483 + 0 = 483

	1	2	3	4	5	6	7				
IV	6	10	42	17	15	12	34	136			
	136	+	26	+	132	+	68	+	43	=	405
	186	299	0								
	180	238	0								
	138	170	0								
	83	130	0								
			0								

GMMIV = 405 + 0 = 405

	1	2	3	4	5	6	7	
V	11	12	48	40	7	20	43	181
	181 + 26 + 132 + 68 + 34 = 441							
	186	299	0					
	180	238	0					
	102	170	0					
	83	130	0					
			0					

GMM V = 441 + 0 = 441

STAGE III. MATRICES FORMED AT INTERMEDIATE STAGES, WITH SUCCESSIVELY DETERMINED PERSPECTIVE MATRICES

	1	2	3	4	5	6	7	
I	15	7	26	22	5	8	26	
II	19	29	0	100	8	30	132	318
	22 + 318 + 68 + 34 + 43 = 485							
	180	299	0					
	138	238	0					
	102	170	0					
			0					

tr I II = max{15 3 0 22 -73 -73 -77} = 22

GMM I II = 485 + 0 = 485

	1	2	3	4	5	6	7	
I	15	7	26	22	5	8	26	
III	10	16	75	35	12	32	68	248
	22 + 248 + 132 + 34 + 43 = 479							
	186	299	0					
	138	238	0					
	102	170	0					
			0					

tr I III = max{15 12 22 -31 -61 -65 -71} = 22

GMM I III = 479 + 0 = 479

	1	2	3	4	5	6	7	
I	15	7	26	22	5	8	26	
IV	6	10	42	17	15	12	34	136
	32 + 136 + 132 + 68 + 43 = 411							
	186	299	0					
	180	238	0					
	138	170	0					
			0					

tr I IV = max{15 16 32 12 0 -7 7} = 32

GMM I IV = 411 + 0 = 411

	1	2	3	4	5	6	7	
I	15	7	26	22	5	8	26	
V	11	12	48	40	7	20	43	181
	25	+	181	+	132	+	68	+ 34 = 440
	186	299	0					
	180	238	0					
	102	170	0					
			0					

tr I V = max{15 11 25 -1 -36 -35 -29} = 25

GMM I V = 440 + 0 = 440

WITH SUCCESSIVELY DETERMINED PERSPECTIVE MATRICES CONT.

	1	2	3	4	5	6	7	
II	19	29	0	100	8	30	132	
V	11	12	48	40	7	20	43	
I	15	7	26	22	5	8	26	109
	180 +		98 +		109	+	68	+ 34 = 489
	102	94	8					
	180	130	50					
			58					

tr II V = max{19 37 25 77 45 68 180} = 180

tr V I = max{11 8 49 63 48 63 98} = 98

GMM II V I = 489+58=547

	1	2	3	4	5	6	7	
II	19	29	0	100	8	30	132	
V	11	12	48	40	7	20	43	
III	10	16	75	35	12	32	68	248
	180	+	45 +		248	+	26	+ 34 = 533
	83	94	0					
	102	130	0					
			0					

tr II V = max{19 37 25 77 45 68 180} = 180

tr V III = max{11 13 45 10 -18 -10 1} = 45

GMM II V III = 533 + 0 =533

	1	2	3	4	5	6	7	
II	19	29	0	100	8	30	132	
V	11	12	48	40	7	20	43	
IV	6	10	42	17	15	12	34	136
	180	+	79 +		136	+	26	+ 68 = 489
	83	94	0					
	180	130	50					
			50					

tr II V = max{19 37 25 77 45 68 180} = 180

tr V IV = max{11 17 55 53 43 48 79} = 79

GMMII V IV = 489 + 50 = 539

Determination of third row's number, assuming that row II is already first row, and row V is second row.

Expansion times for initial sequence of fronts

TrI = max { 15 3 0 22 -73 -73 -77 } = 22
TrII = max { 19 38 22 47 20 38 138 } = 138
TrIII = max { 10 20 85 78 73 90 146 } = 146
TrIV = max { 6 5 35 4 -21 -16 -2 } = 35

	A	B	C	D	E	F	G	Σ*j*	Σ*j*-T_p	Σ*j*-*T*
I	15	7	26	22	5	8	26	109	83	94
II	19	29	0	100	8	30	132	318	186	299
III	10	16	75	35	12	32	68	248	180	238
IV	6	10	42	17	15	12	34	136	102	130
V	11	12	48	40	7	20	43	181	138	170
									hidden	hiding

Table containing data for optimization of work duration matrix

		1	2	3	4	5	6	7					
	I	15	7	26	22	5	8	26		0	-7	7	} =32
<-1	IV	6	10	42	17	15	12	34		-21	-16	-2	} =35
<-2	V	11	12	48	40	7	20	43		-30	-18	-5	} =63
<-3	II	19	29	0	100	8	30	132		20	38	138	} =138
<-4	III	10	16	75	35	12	32	68	Tr I IV V II III = 516				
<-5													

	1	2	3	4	5	6	7					
I	15	7	26	22	5	8	26		0	-7	7	} = 32
IV	6	10	42	17	15	12	34		-21	-16	-2	} = 35
V	11	12	48	40	7	20	43		-18	-10	1	} = 45
III	10	16	75	35	12	32	68		0	24	62	} = 88
II	19	29	0	100	8	30	132	Tr I IV V III II = 518				

Total time of realization of complex of works assuming rows are in this order: I IV V II III and I IV V III II.

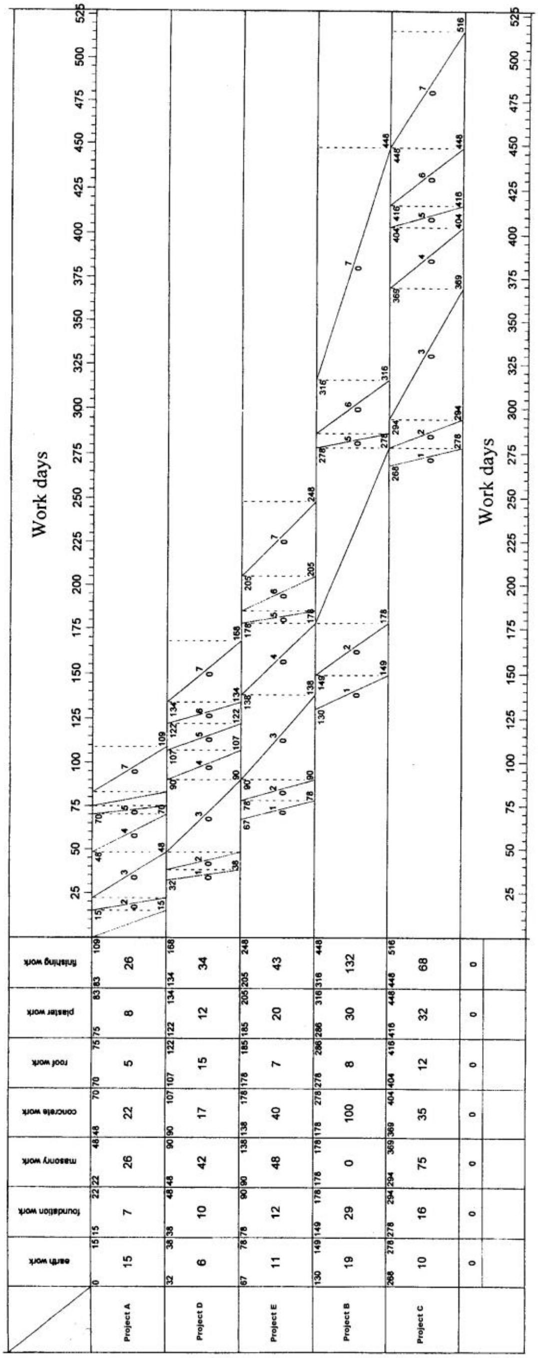


Figure 2.3. Cyclogram of realization of structures for method with zero couplings between projects

On the basis of the optimization calculations a new sequence of work on the structures, i.e. A-D-E-B-C, was proposed. As a result of this change in the sequence a reduction in the cycle (from 522 to 516 units in the considered case) was obtained.

2.4. Algorithms for establishing sequence in construction work organization method which takes into account couplings between realization means and couplings between work fronts

This method differs fundamentally from the other two. It assumes that the realization time is equal to the sum of the durations of the works along the critical path. This is a set of works which must be carried out sequentially, i.e. one after another, whereby it becomes impossible to find a time of the realization of a whole work complex which would be shorter than the critical path.

First the sums of the elements in the particular rows and columns of matrix **T** are calculated, i.e.

$$W_i = \sum_{j=1}^{m} t_{i,j}, \quad i = 1, ..., n, \tag{2.12}$$

$$K_j = \sum_{i=1}^{n} t_{i,j}, \quad j = 1, ..., m. \tag{2.13}$$

Then the highest of these values is determined, i.e.

$$M = \max \langle W_1, ..., W_k, K_1, ..., K_m \rangle, \tag{2.14}$$

which is done for the matrix's row or column. It is assumed that the critical path runs through a selected row or column determined in the above way.

The value of LPM corresponds to the minimum critical path passing through a selected row or column for the established sequence of rows.

If the sequence of rows bearing numbers $u_1, \ldots, u_s$ is established, then for the critical path running through row i_0 we determine LPM as follows:

$$\text{LPM} = \sum_{k=1}^{s} t_{uk,1} + \sum_{j=1}^{m} t_{io,j} + \sum_{\substack{i=1 \\ i \in (u_1,...,u_s)}}^{n} t_{i,m} . \tag{2.15}$$

But if the critical path runs through column j_0, LPM is given by this formula:

$$\text{LPM} = \sum_{j=1}^{j_o - 1} t_{u1,j} + \sum_{i=1}^{n} t_{i,jo} + \min_{\substack{1 \le i \le n \\ i \in (u_1,...,u_s)}} \left\langle \sum_{j=j_o+1}^{m} t_{i,j} \right\rangle . \tag{2.16}$$

If maximum M is realized for a column, the variant tree will grow very much and usually almost a whole subtree (branch), corresponding to a variant with one row's position established, will be expanded. LPM has such a property that as the tree is expanded the LPM value does not decrease and the sought variant will be located at the tree's bottom.

If maximum M is realized for a matrix row, LPM may both increase or decrease as the variant tree's branch is expanded, which makes it impossible to find minimum LPM without searching all the nodes and branches of the tree diagram.

2.4.1. Algorithm for determining minimum LPM

Instructions	**Commentary**
i) For all $i = 1,\ldots, n$, except $i = i_0$, substitute $b_\mathrm{i} = t_{i,1} - t_{i,m}$.	Calculation in each row, except i_0, of differences between first column elements and last column elements.
ii) Substitute $b_{i,0} := 0$	
iii) Put vector (b_i) $i =1,\ldots, n$ into increasing sequence and into first column of matrix $[\mathrm{C}_{i,k}]$ $I = 1,\ldots, n$ $j = 1, 2$ Place list of initial numbers prior to sorting into second column, i.e. $c_{1,2}$ is number of next numeral $c_{i,1}$ in sequence (b_i) $i = 1,\ldots, n$	Above row i_0 there are rows for which first element is smaller than last one and the other rows are below it.

Sequence $(C_{1,2})$ $i =1,\ldots, n$ is the sought sequence of fronts. It should be noted that the same LPM value will be obtained also for other sequences. The rows above the critical row can be reordered in any way and also the rows below the critical row can be reordered. If i_1, $c_{1,1,2} = i_0$ (i.e. the critical row has been placed as i_i), we can rearrange arbitrarily the first $(i_1–1)$ elements in sequence $(c_{1,2})$ $i = 1,\ldots, n$ and also $(c–i_1)$, which will not affect the LPM value.

If the critical path runs through a particular column bearing number j_0, then its length depends only on the established positions of the first and last row. Thus it is reasonable to adopt the following notation: let OPT(i,j) mean that a set of LPM indexes of a sequence of fronts is a set of all sequences whose first and last element is i and j, respectively.

Instructions	Commentary
i) For all *i* = 1,…, *n* make substitutions: $a_i = \sum_{j=1}^{j_O-1} t_{i,j}$ $b_i = \sum_{j=j_O+1}^{m} t_{i,j}$	a_i is time needed to carry out all preceding works, bearing number j_0, on the *i*-th work front. Similarly b_i for following works bearing number j_0
ii) Find such index r that $a_r = \min_{1 \le i \le n} a_i$ and such index s that $b_s = \min_{1 \le i \le n} b_i$	Searching for least elements.
iii) If r ≠ s, then OPT (r, s). If r = s, perform sequence of istructions: α) Find such index r_1 that $a_{r,1} = \min_{\substack{1 \le i \le n \\ i \ne r}} a_i$ and such index s_1 that $b_{s,1} = \min_{\substack{1 \le i \le n \\ i \ne s}} b_i$	Searching elements second in turn.
β) If $a_r + b_{s,1} \ge a_{r,1} + b_s$, then If $a_r + b_{s,1} \le a_{r,1} + b_s$, then OPT (r, s_1)	Testing which pair (minimum element, element second in turn) yields minimum LPM.

The above procedures are applied to establish the sequence of construction works carried out on building structures or their parts (fronts, work sites). The basic problem in the variant-tree calculation is the determination of the value of the tree diagram's nodes (limit possible minimum – LPM). This index expresses the technological and organizational constraints represented by time couplings. By comparing these values one can eliminate the variants which hold no promise of a better result. Thus directions in which the space of solutions will be searched can be determined.

2.4.2. Conclusion

The presented procedures allow us to establish the sequence in which to carry out works, taking into account couplings between means of realization and couplings between work fronts. It has been shown how to establish a sequence of works in organization methods with a critical path. Since a critical path occurs in a large set of construction work organization methods, the presented procedure can be applied to the construction of organizational models.

2.5. Work organization methods taking into account couplings between both realization means and work fronts and diagonal or reverse diagonal couplings

The lead time for a work complex, with a critical path determined taking into account couplings between both realization means and work fronts and diagonal couplings (or reverse diagonal couplings), is calculated similarly as in method III (chap. 4).

Also the rules for determining the optimum sequence used in method III apply also to the present method. Consequently, the "lowest possible minimum" (LPM) is determined similarly as in method III, except that additional diagonal or reverse diagonal constraints should be taken into account when calculating the actual lead time for a work complex in the initial matrix and in the prospective final matrices (after the variant tree has been fully expanded).

To calculate the characteristics the following are determined in turn: the earliest and latest work commencement and completion dates, the critical path, the couplings for the earliest and latest dates, the total time reserves and the minimum lead time for the whole work complex, taking into account all the assumed couplings. An WF matrix for method IV and a WK or WF matrix for method V are used for this purpose.

2.5.1. Calculation of characteristics with additional diagonal couplings taken into account

The earliest commencement and completion dates are determined from the following formulas (fig. 2.4).

$$t_{ij}^{wr} = \max\{t_{i-1,j}^{wz}, t_{i,j-1}^{wz}, t_{i+1,j-1}^{wr}\}, \tag{2.17}$$

$$t_{ij}^{wz} = t_{ij}^{wr} + t_{ij}. \tag{2.18}$$

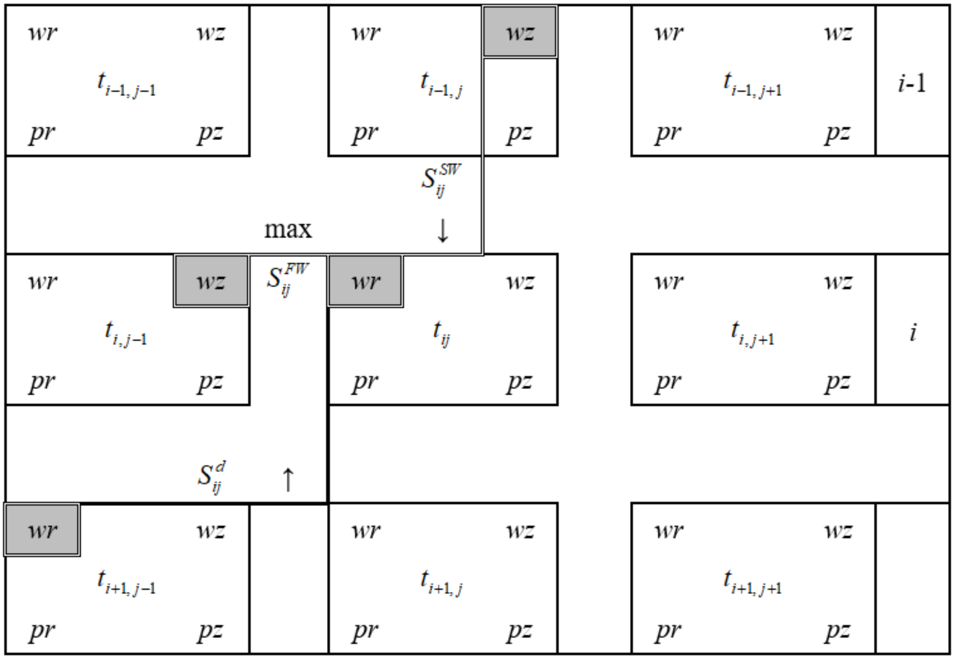


Figure 2.4. Diagram showing how to take into account couplings between realization means, couplings between work fronts and diagonal couplings when calculating earliest work commencement dates

This means that when determining the earliest commencement dates for jobs (i, j) one should take into account:

- couplings between realization means starting with the preceding jobs in a given kind of jobs $(i-1, j)$,
- couplings between work fronts starting with the preceding jobs in a given partial complex $(i, j-1)$,
- diagonal couplings starting with the jobs in the immediately preceding kind of jobs and simultaneously in the immediately following partial work complex $(i+1, j-1)$.

The earliest completion dates for jobs (i, j) are determined by adding up the earliest commencement dates for these jobs and their lead times.

The latest completion dates for jobs (i, j) are determined taking into account (fig. 2.5):

- couplings between realization means starting with the next jobs in given kinds of jobs $(i+1, j)$,
- couplings between work fronts starting with the next jobs in given partial work complexes $(i, j+1)$,

- diagonal couplings (considered simultaneously) starting with the jobs in the preceding partial work complex and in the next kind of jobs ($i - 1, j + 1$).

$$t_{ij}^{pr} = \min\{ t_{i+1,j}^{pr}, t_{i,j+1}^{pr}, t_{i-1,j+1}^{pz} \} . \qquad (2.19)$$

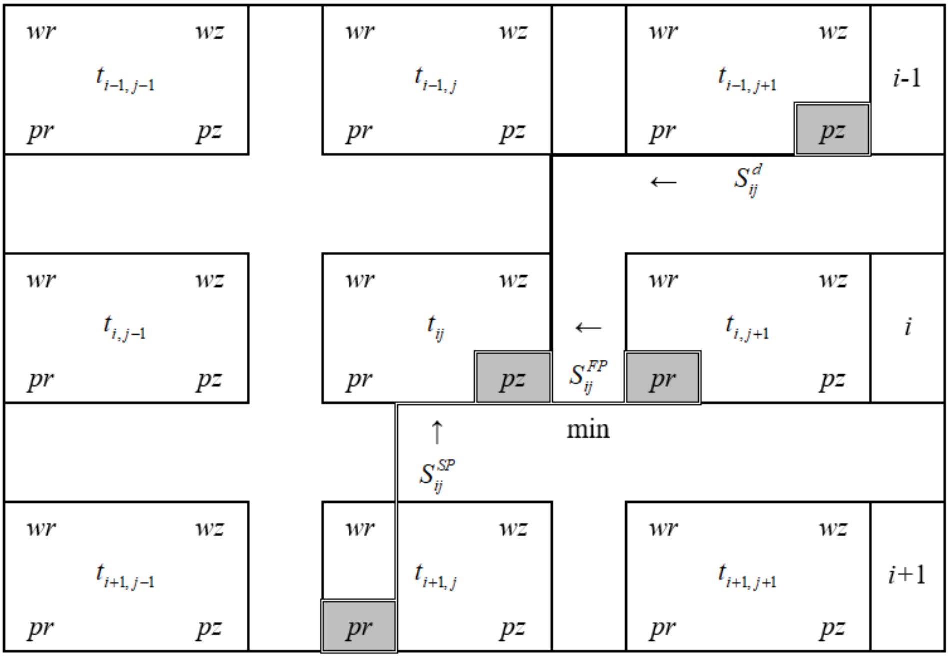


Figure 2.5. Diagram showing how to take into account couplings between realization means, couplings between work fronts and diagonal couplings when calculating latest work completion dates

The latest work commencement dates are determined from the following formula:

$$t_{ij}^{pr} = t_{ij}^{pz} - t_{ij} . \qquad (2.20)$$

Couplings between realization means, couplings between work fronts and total time reserves are determined similarly as in method III (chap. 4). One should note that the earliest and latest work commencement and completion dates which were determined taking into account also diagonal couplings are used here.

The coupling for the earliest dates are:

$$S_{ij}^{SW} = t_{i-1,j}^{wz} - t_{ij}^{wr} . \qquad (2.21)$$

$$S_{ij}^{FW} = t_{ij}^{wr} - t_{i,j-1}^{wz} , \qquad (2.22)$$

The couplings for the latest dates are:

$$S_{ij}^{SP} = t_{i-1,j}^{pr} - t_{ij}^{pz} , \qquad (2.23)$$

$$S_{ij}^{FP} = t_{ij}^{pr} - t_{i,j-1}^{pz}. \tag{2.24}$$

2.5.1. Calculation of characteristics with additional reverse diagonal couplings taken into account

The earliest commencement and completion dates are determined from the following formulas (fig. 2.6):

$$t_{ij}^{wr} = \max\{t_{i-1,j}^{wz}, t_{i,j-1}^{wz}, t_{i-1,j+1}^{wr}\}, \tag{2.25}$$

$$t_{ij}^{wz} = t_{ij}^{wr} + t_{ij}. \tag{2.26}$$

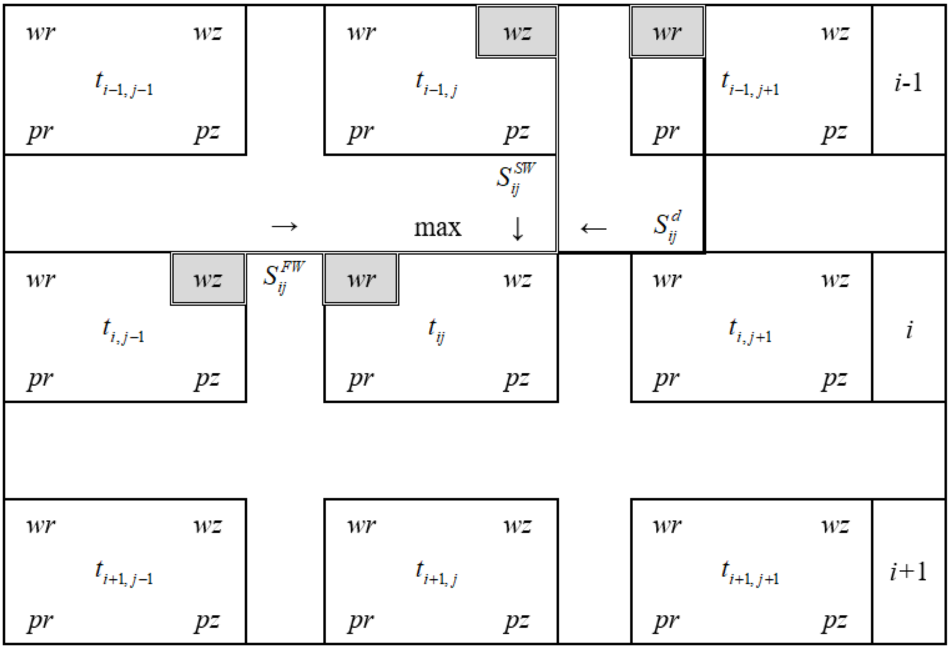


Figure 2.6. Diagram showing how to take into account couplings between realization means, couplings between work fronts and reverse diagonal couplings when calculating earliest work commencement dates

Thus when determining the earliest commencement dates (i, j) one should take into account:

- couplings between realization means starting from the jobs preceding them in a given kind of jobs ($i-1$, j),
- couplings between fronts starting from the jobs preceding them in a given partial complex (i, $j-1$),
- diagonal couplings starting from the jobs in the next kind of jobs and simultaneously in the directly preceding partial complex ($i-1$, $j+1$).

The earliest completion dates for jobs are determined as the total of the earliest commencement dates for the jobs and their lead times.

The latest completion dates for jobs (i, j) are determined taking into account (fig. 2.7):

- couplings between realization means starting from the jobs directly following the considered jobs in given kinds of jobs $(i+1, j)$,
- couplings between fronts starting from the jobs directly following them in given partial complexes $(i, j + 1)$,
- diagonal couplings starting from the jobs in the directly following partial complex and simultaneously in the directly preceding kind of jobs.

$$t_{ij}^{pr} = \min\{ t_{i+1,j}^{pr}, t_{i,j+1}^{pr}, t_{i+1,j-1}^{pz} \}. \tag{2.27}$$

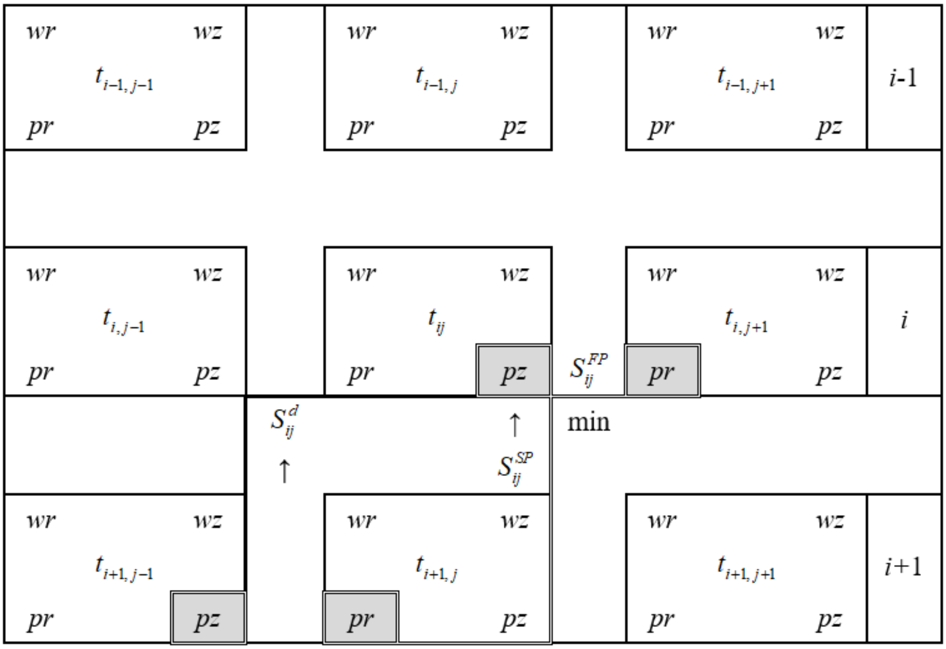


Figure 2.7 Diagram showing how to take into account couplings between realization means, couplings between work fronts and reverse diagonal couplings when calculating latest work commencement dates

The latest job commencement dates are determined from this formula:

$$t_{ij}^{pr} = t_{ij}^{pz} - t_{ij}. \tag{2.28}$$

The characteristics of an unrythmic complex of building structures are calculated (similarly as in method III) from a table as shown in fig. 2.7.

3. Variants of time couplings between work processes in matrix models

3.1. Variants of construction work organization method taking into account zero reverse diagonal couplings

Zero reverse diagonal couplings link together jobs located in the matrix diagonals. They differ from diagonal couplings in the direction of their action, i.e. in the sequence of jobs (in terms of the calculus of vectors, they have a direction in common with diagonal couplings but differ in their sense).

Zero reverse diagonal couplings represent the technological and organizational dependence between a given job and the preceding job in a technological sequence on two neighboring work fronts. This situation arises in building practice when, for example, given firms (gangs) are carrying out two technologically different jobs on two objects or their parts at the same time. The mutual dependence between the two jobs may be connected with the use of a common vertical transport, which limits the simultaneous starting of jobs on different fronts of a given building structure. A significant feature is the dependence between jobs j of front i and jobs $j + 1$ of front $i - 1$.

Jobs of different kinds performed on adjacent work fronts can be realized in parallel depending on the conditions. This is defined as follows:

$$\bigwedge_{a,b \in W} \bigvee_{X \subset W} f(x_1) = a \wedge f(x_2) = b \Rightarrow a = b \, , \tag{3.1}$$

where: $X = \{a, b\}$ and $S^{OD} = 0$.

3.1.1. Variants of method

Variant 1 – couplings linking job commencement times for the earliest dates:

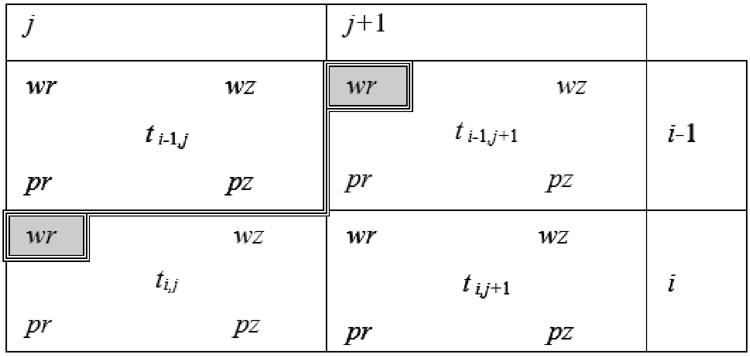
$$t_{ij}^{wr} = t_{i-1, j+1}^{wr} \, . \tag{3.2}$$

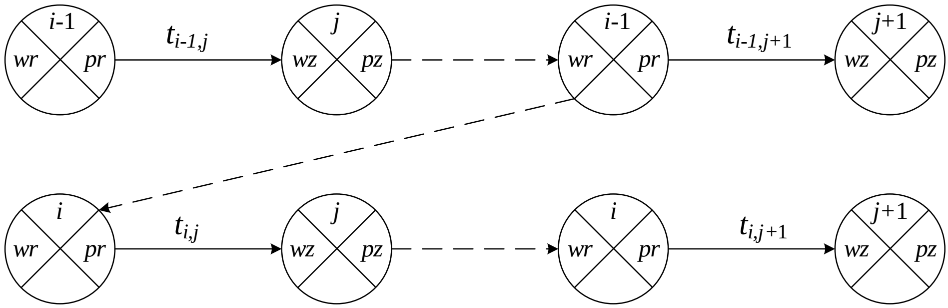
Formula (3.2) specifies that jobs on neighboring work fronts (building structures) must be started simultaneously. It imposes the constraint of jobs sequentially, making starting a given job conditional on the date of commencement of the job next in the technological order on the preceding building structure.

In building practice an organizational situation often arises when jobs j and $j - 1$ of different kinds must be performed on two independent work fronts i and $i - 1$. This is defined as follows:

$$\bigwedge_{c,d \in P} \bigvee_{Y \subset P} f(y_1) = c \wedge f(y_2) = d \Rightarrow c = d \, , \tag{3.3}$$

where: $Y = \{c, d\}$.



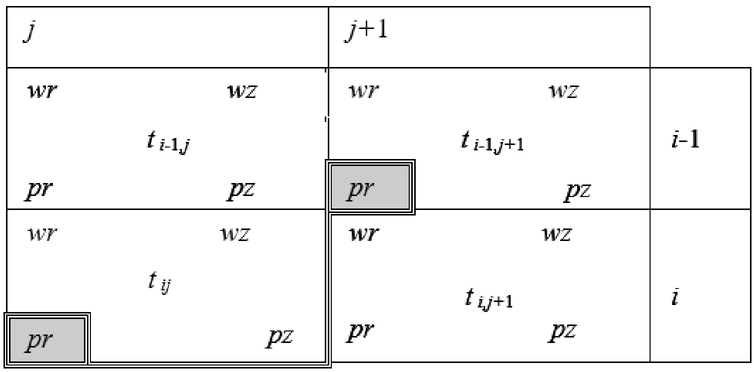


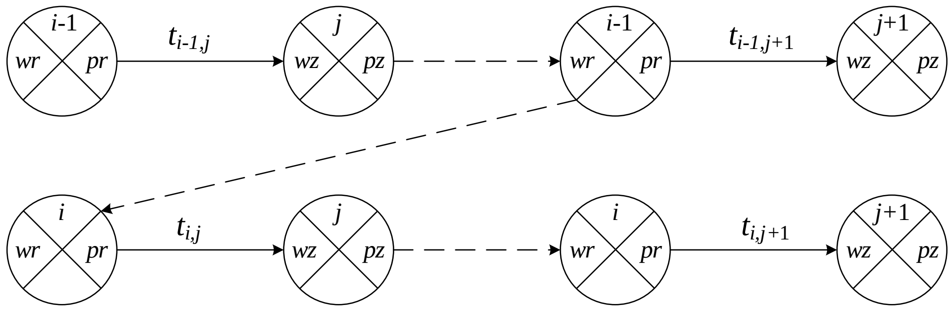
The possibilities are:

$$1)\quad y_1 = c \Rightarrow t^{wr}_{i-1,\,j+1} = t^{wr}_{ij} \Leftrightarrow S^D = 0$$
$$2)\quad y_2 = d \Rightarrow t^{wr}_{ij} = t^{wr}_{i-1,\,j+1} \Leftrightarrow S^{OD} = 0 \qquad (3.4)$$

Variant 2 – couplings linking job commencement times for the latest dates.

$$t^{pr}_{ij} = t^{pr}_{i-1,\,j+1}\,. \qquad (3.5)$$





Relation (3.5) specifies that jobs *j* and *j*–1 of different kinds must be started simultaneously. Then the latest dates of starting the jobs should be considered. The condition applies to jobs which are not on the critical path. It is possible to plan the dates of starting the jobs so as to use the time reserves to a maximum degree.

This situation arises when due to necessary technological breaks (for e.g. cement setting, anticorrosion protection of the structure and so on) the available time must be used – the starting of jobs is planned for the maximum time between the earliest and latest date of starting them, i.e. t_{ij}^{wr} and t_{ij}^{pr}, then $t_{ij}^{wr} - t_{ij}^{pr} \geq 0$.

Also a relation holds which ensures the parallel realization of technologically linked jobs on two adjacent building structures.

This is defined as follows:

$$\bigwedge_{e,f \in P} \bigvee_{U \subset P} f(u_1) = e \wedge f(u_2) = f \Rightarrow e = f \ , \tag{3.6}$$

where: $U = \{e, f\}$.

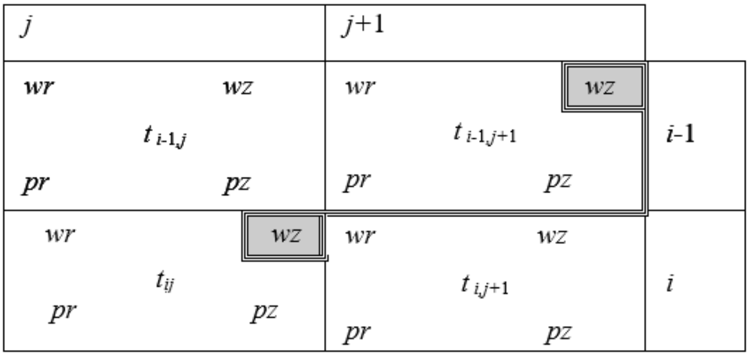
The possibilities are:

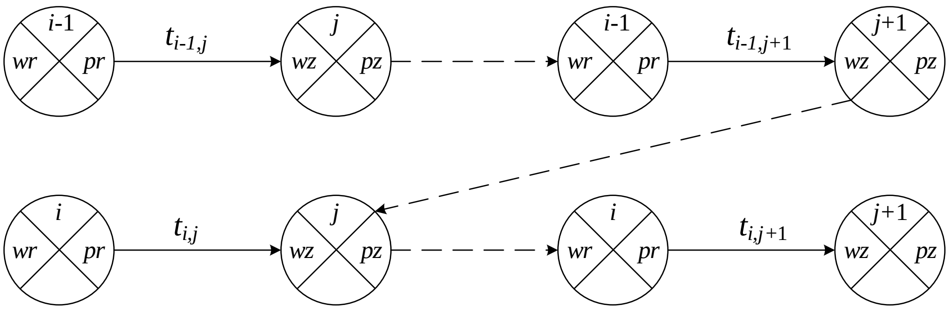
$$\begin{aligned} &1) \quad u_1 = e \Rightarrow t_{ij}^{pr} = t_{i-1,j+1}^{pr} \Leftrightarrow S^{OD} = 0 \\ &2) \quad u_2 = f \Rightarrow t_{i-1,j+1}^{pr} = t_{ij}^{pr} \Leftrightarrow S^{D} = 0 \end{aligned} . \tag{3.7}$$

Variant 3 – couplings linking job completion times for the earliest dates.

$$t_{ij}^{wz} = t_{i-1,j+1}^{wz} . \tag{3.8}$$

Formula (3.8) specifies that jobs *j* and *j*+1 of different kinds on neighboring work fronts must be completed at the same time. This is a common technological constraint stemming from the necessity of putting into service a part of a building structure or the whole of it or from the necessity of carrying out other jobs (e.g. pluming). The condition of the parallel carrying out of jobs of different kinds and their simultaneous completion must be fulfilled. At the same time, reverse diagonal couplings determine the sequence of the jobs, i.e. $t_{i-1,j+1} \prec t_{ij}$.





This is defined as follows:

$$\bigwedge_{g,h\in w}\ \bigvee_{Z\subset W} f(z_1)=g \wedge f(z_2)=h \Rightarrow g=h\,, \tag{3.9}$$

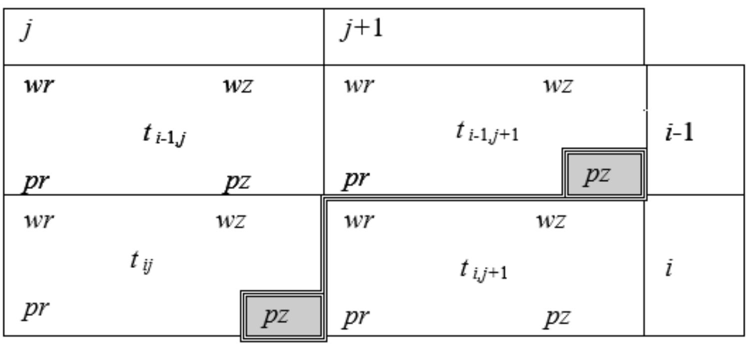
where: $Z = \{z_1, z_2\}$.

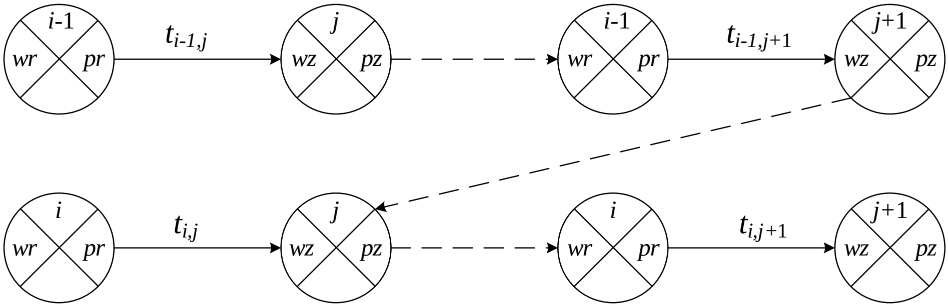
The possibilities are:

$$\begin{array}{l} 1)\quad z_1 = g \Rightarrow t_{ij}^{wz} = t_{i-1,\,j+1}^{wz} \Leftrightarrow S^{OD} \\ 2)\quad z_2 = h \Rightarrow t_{i-1,\,j+1}^{wz} = t_{ij}^{wz} \Leftrightarrow S^{D} \end{array}. \tag{3.10}$$

Variant 4 – couplings linking job completion times for the latest dates.

$$t_{ij}^{pz} = t_{i-1,\,j+1}^{pz}\,. \tag{3.11}$$





Relation (3.11) ensures that jobs *j* and *j*+1 of different kinds are completed at the same time. As a result, some parts of the building structure can be used once the jobs are finished. Roof hydroinsulation and draining work may serve as an example here. Once this is done (the roof is made leakproof), it is possible to carry out floor work. Numerous examples of such a sequence of jobs can be found in industrial building. In addition, formula (3.11) imposes a condition of the latest realization dates, which rules out any rescheduling. Since the jobs are to be carried out in parallel, their commencement dates must be precisely fixed to ensure that the jobs are completed on time.

$$t_{ij}^{pz} = t_{ij}^{pr} + t_{ij}\,, \tag{3.12}$$

and

$$t_{i-1,\,j+1}^{pz} = t_{i-1,\,j+1}^{pr} + t_{i-1,\,j+1}\,. \tag{3.13}$$

This is defined as follows:

$$\bigwedge_{k,l\in w}\ \bigvee_{K\subset W} f(x_1) = k \wedge f(x_2) = l \Rightarrow k = l\,, \tag{3.14}$$

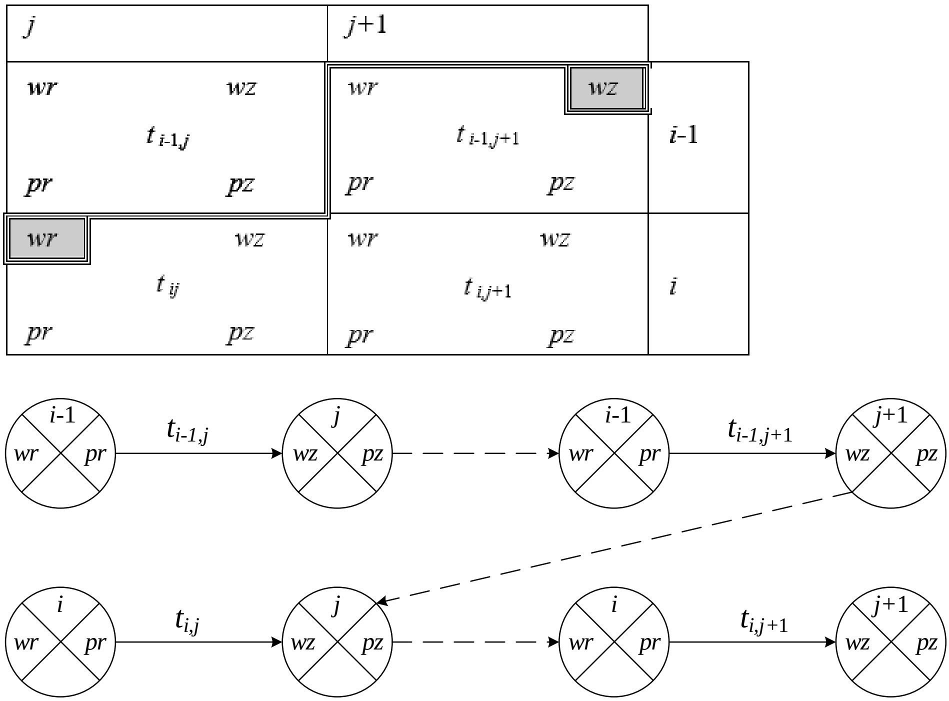
where $K = \{k, l\}$.
The possibilities are:

$$\begin{aligned} &1)\quad x_1 = k \Rightarrow t_{ij}^{pz} = t_{i-1,\,j+1}^{pz} \Leftrightarrow S^{OD} = 0 \\ &2)\quad x_2 = l \Rightarrow t_{i-1,\,j+1}^{pz} = t_{ij}^{pz} \Leftrightarrow S^{D} = 0 \end{aligned}\,. \tag{3.15}$$

Variant 5 – couplings linking job commencement and completion times for the earliest dates.

$$t_{ij}^{wr} = t_{i-1,\,j+1}^{wz}\,. \tag{3.16}$$

Formula (3.16) imposes the condition that job *j* on front *i* should be started depending on the date of completion of job *j*+1 on front *i*–1. Then a sequential relationship becomes apparent: first jobs *j*+1 on front *i*–1 and then jobs *j* on front *i* should be realized. Construction jobs of different kinds should be performed in a fixed sequence for the early dates on the neighboring work fronts. This relationship is important for the organization of construction work since it enables the performance of different jobs by the same means (e.g. by commissioning one contractor to do the jobs).



It applies to the earliest realization dates and makes it possible to determine the time reserves for carrying out jobs on given work fronts (S^F). For a given job the following relations hold:

$$t^{wr}_{i-1,\,j+1} + t_{i-1,\,j+1} = t^{wz}_{i-1,\,j+1}, \tag{3.17}$$

and

$$t^{wz}_{i-1,\,j+1} = t^{wr}_{ij}\,. \tag{3.18}$$

This is defined as follows:

$$\bigwedge_{m,n\in p}\ \bigvee_{M\subset P} f(u_1) = m \wedge f(u_2) = n \Rightarrow m = n\,, \tag{3.19}$$

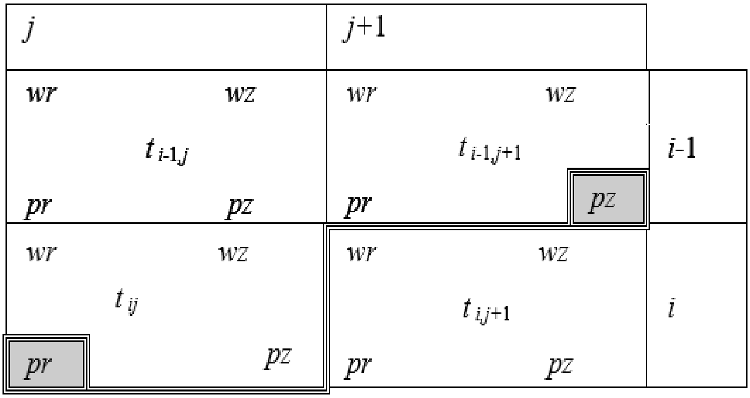
where $M = \{u_1, u_z\}$.

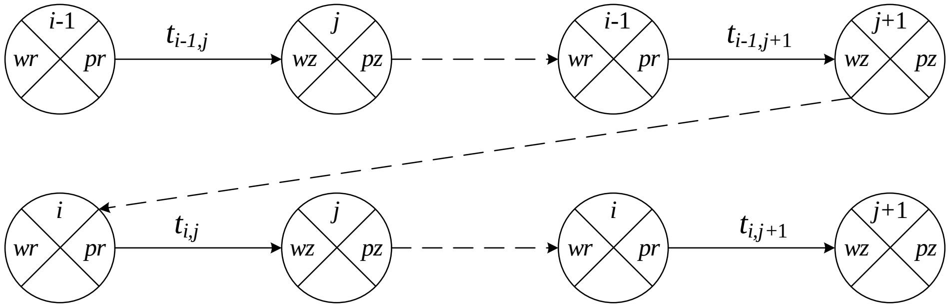
The possibilities are:

$$\begin{aligned} &1)\quad u_1 = m \Rightarrow t^{wr}_{ij} = t^{wz}_{i-1,\,j+1} \Leftrightarrow S^{OD} = 0 \\ &2)\quad u_2 = n \Rightarrow t^{wz}_{i-1,\,j+1} = t^{wr}_{ij} \Leftrightarrow S^{D} = 0 \end{aligned}\ . \tag{3.20}$$

Variant 6 – couplings linking job commencement and completion times for the latest dates.

$$t^{pr}_{ij} = t^{pz}_{i-1,\,j+1}\,. \tag{3.21}$$





Relation (3.21) ensures that given job t_{ij} is started after job $t_{i-1,j+1}$ for neighboring fronts. According to the adopted rule, construction jobs should be realized in a fixed sequence, by carrying out successive jobs on successive fronts. This allows a given construction firm or a gang to perform successive jobs on different building structures. The only condition here is the availability of work fronts to carry out the planned jobs. In this organizational variant, the particular condition applying to the latest dates of completion of jobs j+1 on front i–1 and commencement of jobs j on front i should be taken into account. Also the relation of preceding is important. This is defined as follows:

$$\bigwedge_{p,s\in w} \bigvee_{L\subset W} f(z_1) = p \wedge f(z_2) = s \Rightarrow p = s \,, \tag{3.22}$$

where $L = \{p, s\}$.

The possibilities are:

$$\begin{aligned} &1) \quad z_1 = p \Rightarrow t_{ij}^{pr} = t_{i-1,\,j+1}^{pz} \Leftrightarrow S^{OD} = 0 \\ &2) \quad z_2 = s \Rightarrow t_{i-1,\,j+1}^{pz} = t_{ij}^{pr} \Leftrightarrow S^{D} = 0 \end{aligned} \,. \tag{3.23}$$

3.1.2. Algorithm for scheduling construction jobs with zero reverse diagonal couplings

Basic assumptions

The investment problem is given as a set of building structures $O = \{O_1, O_2, ..., O_n\}$. They can be realized by multitrade gangs $B = \{B_1, B_2, ..., B_p\}$.The anticipated work processes form this sequence $P_i = [P_{i1}, P_{i2}, ..., P_{im}]$.
The work processes can be realized on the building structures in a prescribed technological order, i.e. each process P_{ik} can be realized after process P_{ik-1} ends but before process P_{ik+1} ends.

The following requirements are imposed on the realization: gangs (construction firms) move from one building structure to another in a continuous way, the building processes should be carried out in a fixed order, the values of the couplings reverse to the diagonal ones should be equal to zero, i.e. the reverse diagonal couplings will link the dates of completion of given process P_j on building structure O_i with the date of commencement of process P_{j-1} on structure O_{i+1}. The problem is to harmonize the construction work in such a way that $T \rightarrow$min, where T – the duration of the realization of the building complex.

Scheme of algorithm

The organizational model, which takes into account the influence of reverse diagonal couplings, is constructed according to the scheme presented below. Construction jobs are synchronized on the basis of a matrix of job lead times.

Stage 1 – fix the dates of commencement and completion for a job in the first diagonal of matrix τ.

Stage 2 – fix the dates of commencement and completion for the jobs in the second diagonal. The date of commencement of the second job on the first building structure (indices 1, 2) is the same as the dates of completion of the preceding job on this structure (1, 1). The other dates are determined from the following conditions:

$$\begin{aligned} &t_{1,2}^{wr} = t_{1,1}^{wz}; \quad t_{1,2}^{wz} = t_{1,2}^{wr} + t_{1,2}; \\ &t_{2,1}^{wr} = t_{1,2}^{wz}; \quad t_{2,1}^{wz} = t_{2,1}^{wr} + t_{2,1}; \end{aligned} \tag{3.24}$$

Stage 3 – fix commencement and completion dates for jobs in the third diagonal. Start from the third job on the first structure (matrix indices 1, 3) and proceed as above, i.e.

$$\begin{aligned} &t_{1,3}^{wr} = t_{1,2}^{wz}; \quad t_{1,3}^{wz} = t_{1,2}^{wr} + t_{1,3}; \quad t_{2,2}^{wr} = t_{1,3}^{wz}; \\ &t_{2,2}^{wz} = t_{2,2}^{wr} + t_{2,2}; \quad t_{3,1}^{wr} = t_{2,2}^{wz}; \quad t_{3,1}^{wz} + t_{1,3}; \end{aligned} . \tag{3.25}$$

After a sequence of the values is determined, create sets of pairs of commencement and completion dates for successive (in a technological sequence) jobs on the building structures in the adjacent diagonals, except for the first and last job in the sequence:

$$R = \left\{ t_{2,1}^{wz}; t_{2,2}^{wr} \right\}, \tag{3.26}$$

if $t_{2,1}^{wz} > t_{2,2}^{wr}$, calculate the difference $t_{2,1}^{wz} - t_{2,2}^{wr} = a$. Add the obtained value to the value of the commencement dates for the jobs in the diagonal and fix new dates of their completion.

This adjustment is necessary to fulfil the condition of collision less work of the gangs (construction firms). Proceed according to this scheme until there are no diagonals left in the set. When creating sets of differences between the values of commencement and completion dates for the successive jobs on the given building structures, one should remember that only the maximum value in the set of differences is taken into account and used to adjust the commencement dates for the jobs.

Time characteristics of construction jobs

A matrix of durations of the jobs involved in the realization of the underground parts of four buildings is constructed. The set of processes includes in succession: carpentry, reinforcement work, concreting and insulation work. The durations of the jobs are specified in the matrix:

$$\tau = \begin{bmatrix} 9 & 6 & 7 & 4 \\ 7 & 4 & 10 & 4 \\ 12 & 9 & 11 & 6 \\ 6 & 4 & 5 & 3 \end{bmatrix}.$$

The rows in the matrix correspond to the successive building structures and the columns to the work processes. The matrix elements specify the lead time for a given job to be performed by a gang (construction firm) on a building structure. The algorithm was used to form a work flow and determine its time characteristics.

Table 3.1. Time characteristics of jobs (reverse diagonal couplings)

Project	**Diagonals**						
	1	2	3	4	5	6	7
A	0/9/9	9/6/15	15/7/22	24/4/28			
B		15/7/22	22/4/26	28/10/38	43/4/47		
C			26/12/38	38/9/47	47/11/58	58/6/64	
D				47/6/53	58/4/62	64/5/69	69/3/72

Note: the numerals in the fields represent in turn: the date of commencement of a job, its duration and its completion date.

3.2. Variants of construction work organization method taking into account couplings perpendicular to diagonal couplings

In a complex of (technologically) different construction jobs carried out on work fronts couplings perpendicular to diagonal couplings (CPDCs) occur. They are perpendicular to the direction in which the diagonal couplings act in the matrix of job durations.

CPDCs link jobs *j* of a given kind on front *i* with the preceding (in technological order) jobs *j*–1 on preceding front *i*–1.

Table 3.2. The matrix of job durations in CPDC complex

j-1	j	j+1	
$t_{i-1,j-1}$	$t_{i-1,j}$	$t_{i+1,j+1}$	i-1
$t_{i,j-1}$	t_{ij}	$t_{i,j+1}$	i
$t_{i+1,j}$	$t_{i+1,j}$	$t_{i+1,j+1}$	i+1

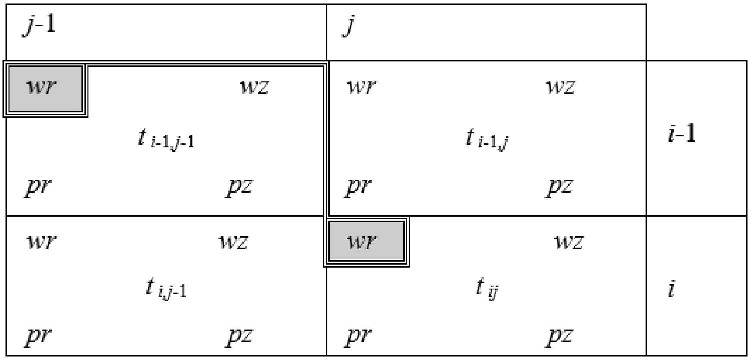
The action of CPDCs on the construction jobs with the earliest and latest commencement and completion dates can be varied. Several cases of the action of CPDCs on construction jobs in a complex can be distinguished. Also a set of variants of construction work organization methods can be created. Jobs of one kind can be linked, e.g. jobs *j*–1, *j*+1, to form chains of technologically similar jobs on work fronts. As a result, a group of organization methods with properties intermediate between classic flow organization methods and consecutive execution methods is created.

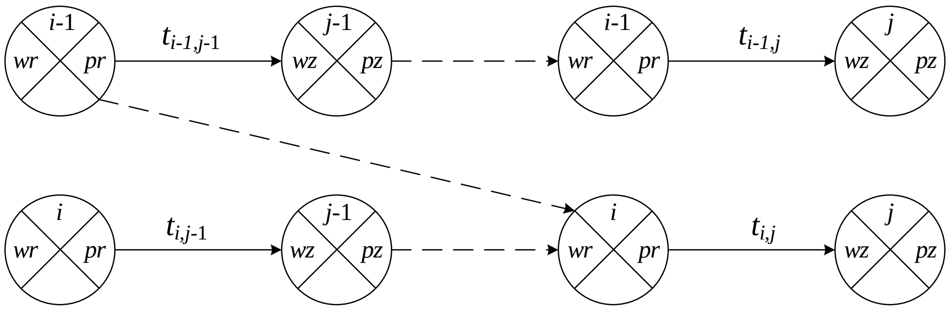
By transposing the initial matrix of job durations from the WF (ordinates/work fronts) arrangement to the TP (ordinates/technological processes) arrangement and using CPDCs we can create a new group of organizational and technological relationships. The couplings make it possible to link together work fronts in a sequential arrangement, i.e. fronts *i*–1, *i*, *i*+1. In this way a new group of relationships between the construction jobs can be created, taking into account the earliest and latest dates of commencement and completion of the jobs.

3.2.1. Variants of method – overview

Variant 1 – for the earliest dates.

$$t_{ij}^{wr} = t_{i-1,\,j-1}^{wr}. \tag{3.27}$$





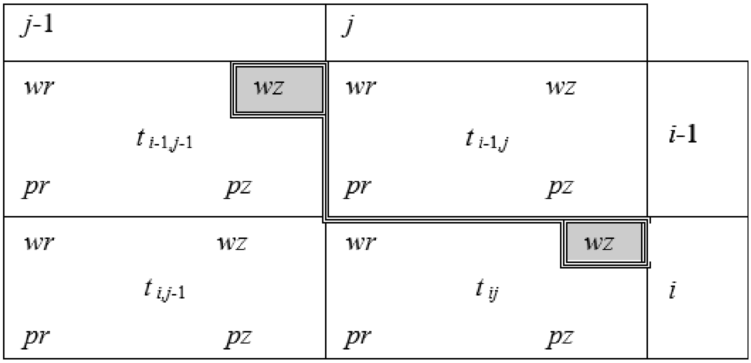
$$\bigwedge_{a,b\in W}\ \bigvee_{X\subset W} f(x_1)=a \wedge f(x_2)=b \Rightarrow a=b, \tag{3.28}$$

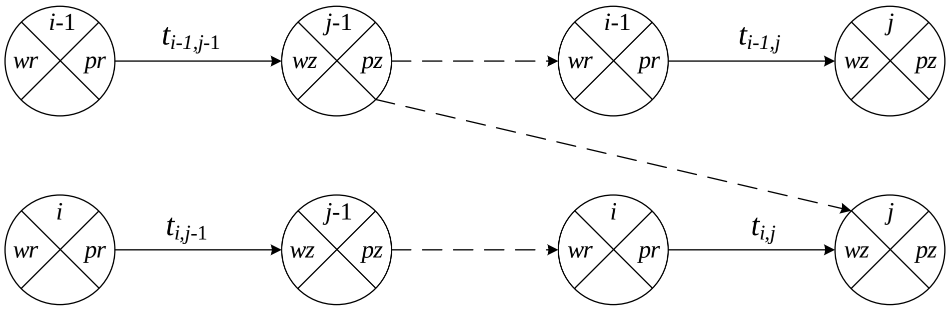
where $X = \{a, b\}$.
The possibilities are:

$$\begin{array}{l} 1)\quad x_1 = a \Rightarrow t_{ij}^{wr} = t_{i-1,\,j-1}^{wr}, \quad S^P = 0 \\ 2)\quad x_2 = b \Rightarrow t_{i-1,\,j-1}^{wr} = t_{ij}^{wr}, \quad S^P = 0 \end{array}. \tag{3.29}$$

Variant 2 – for the earliest dates.

$$t_{ij}^{wz} = t_{i-1,\,j-1}^{wz}. \tag{3.30}$$





This is defined as follows:

$$\bigwedge_{c,d\in P}\ \bigvee_{Y\subset P} f(y_1)=c \wedge f(y_2)=d \Rightarrow c=d \ , \tag{3.31}$$

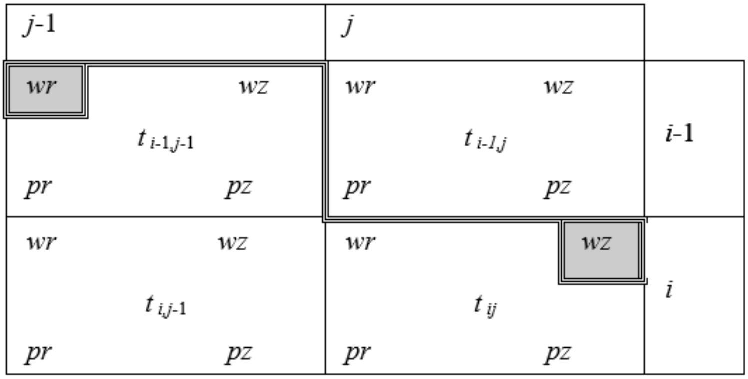
where $Y = \{c, d\}$.

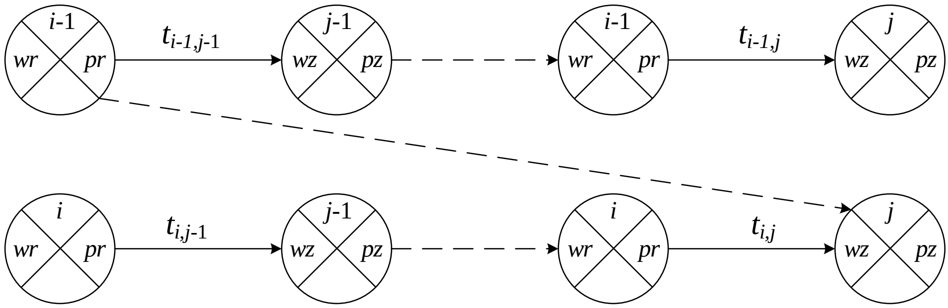
The possibilities are:

$$\begin{aligned} &1) \quad y_1 = c \Rightarrow t_{ij}^{wz} = t_{i-1,\,j-1}^{wz}, \quad S^P = 0 \\ &2) \quad y_2 = d \Rightarrow t_{i-1,\,j-1}^{wz} = t_{ij}^{wz}, \quad S^P = 0 \end{aligned} . \tag{3.32}$$

Variant 3 – for the earliest dates.

$$t_{ij}^{wz} = t_{i-1,\,j-1}^{wr} . \tag{3.33}$$





This is defined as follows:

$$\bigwedge_{e,f\in P}\ \bigvee_{U\subset P} f(u_1)=e \wedge f(u_2)=f \Rightarrow e=f\ , \tag{3.34}$$

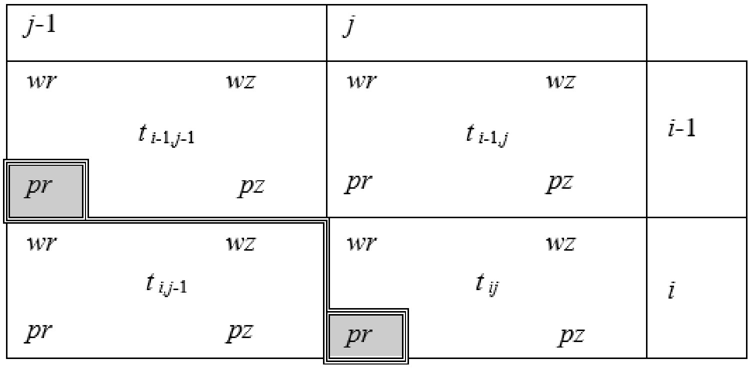
where $U = \{e, f\}$.

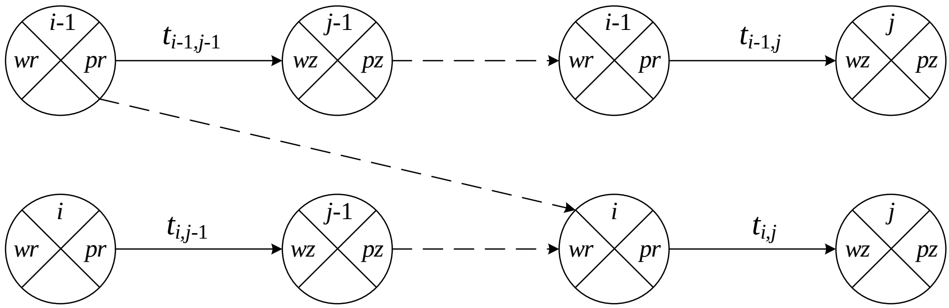
The possibilities are:

$$\begin{aligned} &1)\quad u_1 = e \Rightarrow t_{ij}^{wz} = t_{i-1,\,j-1}^{wr}, \quad S^P = 0 \\ &2)\quad u_2 = f \Rightarrow t_{i-1,\,j-1}^{wr} = t_{ij}^{wz}, \quad S^P = 0 \end{aligned}. \tag{3.35}$$

Variant 4 – for the latest dates.

$$t_{ij}^{pr} = t_{i-1,\,j-1}^{pr}. \tag{3.36}$$





This is defined as follows:

$$\bigwedge_{g,h\in W}\bigvee_{Z\subset W} f(z_1)=g \wedge f(z_2)=h \Rightarrow g=h\,, \tag{3.37}$$

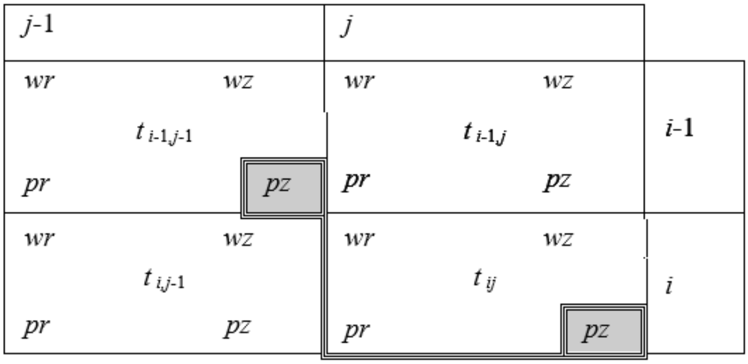
where $Z = \{g, h\}$.

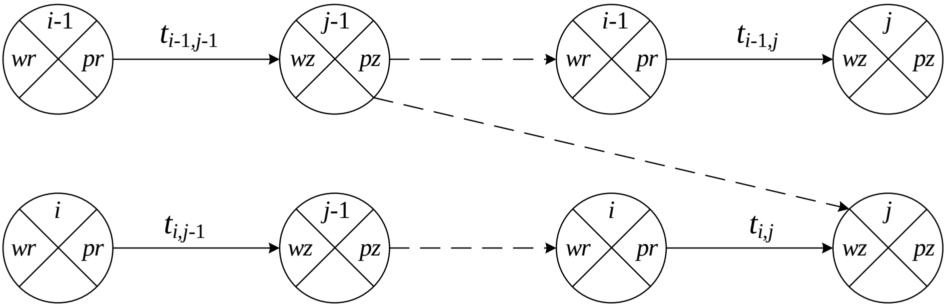
The possibilities are:

$$\begin{matrix} 1) & z_1 = g \Rightarrow t_{ij}^{pr} = t_{i-1,\,j-1}^{pr}, & S^P = 0 \\ 2) & z_2 = h \Rightarrow t_{i-1,\,j-1}^{pr} = t_{ij}^{pr}, & S^P = 0 \end{matrix}\,. \tag{3.38}$$

Variant 5 – for the latest dates.

$$t_{ij}^{pz} = t_{i-1,\,j-1}^{pz}\,. \tag{3.39}$$





This is defined as follows:

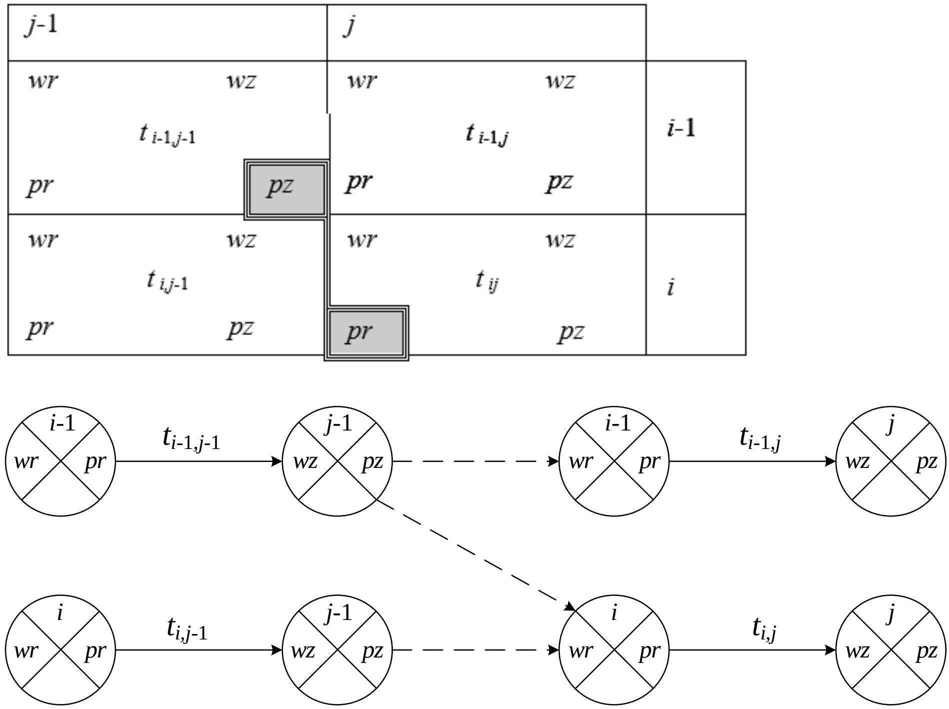
$$\bigwedge_{k,l\in W}\bigvee_{K\subset W} f(x_1)=k \wedge f(x_2)=l \Rightarrow k=l\,, \tag{3.40}$$

where $K = \{k, l\}$.
The possibilities are:

$$\begin{aligned} &1)\quad x_1 = k \Rightarrow t_{ij}^{pz} = t_{i-1,\,j-1}^{pz}, \quad S^P = 0 \\ &2)\quad x_2 = l \Rightarrow t_{i-1,\,j-1}^{pr} = t_{ij}^{pz}, \quad S^P = 0 \end{aligned}\,. \tag{3.41}$$

Variant 6 – for the latest dates.

$$t_{ij}^{pr} = t_{i-1,\,j-1}^{pz}\,. \tag{3.42}$$



This is defined as follows:

$$\bigwedge_{m,n\in P}\bigvee_{U\subset P} f(y_1)=m\wedge f(y_2)=n\Rightarrow m=n\,, \qquad (3.43)$$

where $U = \{m, n\}$.

The possibilities are:

$$\begin{aligned} &1)\quad y_1=m\Rightarrow t_{ij}^{pr}=t_{i-1,\,j-1}^{pz},\quad S^P=0\\ &2)\quad y_2=n\Rightarrow t_{i-1,\,j-1}^{pz}=t_{ij}^{pr},\quad S^P=0 \end{aligned}\,. \qquad (3.44)$$

The CPDCs which link jobs on neighboring work fronts i–1, i, taking into account next (in the technological order) jobs j–1, j, act on the time characteristics in an indirect way. They usually take on values > 0. In building practice situations arise when adjacent construction jobs (in a technological sequence) are regarded as “zero” jobs (they do not occur on a particular work front). Then the CPDCs act in an indirect way, e.g. for job t_{ij}, $t_{i-1,j} = 0$ and $t_{i,j-1} = 0$, then $S^P = 0$, and one of the above variants appears.

3.2.2. Algorithm for scheduling jobs with CPDCs taken into account

Basic assumptions

The investment task is given as a set of building structures $O = \{O_1, O_2, ..., O_n\}$. They are to be realized by construction firms (gangs) $B = \{B_1, B_2, ..., B_p\}$.The planned technological processes form this sequence $P_i = <P_{i1}, P_{i2}, ..., P_{im}>$. The technological processes should be carried out in this fixed order $P_{i1} \prec P_{i2} \prec ... \prec P_{im}$. Constraints, expressed through CPDCs, are imposed on the realization $t_{i-1,j-1} \prec t_{ij} \prec t_{i+1,j+1}$.

The scheduling consists in synchronizing construction jobs of one kind to ensure the shortest lead time for the investment task, i.e. $T \mapsto \min$.

Scheme of algorithm

The time characteristics of construction jobs are calculated on the basis of the matrix of job durations.

Step 1 – fix the dates of commencement and completion of the job with index 1,1, i.e. $t_{1,1}^{wr} = 0$, $t_{1,1}^{wz} = t_{1,1}^{wr} + t_{1,1}$.

Step 2 – fix the date of commencement of jobs $t_{2,2}^{wr}$, assuming that $t_{2,2}^{wr} = t_{1,1}^{wz}$, and then the date of completion of job $t_{2,2}^{wz} = t_{2,2}^{wr} + t_{2,2}$. The commencement and completion dates for the other matrix elements are fixed in a similar way.

Step 3 – for the jobs in the second and next diagonals of the matrix fix the dates of commencement of the first jobs in the technological sequence. This consists in calculating the time between the dates of commencement of the jobs in neighboring technological sequences (t_{rj}). Create two-column submatrices encompassing the adjacent technological sequences: $t_{rj} = \max\{(t_{i-1,j-1}); (t_{i,j} + t_{i-1,j-1} - t_{i-1,j}); (t_{i-1,j-1} + t_{i,j} + t_{i+1,j+1} - t_{i,j-1} - t_{i+1,j}) ...\}$, where: $i = 1, 2, ..., n$; j = 1,2, ..., m.

Assuming that $t_{r,j} = t_{i-1,j}^{wr}$, calculate the other time characteristics of the construction jobs as above.

Step 4 – repeat the procedure until the set of technological sequences connected by the CPDCs in the matrix is exhausted.

Numerical example

A matrix of job (renovation of paint coatings) durations for four apartment buildings was constructed. The set of jobs determined on the basis of a survey includes: repairing the internal and external plaster, painting, and repairing the facade. The durations were written in the matrix:

$$\boldsymbol{\tau} = \begin{bmatrix} 9 & 6 & 7 & 4 \\ 7 & 4 & 10 & 4 \\ 12 & 9 & 11 & 6 \\ 6 & 4 & 5 & 3 \end{bmatrix}.$$

Matrix **τ**’ was constructed, taking into account CPDCs, to link organizationally the different jobs to be performed on the building structures, e.g. $t_{11} \prec t_{22} \prec t_{33} \prec t_{44}$. Then the time characteristics of the jobs were calculated.

Table 3.3. Time characteristics of the job

9 9	15 6	27 7	31 4
16 7	13 4	25 10	31 4
28 12	25 9	24 11	31 6
34 6	32 4	30 5	27 3

Auxiliary two-column submatrices were constructed to determine the appropriate commencement dates for the jobs in the organizational sequences.

$$\begin{vmatrix} 9 & 6 \\ 4 & 10 \\ 11 & 6 \\ 3 & \end{vmatrix} \quad \begin{vmatrix} 6 & 7 \\ 10 & 4 \\ 6 & \\ & \end{vmatrix} \quad \begin{vmatrix} 7 & 4 \\ 4 & \\ & \\ & \end{vmatrix} \quad \begin{vmatrix} 9 & 7 \\ 4 & 9 \\ 11 & 5 \\ 3 & \end{vmatrix} \quad \begin{vmatrix} 7 & 12 \\ 9 & 4 \\ 5 & \\ & \end{vmatrix} \quad \begin{vmatrix} 12 & 6 \\ 4 & \\ & \\ & \end{vmatrix}$$

$t_{r1} = 9 \quad t_{r2} = 11 \quad t_{r3} = 7 \quad t_{r4} = 9 \quad t_{r5} = 7 \quad t_{r6} = 12$

Table 3.4. Summary of calculation results of job durations in CPDC complex – perpendicular couplings

Project	**Diagonals**						
	1	2	3	4	5	6	7
A	0/9/9	9/6/15	20/7/27	27/4/31			
B	9/4/13	15/10/25			9/7/16		
C	13/11/24	25/6/31			16/9/25	16/12/28	
D	24/3/27				25/5/30	28/4/32	28/6/34

If the diagonals and the actions of the time couplings are formed starting from the last bottom element of the matrix, another organizational model can be obtained. The following two-column matrices are created:

$$\begin{vmatrix} 6 & 12 \\ & 4 \\ & \\ & \end{vmatrix} \mapsto 6 \qquad \begin{vmatrix} 12 & 7 \\ 4 & 9 \\ & 5 \\ & \end{vmatrix} \mapsto 12 \qquad \begin{vmatrix} 7 & 9 \\ 9 & 4 \\ 5 & 11 \\ & 3 \end{vmatrix} \mapsto 7$$

$$\begin{vmatrix} 9 & 6 \\ 4 & 10 \\ 11 & 6 \\ 3 & \end{vmatrix} \mapsto 9 \qquad \begin{vmatrix} 6 & 7 \\ 10 & 4 \\ 6 & \\ & \end{vmatrix} \mapsto 11 \qquad \begin{vmatrix} 7 & 4 \\ 4 & \\ & \\ & \end{vmatrix} \mapsto 4$$

Table 3.5. Time characteristics of jobs

34 9	40 6	52 7	56 4
25 7	38 4	50 10	56 4
18 12	34 9	49 11	56 6
6 6	22 4	39 5	52 3

Table 3.6. Summary of calculation results of job durations in CPDC complex – diagonal couplings

Project	**Diagonals**						
	1	2	3	4	5	6	7
A				25/9/34	34/6/40	45/7/52	52/4/56
B			18/7/25	34/4/38	40/10/50	52/4/56	
C		6/12/18	25/9/34	38/11/49	50/6/56		
D	0/6/6	18/4/22	34/5/39	49/3/52			

3.3. Variants of construction work organization method taking into account reverse CPDCs

Reverse CPDCs (acting in the direction opposite to that of CPDCs) link jobs of given kind *j* on given front *i* with the jobs following in technological order

j+1 on next front i+1. If we consider the possible actions of CPDCs we can infer by analogy that reverse CPDCs occur between the relevant jobs.

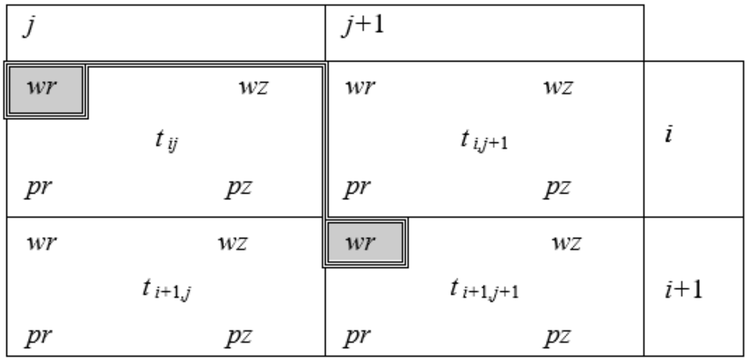
This can be exploited in practice to link jobs j and j+1 of different kinds on neighboring work fronts if technological constraints occur. A proper sequence of jobs must be established (e.g. erecting brick walls, plastering, painting, etc.).

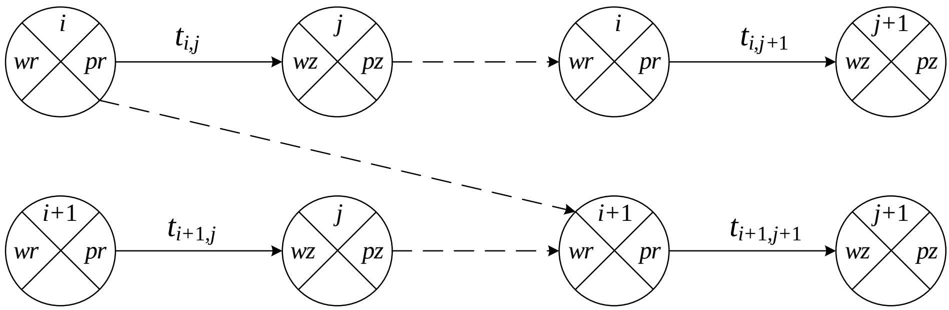
Reverse CPDCs whose value is zero impose an additional condition on jobs – the latter must be carried out in parallel or in turn. Thus they provide a basis for the creation of variants of work organization methods for construction jobs with complex internal structures: parallel, series, parallel-flow, series-flow and flow.

The following variants of the action of reverse CPDCs can be distinguished:

Variant 1 – the earliest job commencement dates.

$$t_{ij}^{wr} = t_{i+1,\,j+1}^{wr}\,. \tag{3.45}$$





This is defined as follows:

$$\bigwedge_{a,b\in z}\ \bigvee_{x\subset Z} f(x_1)=a \wedge f(x_2)=b \Rightarrow a=b\,, \tag{3.46}$$

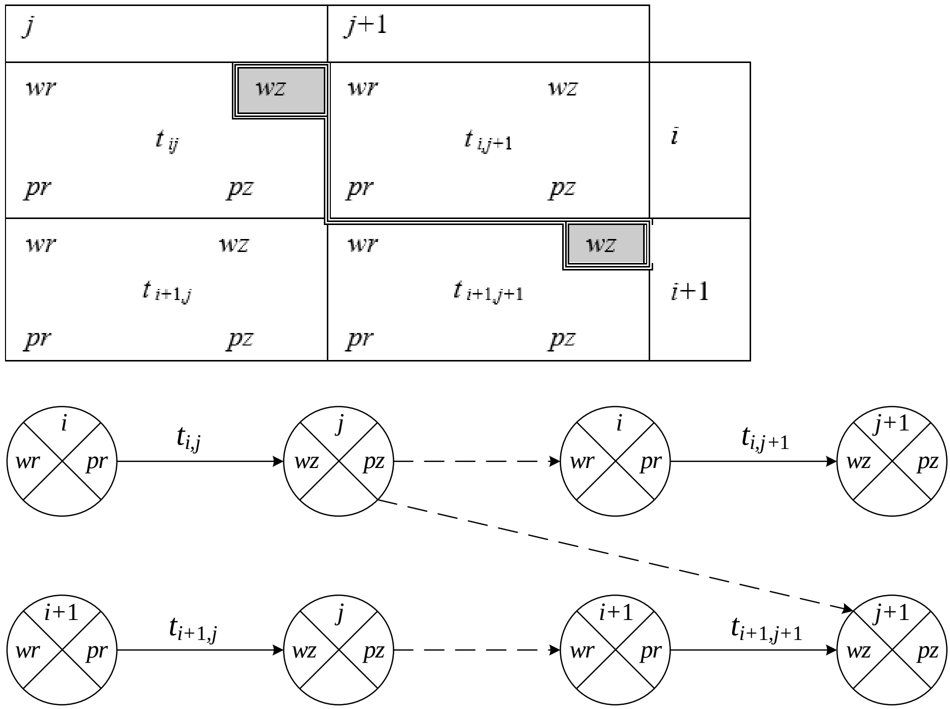
where $X = \{a, b\}$.

The possibilities are:

$$\begin{aligned} &1)\quad x_1 = a \Rightarrow t_{ij}^{wr} = t_{i+1,\,j+1}^{wr}, \quad S^{OP} = 0 \\ &2)\quad x_2 = b \Rightarrow t_{i+1,\,j+1}^{wr} = t_{ij}^{wr}, \quad S^{P} = 0 \end{aligned}\,. \tag{3.47}$$

Variant 2 – the earliest job completion dates.

$$t_{ij}^{wz} = t_{i+1,\,j+1}^{wz}\,. \tag{3.48}$$



This is defined as follows:

$$\bigwedge_{b,c\in p}\ \bigvee_{Y\subset P} f(y_1)=b \wedge f(y_2)=c \Rightarrow b=c\,, \tag{3.49}$$

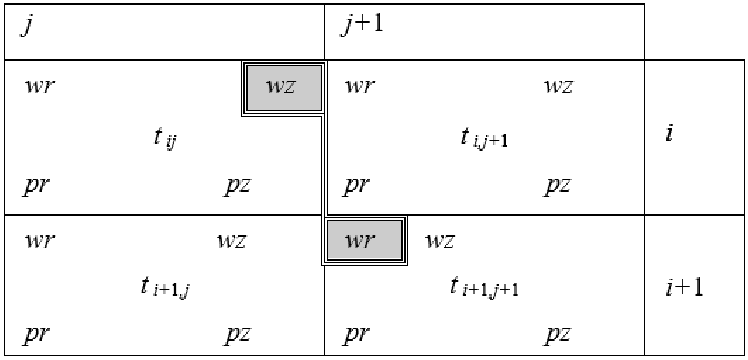
where $Y = \{b, c\}$.

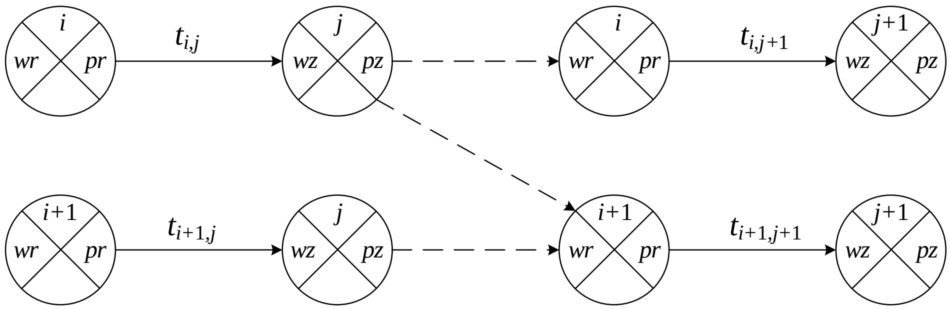
The possibilities are:

$$\begin{matrix} 1) \quad y_1 = b \Rightarrow t_{ij}^{wz} = t_{i+1,\,j+1}^{wz}, \quad S^{OP} = 0 \\ 2) \quad y_2 = c \Rightarrow t_{i+1,\,j+1}^{wz} = t_{ij}^{wz}, \quad S^{P} = 0 \end{matrix}\,. \tag{3.50}$$

Variant 3 – the earliest job commencement and completion dates.

$$t_{ij}^{wz} = t_{i+1,\,j+1}^{wr}\,. \tag{3.51}$$





This is defined as follows:

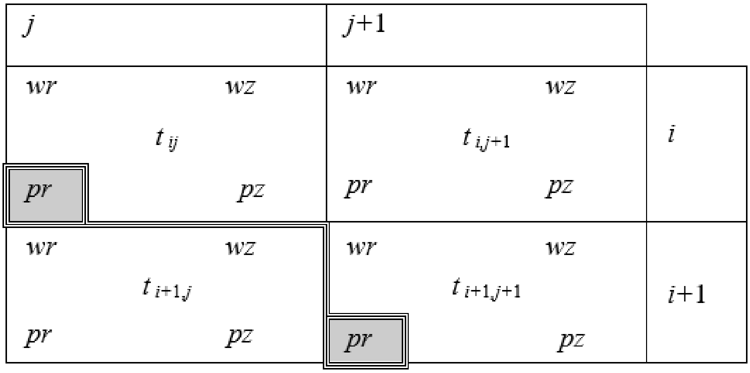
$$\bigwedge_{c,d\in U}\bigvee_{P\subset U} f(u_1)=c \wedge f(u_2)=d \Rightarrow c=d\,, \tag{3.52}$$

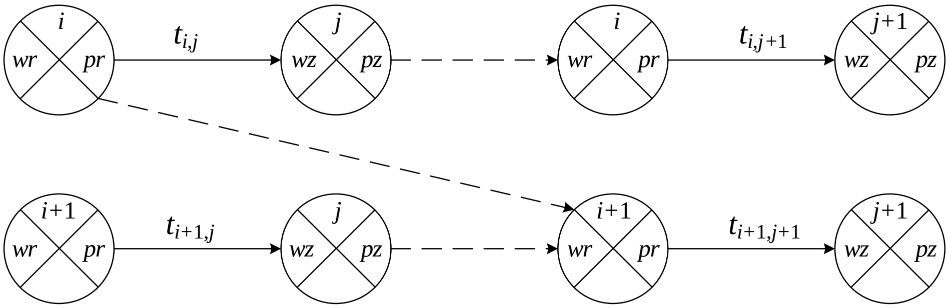
where $P = \{c, d\}$.
The possibilities are:

$$\begin{aligned} &1)\quad u_1=c \Rightarrow t_{ij}^{wz}=t_{i+1,\,j+1}^{wr}, \quad S^{OP}=0 \\ &2)\quad u_2=d \Rightarrow t_{i+1,\,j+1}^{wr}=t_{ij}^{wz}, \quad S^{P}=0 \end{aligned}. \tag{3.53}$$

Variant 4 – the latest job commencement dates.

$$t_{ij}^{pr}=t_{i+1,\,j+1}^{pr}\,. \tag{3.54}$$





This is defined as follows:

$$\bigwedge_{d,f\in W}\ \bigvee_{Z\subset W} f(z_1)=d \wedge f(z_2)=f \Rightarrow d=f\ , \tag{3.55}$$

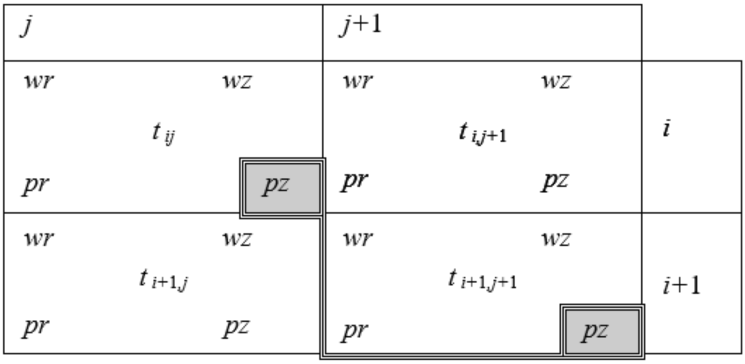
where $Z = \{d, f\}$.

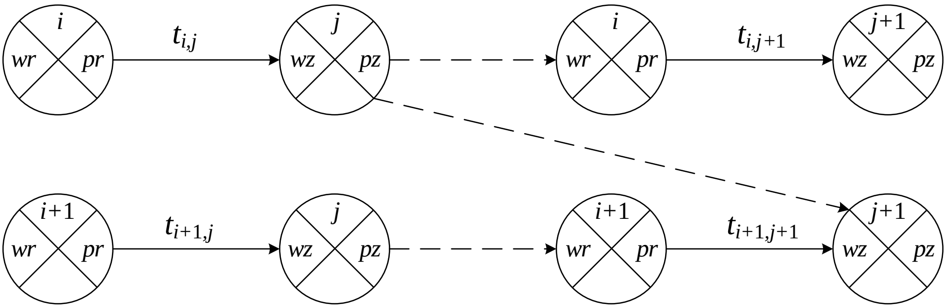
The possibilities are:

$$\begin{array}{ll} 1) & z_1 = d \Rightarrow t_{ij}^{pr} = t_{i+1,\,j+1}^{pr}, \quad S^{OP} = 0 \\ 2) & z_2 = f \Rightarrow t_{i+1,\,j+1}^{pr} = t_{ij}^{pr}, \quad S^{P} = 0 \end{array}. \tag{3.56}$$

Variant 5 – the latest job commencement dates.

$$t_{ij}^{pz} = t_{i+1,\,j+1}^{pz}\,. \tag{3.57}$$





This is defined as follows:

$$\bigwedge_{g,h\in K}\ \bigvee_{W\subset K} f(x_1)=g \wedge f(x_2)=h \Rightarrow g=h\,, \tag{3.58}$$

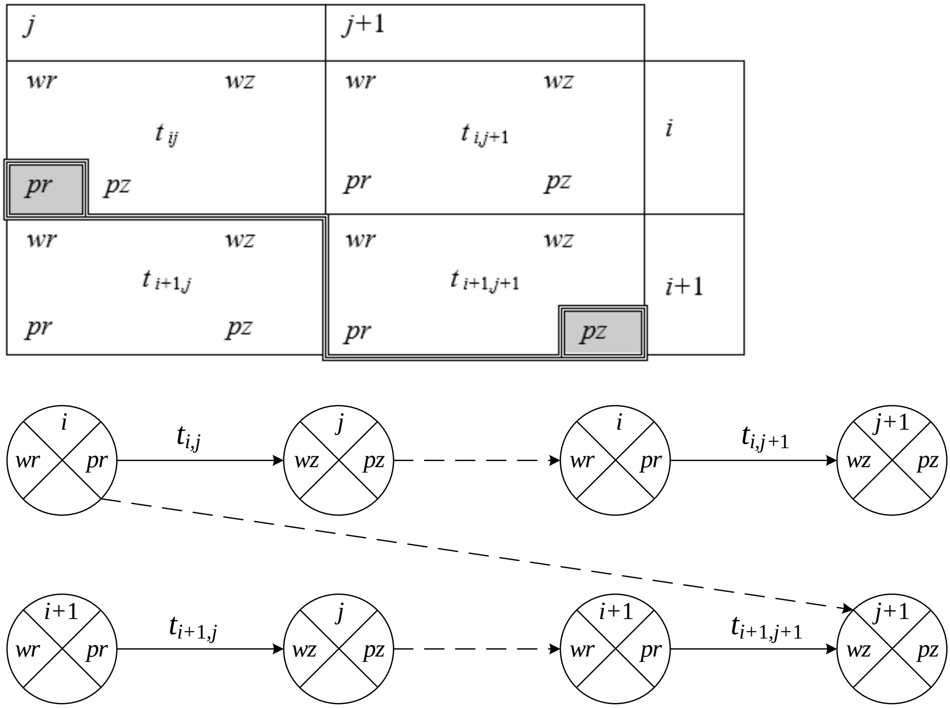
where $W = \{g, h\}$.

The possibilities are:

$$\begin{array}{l} 1)\quad x_1 = g \Rightarrow t_{ij}^{pz} = t_{i+1,\,j+1}^{pz}, \quad S^{OP} = 0 \\ 2)\quad x_2 = h \Rightarrow t_{i+1,\,j+1}^{pz} = t_{ij}^{pz}, \quad S^{P} = 0 \end{array}. \tag{3.59}$$

Variant 6 – the latest job commencement and completion dates.

$$t_{ij}^{pr} = t_{i+1,\,j+1}^{pz}\,. \tag{3.60}$$



This is defined as follows:

$$\bigwedge_{k,l\in P}\bigvee_{U\subset P} f(y_1)=k \wedge f(y_2)=l \Rightarrow k=l\,, \tag{3.61}$$

where $U = \{k, l\}$.

The possibilities are:

$$\begin{array}{l} 1)\quad y_1=k \Rightarrow t_{ij}^{pr}=t_{i+1,\,j+1}^{pz}, \quad S^{OP}=0 \\ 2)\quad y_2=l \Rightarrow t_{i+1,\,j+1}^{pz}=t_{ij}^{pr}, \quad S^{P}=0 \end{array}. \tag{3.62}$$

The above relations between technologically consecutive jobs actually occur in practice. Situations in which relations link technologically ordered jobs on neighboring front often arise. They can link job commencement and completion dates, both the early and late ones. An example is the common in industrial building relation between the date of completion of roof waterproofing and the starting of flooring. This technological constraint is due to the need to protect the floor work against precipitation and to obtain a suitable temperature (> 5°C) during concreting. Another example is the relation between roof drainage, flooring and plumbing. One could give many such examples.

4. Methods of organizing construction work with series internal structure

Some problems of planning construction work with complex internal structure are described in [7, 10, 38, 40, 44, 51, 75, 76]. They are connected with the synchronization of jobs, the use of different work organization methods in a complex, the parallelism of jobs of one kind and the parallelism of complexes of jobs of different kinds.

If the classification of construction work organization methods given in [1–3] and the systematics of terms and their definitions given in [79, 81] are used, one can determine the area of the identified relations representing the interactions of time couplings.

Linking different kinds of jobs carried out by different work organization methods is a key problem in scheduling. The use of couplings between, for example, realization means and work fronts is described in [1–3, 81, 83]. But if the couplings are to be used independently in one work complex, one must apply algorithms ensuring faultless synchronization of jobs [43, 49].

4.1. Synchronization of construction jobs in complex by means of different organization methods

Basic assumptions

A set of work fronts (building sites, building structures): $O = \{O_i: i = 1, 2, ..., n\}$, on which a sequence of technological processes forming set $R = \{R_j: j = 1,2,..., m\}$, must be carried out, is given. The processes in set R should be carried out in a fixed technological order: $R_1 < R_2 <...< R_m$. The particular processes will be realized by construction firms (work teams) forming set $B = \{B_j\}$, where $j = 1,..., m$. Groups of technological processes from set R will be carried out using different work methods for which job duration matrices: $\mathbf{T}_1 = [t_{ij}]$; $\mathbf{T}_2 = [t_{i,j+1}]$, where: t_{ij} – the duration of the j-th job on the i-the building structure, were constructed. The problem is to synchronize the jobs so that: $T \rightarrow \min$.

Scheme of algorithm

The problem of linking work complexes (formed by means of different methods) was solved for the minimum time criterion ensuring the shortest gang work downtimes at the junction of some work flows.

Step 1 – having determined the partial flows in the zone of their junction, create a set of the latest dates of completion of jobs on the fronts in the last partial flow of the first flow. The elements of the set form a sequence of numerical values in the order determined through the scheduling of tasks:

$$t^k = \{t^{pz}_{1m},\ t^{pz}_{2m},...,\ t^{pz}_{nm}\}\ , \tag{4.1}$$

where:

k – the number of a flow,

t^{pz}_{lm} – the latest date of completion of a process on the i-th front,

n – a number of work fronts,

m – the last process in flow k.

From the elements of the matrix column corresponding to the first partial flow of the second flow create a set of estimated lead times for the jobs in the previously established order:

$$t^{k+1} = \{t_{1,m+1},\ t_{2,m+1},...,\ t_{i,m+1},\ t_{n,m+1}\}\ . \tag{4.2}$$

Having ordered the jobs on the building lots (structures), one often finds their order in the two flows is different. Then it becomes necessary to adjust the work organization method and the established sequences of realization of the building structures in the flows. This can be done through the next steps in the algorithm.

Step 2 – fix the dates of completion for the jobs in the first partial flow of the second flow, taking into account the preceding jobs. Create a set of dates of completion for the jobs on the appropriately ordered building structures. The differences between the job completion date values for the last partial process (partial flow) and the appropriate job commencement date values for the first partial process (the first partial flow) of the second flow form a set of minimum breaks needed to ensure collisionless work of the crews.

$$U = \left\{U_1,...,U_2,...,U_i,...,U_n\right\}\ . \tag{4.3}$$

$$U_i = t^{pz}_{i,m} - \sum_{k=1}^{p} t_{i,m+1}\ ,\ k = 1, 2,..., p,\ p = i - 1. \tag{4.4}$$

where:

$\sum_{i=1}^{p} t_{i,m+1}$ – the sum of the lead times for the jobs on the preceding building lots (structures) in the established order.

Step 3 – having determined the required breaks in the work of the gangs on the building lots (structures) – U_i, determine the adjusted dates of commencement of jobs on the appropriate fronts for the maximum numerical value from set U_i. For the first job in the first partial flow in the second flow we get:

$$t^{wr}_{i,m+1} = U_i + \sum_{k=1}^{p} t_{i,m+1}\ . \tag{4.5}$$

<u>Illustrative application of algorithm</u>

Matrix **T** of lead times for jobs on fronts corresponding to apartment buildings was constructed. The construction jobs were consolidated to form eight partial processes (partial flows), i.e. humus removal, earth work, foundation work, assembly, corrections, plaster work, masonry work and floor and parquet work. The matrix was constructed for a complex of buildings in a housing estate in Wrocław, taking into account the standards in force and the contractor's resources.

$$\mathbf{T} = \begin{vmatrix} 2 & 7 & 5 & 54 & 12 & 13 & 15 & 13 \\ 5 & 20 & 15 & 141 & 34 & 37 & 35 & 38 \\ 2 & 7 & 5 & 54 & 12 & 13 & 15 & 13 \\ 5 & 20 & 15 & 141 & 34 & 37 & 25 & 18 \end{vmatrix}$$

The realization of the first four partial processes was planned using the flow organization method with zero couplings between realization means and the other processes were planned by the method which takes into account both couplings between realization means and couplings between work fronts. Computations yielded submatrices with new sequences of realization of the buildings.

$$\mathbf{T}_1 = \begin{vmatrix} 2 & 7 & 5 & 54 \\ 5 & 20 & 15 & 141 \\ 5 & 20 & 15 & 141 \\ 2 & 7 & 5 & 54 \end{vmatrix} \qquad \mathbf{T}_2 = \begin{vmatrix} 24 & 37 & 35 & 38 \\ 12 & 13 & 15 & 13 \\ 12 & 13 & 15 & 13 \\ 34 & 37 & 35 & 38 \end{vmatrix}$$

Assuming the order of the rows in matrix **T** as the initial one, optimization was performed and new sequences of realization of the buildings were obtained: 1, 2, 4, 3 for matrix $\mathbf{T_1}$ and 2, 1, 3, 4 for matrix $\mathbf{T_2}$.

Step 1 – the sequence of job completion date values is t^1 = [88, 229, 370, 424], and the sequence of estimated lead times for the jobs in the first process of the second flow is t^2 = [34, 12, 12, 34].

Step 2

Fixed sequence of realization of building structures	t_{im}^{pz}	Fixed sequence of realization of building structure	$t_{i,m+1}$	Auxiliary job commencement dates for sequence	U_i
1	88	2	34	1÷34	54
2	229	1	12	2÷0	229
4	370	3	12	4÷24 + 12 + 12	312
3	424	4	34	3÷34 + 12	378

$$U_{i,\max} = 378$$

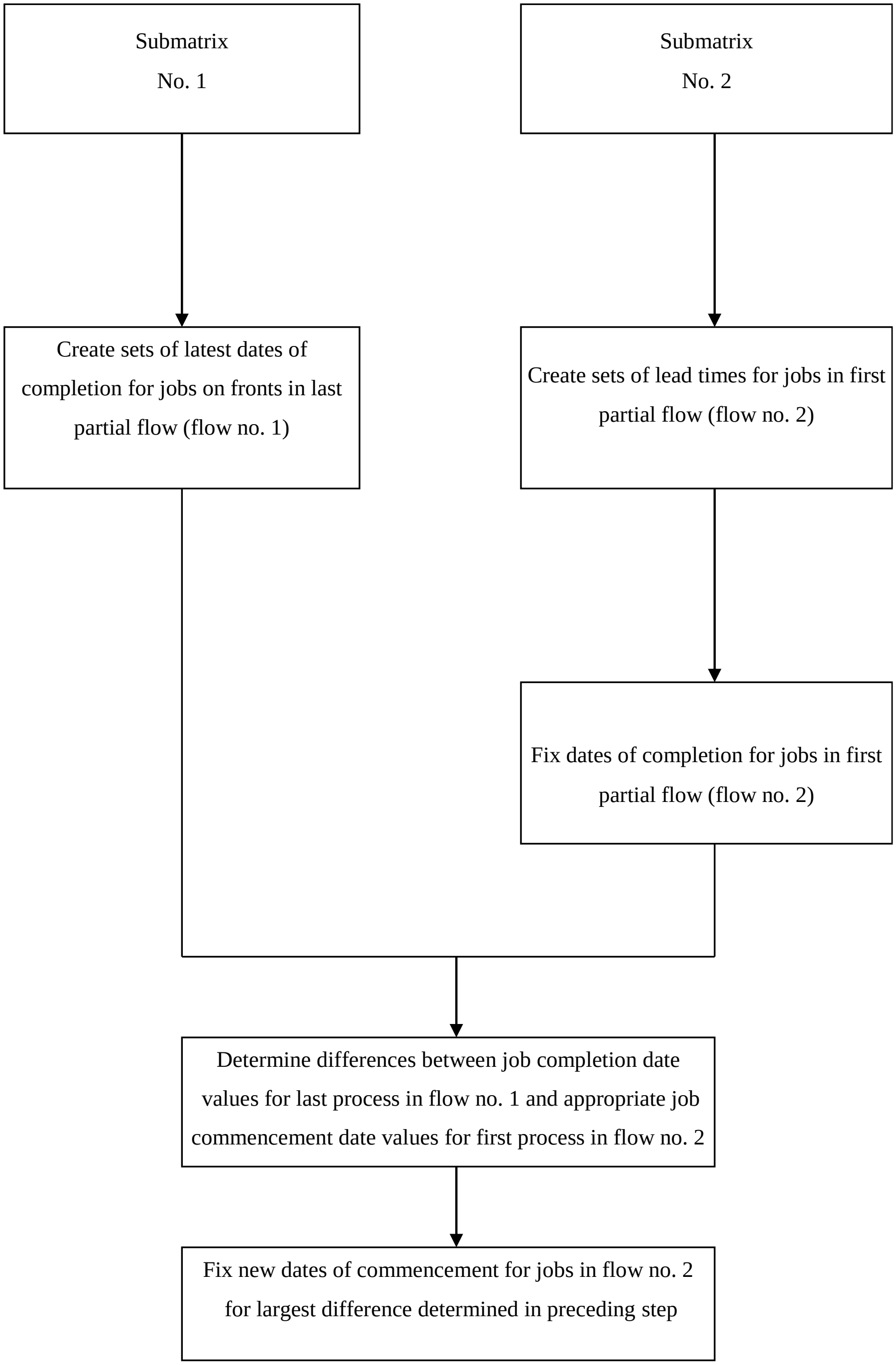


Figure 4.1. Scheme of algorithm for synchronization of work complexes (or their parts) formed by different work organization methods

Step 3 – fix the dates of commencement and completion for the jobs in the first partial complex.

Fixed sequence of realization of building structure	$t^{wr}_{i,m+1}$	$t^{wz}_{i,m+1}$
2	378 + 0 = 378	378 + 34 = 412
1	378 + 34 = 412	412 + 12 = 424
3	378 + 34 + 12 = 424	424 + 12 = 436
4	378 + 34 + 12 + 12 = 436	436 + 34 = 470

4.2. Variants of construction work organization method taking into account CPDCs and zero reverse diagonal couplings

Within the area of organization methods enabling the consecutive or parallel realization of construction jobs there is a set of possible intermediate variants of the methods. The possible time couplings and their combinations can be used to create new ways of synchronizing construction jobs and new tools for scheduling them.

If one takes into account zero diagonal couplings and CPDCs, representing the technological relationships within the same kind of jobs, a qualitatively new way of organizing construction jobs becomes apparent. By shifting the jobs on a given front by 1, 2, … to *n*-2 we bring the organizational relationships closer to the properties of the consecutive execution method. One can notice that the shifts can apply to jobs in relation to work fronts and to work fronts in relation to kinds of jobs.

By employing CPDCs we shift the jobs in the job duration matrix so that the jobs will be carried out collisionless on different fronts of neighboring building structures. The couplings between realization means, which ran vertically in the matrix, now run obliquely and in the extreme case lead to the formation of chains of jobs of one kind.

When constructing a job duration matrix with values relating to work fronts placed in its rows, we can create several variants.

Variant 1 – the earliest dates:

$$t^{wr}_{ij} = \max\{t^{wr}_{i-1,j+1}; t^{wz}_{i-1,j-1}\}, \tag{4.6}$$

and:

$$t^{wz}_{i-1,j-1} < t^{wr}_{i-1,j+1}, \tag{4.7}$$

i.e.:

$$t^{wr}_{ij} = t^{wr}_{i-1,j+1}. \tag{4.8}$$

Jobs of one kind performed on neighboring building structures can be carried out in series depending on the conditions: $t_{i-1,j-1} < t_{ij} < t_{i+1,j+1}$. This is defined as follows:

$$\bigwedge_{x\in W}\bigvee_{x\subset W} f(z_1)=x \vee f(z_2)=x \Leftrightarrow z_2 < z_1 > z_3 \vee Z < z_2 > z_3 \,, \qquad (4.9)$$

where: $X = \{z_1, z_2\}$.
The possibilities are:

$$\begin{aligned} z_2 \le z_1 \Rightarrow z_1 = X, &\quad \text{i.e.} \quad t^{wr}_{i-1,j+1} = t^{wr}_{ij} \\ z_1 < z_2 \Rightarrow z_2 = X, &\quad \text{i.e.} \quad t^{wr}_{i-1,j-1} < t^{wr}_{ij} \end{aligned} \, . \qquad (4.10)$$

wr *wz* $t_{i\text{-}1,j\text{-}1}$ *pr* *pz*	*wr* *wz* $t_{i\text{-}1,j}$ *pr* *pz*	*wr* *wz* $t_{i\text{-}1,j+1}$ *pr* *pz*	*i*-1
	wr *wz* t_{ij} *pr* *pr*	*wr* *wz* $t_{i,j+1}$ *pr* *pz*	*i*
		wr *wz* $t_{i+1,j+1}$ *pr* *wz*	*i*+1

Formula (4.6) expresses the relationship between the jobs in the matrix diagonals, including the action of CPDCs linking jobs of one kind shifted in series on work fronts.

Variant 2 – the earliest dates:

$$t^{wr}_{ij} = t^{wz}_{i-1,j+1}\,, \qquad (4.11)$$

and:

$$t^{wr}_{ij} > t^{wz}_{i-1,j-1}\,. \qquad (4.12)$$

Formula (4.11) defines the relationship between the completion of the preceding job and the following job on two different building structures. It is also assumed that jobs of one kind will be performed consecutively, forming sequences of jobs (the consecutive execution method). A shift of jobs of one kind on a front changes the character of the action of the diagonal couplings and links jobs of one kind, i.e. job j–1, job j and job j+1 are jobs carried out in one technology on different building structures.

wr *wz* $t_{i\text{-}1,j\text{-}1}$ *pr* *pz*	*wr* *wz* $t_{i\text{-}1,j}$ *pr* *pz*	*wr* *wz* $t_{i\text{-}1,j+1}$ *pr* *pz*	*i*-1
	wr *wz* t_{ij} *pr* *pr*	*wr* *wz* $t_{i,j+1}$ *pr* *pz*	*i*
		wr *wz* $t_{i+1,j+1}$ *pr* *wz*	*i+1*

This is defined as follows:

$$\bigwedge_{u\in W}\bigvee_{c\subset W} f(z_1)=u \vee f(z_2)=u \Leftrightarrow z_1 \geq z_2\,, \tag{4.13}$$

where: $C = \{z_1, z_2\}$.

The possibilities are:

$$\begin{array}{l} z_1 \geq z_2 \Rightarrow z_1 = u, \quad \text{i.e.} \quad t_{ij}^{wz} = t_{i-1,\,j-1}^{wz}, S^D = 0 \\ z_2 > z_1 \Rightarrow z_2 = x, \quad \text{i.e.} \quad t_{i-1,\,j-1}^{wz} < t_{ij}^{wr}, S^P > 0 \end{array}. \tag{4.14}$$

Variant 3 – the earliest dates:

$$t_{ij}^{wz} = t_{i-1,\,j+1}^{wr}\,, \tag{4.15}$$

and:

$$t_{ij}^{wz} = t_{i-1,\,j-1}^{wz}\,. \tag{4.16}$$

wr *wz* $t_{i\text{-}1,j\text{-}1}$ *pr* *pz*	*wr* *wz* $t_{i\text{-}1,j}$ *pr* *pz*	*wr* *wz* $t_{i\text{-}1,j+1}$ *pr* *pz*	*i*-1
	wr *wz* t_{ij} *pr* *pr*	*wr* *wz* $t_{i,j+1}$ *pr* *pz*	*i*
		wr *wz* $t_{i+1,j+1}$ *pr* *wz*	*i*+1

This is defined as follows:

$$\bigwedge_{x \in W} \bigvee_{E \subset W} f(z_1) = x \vee f(z_2) = x \Leftrightarrow z_1 \geq z_2 \,, \tag{4.17}$$

where: $E = \{z_1, z_2\}$.
The possibilities are:

$$\begin{aligned} & z_1 \geq z_2 \Rightarrow z_1 = x, \quad \text{i.e.} \quad t^{wr}_{i-1,\,j+1} = t^{wz}_{ij}\,, S^P = 0 \\ & z_2 < z_1 \Rightarrow z_2 = x, \quad \text{i.e.} \quad t^{wz}_{i-1,\,j-1} = t^{wz}_{ij}\,, S^P > 0 \end{aligned}. \tag{4.18}$$

Formula (4.15) defines the diagonal relationship between jobs of different kinds performed on different work fronts, linking not proximate jobs but every other job. At the same, jobs of one kind still form a sequence, but a shifted one. Such a situation sometimes arises when jobs comprising parts of building structures appear on the latter. The situation can be represented by CPDCs and diagonal couplings with values higher than or equal to zero.
Variant 4 – the earliest dates:

$$t^{wz}_{ij} = t^{wz}_{i-1,\,j-1}\,, \tag{4.19}$$

and:

$$t^{wz}_{ij} > t^{wz}_{i-1,\,j-1}\,. \tag{4.20}$$

wr *wz* $t_{i\text{-}1,j\text{-}1}$ *pr* *pz*	*wr* *wz* $t_{i\text{-}1,j}$ *pr* *pz*	*wr* *wz* $t_{i\text{-}1,j+1}$ *pr* *pz*	*i*-1
	wr *wz* t_{ij} *pr* *pr*	*wr* *wz* $t_{i,j+1}$ *pr* *pz*	*i*
		wr *wz* $t_{i+1,j+1}$ *pr* *pz*	*i*+1

This is defined as follows:

$$\bigwedge_{z \in W} \bigvee_{Z \subset W} f(z_1) = a \vee f(z_2) = b \Leftrightarrow z_1 \geq z_2 \,, \tag{4.21}$$

where: $E = \{z_1, z_2\}$.
The possibilities are:

$$\begin{aligned} & z_1 \geq z_2 \Rightarrow z_1 = a, \quad \text{i.e.} \quad t^{wz}_{i-1,\,j+1} = t^{wz}_{ij}\,, S^P = 0 \\ & z_2 < z_1 \Rightarrow z_2 = b, \quad \text{i.e.} \quad t^{wz}_{i-1,\,j-1} = t^{wz}_{ij}\,, S^P > 0 \end{aligned}. \tag{4.22}$$

Formula (4.19) while taking the above constraint into account links jobs, interrelating their earliest dates of completion. The diagonal relationship makes it possible to complete simultaneously jobs of different kinds on neighboring fronts. This applies not to jobs that follow one another other but to every other job. CPDCs make it possible to link jobs of one kind performed in sequence on adjacent work fronts.

Variant 5 – the latest dates:

$$t_{ij}^{pr} = t_{i-1,j+1}^{pr}, \tag{4.23}$$

and:

$$t_{ij}^{pr} > t_{i-1,j-1}^{pr}. \tag{4.24}$$

wr *wz* $t_{i\text{-}1,j\text{-}1}$ *pr* *pz*	*wr* *wz* $t_{i\text{-}1,j}$ *pr* *pz*	*wr* *wz* $t_{i\text{-}1,j+1}$ *pr* *pz*	*i*-1
	wr *wz* t_{ij} *pr* *pr*	*wr* *wz* $t_{i,j+1}$ *pr* *pz*	*i*
		wr *wz* $t_{i+1,j+1}$ *pr* *pz*	*i*+1

This is defined as follows:

$$\bigwedge_{x \in W} \bigvee_{A \subset W} f(x_1) = a \vee f(x_2) = b \Leftrightarrow x_1 \geq x_2, \tag{4.25}$$

where: $A = \{x_1, x_2\}$.

The possibilities are:

$$\begin{aligned} x_1 \geq x_2 \Rightarrow x_1 = a, \quad \text{i.e.} \quad t_{i-1,j+1}^{pr} = t_{ij}^{pr}, S^D = 0 \\ x_2 < x_1 \Rightarrow x_2 = b, \quad \text{i.e.} \quad t_{i-1,j-1}^{pr} < t_{ij}^{pr}, S^P > 0 \end{aligned}. \tag{4.26}$$

Formula (4.23) allows one to take into account zero diagonal couplings linking jobs (of different kinds) written in the diagonals of the matrix. This applies to jobs that follow immediately one another. CPDCs guarantee the realization of jobs of one kind carried out successively on neighboring work fronts.

Variant 6 – the latest dates:

$$t_{ij}^{pz} = t_{i-1,j+1}^{pz}, \tag{4.27}$$

and:

$$t_{ij}^{pz} > t_{i-1,j-1}^{pz}. \tag{4.28}$$

This is defined as follows:

$$\bigwedge_{u\in P}\bigvee_{U\subset P} f(u_1)=u \vee f(u_2)=u \Leftrightarrow u_1 \geq u_2\,, \tag{4.29}$$

where: $U = \{u_1, u_2\}$.

wr *wz* $t_{i\text{-}1,j\text{-}1}$ *pr* *pz*	*wr* *wz* $t_{i\text{-}1,j}$ *pr* *pz*	*wr* *wz* $t_{i\text{-}1,j+1}$ *pr* *pz*	*i*-1
	wr *wz* t_{ij} *pr* *pz*	*wr* *wz* $t_{i,j+1}$ *pr* *pz*	*i*
		wr *wz* $t_{i+1,j+1}$ *pr* *pz*	*i*+1

The possibilities are:

$$\begin{array}{l} u_1 \geq u_2 \Rightarrow u_1 = u, \quad \text{i.e.} \quad t_{ij}^{pz} = t_{i-1,\,j+1}^{pz}, S^D = 0 \\ u_2 < u_1 \Rightarrow u_2 = u, \quad \text{i.e.} \quad t_{i-1,\,j-1}^{pz} > t_{ij}^{pz}, S^P > 0 \end{array}. \tag{4.30}$$

Formula (4.27) links jobs located in the matrix diagonals, ensuring the simultaneous completion of jobs of different kinds on different work fronts. This applies to consecutive jobs in a technological sequence. CPDCs guarantee the realization of jobs of one kind on neighboring fronts. Similarly as the above cases, this variant takes into account technological and organizational constraints but it links the latest job completion dates.

4.3. Algorithms for scheduling construction work

<u>Basic assumptions</u>

An investment task in the form of a set of building structures $O = \{O_i$: $i = 1, 2, ..., n\}$ which are to be realized by work teams (construction firms) $B = \{B_p$: $p = 1, 2, ..., p\}$ is given. The following technological processes are to be performed on each building structure. $P_i = \langle P_j, P_{i2}, ..., P_{im}\rangle$.

They should be carried out in a prescribed technological order, i.e. each process P_{ik} can be performed after process $P_{i,k-1}$ and before process $P_{i,k+1}$ are carried out. The following conditions are imposed on the realization of the building structures:

1. Work teams (construction firms) move on from one building structure to another in a fixed order.

2. Construction processes should be carried out consecutively.
3. The value of the diagonal couplings should be equal to zero.
4. Process P_{ik} can on given building structure O_i can be started after process $P_{i,k-1}$ has been completed on this building structure.

The problem is to determine such a value of the time characteristics which would eliminate technological collisions and ensure the shortest lead time: $T \rightarrow \min$.

Scheme of algorithm

An organizational model in which the basic time characteristics of the jobs should be determined is constructed according to the principles described below. The computations are based on a job duration matrix.

Step 1 – determine the matrix diagonals, taking into account the CPDCs and the diagonal couplings. Fix the dates of commencement and completion of the jobs which occur at the beginning and end in the successive diagonals. This applies to all the jobs on the first work front.

Step 2 – fix the commencement dates for the jobs in a diagonal where the next jobs appear, using this relation:

$$t_{2,j}^{wr} = t_{1,j}^{wz} , \tag{4.31}$$

and then:

$$t_{2,j}^{wr} + t_{2,j} = t_{1,j}^{wz} , \tag{4.32}$$

and:

$$t_{2,j}^{wz} = t_{2,j+1}^{wr} . \tag{4.33}$$

Fix the commencement and completion dates for the jobs in the second work front.

Step 3 – fix the commencement of the jobs in a diagonal where double jobs occur and apply the procedure of fixing commencement and completion dates for the other jobs on the building structure.

Step 4 to *n* – in a similar way determine the time characteristics for the jobs on the next building structures until all the values are determined.

Illustrative application of algorithm

A job duration matrix for four building structures was constructed. The set of jobs to be carried out included: repairing the interior and exterior plasters, painting and repairing the facades. The relevant times were written in matrix **T**.

$$\mathbf{T} = \begin{vmatrix} 9 & 6 & 7 & 4 \\ 7 & 4 & 10 & 4 \\ 12 & 3 & 11 & 6 \\ 6 & 4 & 5 & 3 \end{vmatrix}$$

The applied algorithm yielded the time characteristics of the jobs given in the table below.

Table 4.1. Time characteristics of construction jobs

Project	Diagonals												
	1	2	3	4	5	6	7	8	9	10	11	12	13
I	0/9/9	9/6/15	15/7/22	22/4/29									
II			29/7/36	36/4/40	30/0/50	40/0/50	50/4/54						
III							54/12/66	66/9/75	75/11/86	86/6/93			
IV										92/6/98	98/4/102	102/5/107	107/3/110

The number in the fields stand for:

- a job commencement date,
- a job duration,
- a job completion date.

5. Methods of organizing construction works with sequence-stream internal structure

5.1. Construction work organization method variants incorporating diagonal couplings linking works of different kinds

This chapter pursues the subject introduced in several studies concerned with the application of time couplings to organizational models. It opens a series of publications devoted to the modification of construction work organization methods by means of time couplings which express technological and organizational relationships.

As a result of the application of diagonal and inversely diagonal couplings to organizational models it has become possible to create groups of methods having properties characteristic for stream methods (stages of: expansion, full run and diminishment of the stream) and for consecutive execution methods (realization of sequentially occurring work groups). A group of construction work organization methods having sequence-stream structure can be distinguished. Works on a given front (in a row of the work duration matrix) can be rearranged so that in the extreme case they will occur sequentially. The positions of the works in matrix rows prior to the above case will be arranged in a way characteristic for sequence-stream methods.

The basic property of this group of construction work organization methods is the effect of diagonal couplings on various works. In addition, the imposed zero-value inversely diagonal couplings ensure sequential technological dependence of the works.

If we write a work duration matrix in the RP system, with kinds of work (technological processes) put on the Y-axis, we obtain a new way in which time couplings can function. Diagonal couplings in a matrix link works situated on adjacent work fronts. They are equivalent to couplings between work fronts – time couplings peculiar to classic stream work organization methods.

Diagonal couplings tie construction works to one another so that sequences of various works carried out successively on the particular work fronts are formed, i.e.:

$$t_{i-1,\,j-1} \prec t_{ij} \prec t_{i+1,\,j+1}\,. \tag{5.1}$$

Zero-value inversely diagonal couplings imposed on works link a given work with the preceding work in a technological order on a given work front and the next work front, i.e.:

$$t_{ij} \prec t_{i+1,\,j-1}\,. \tag{5.2}$$

Such conditions determine a new kind of links (of a technological nature) between works, ensuring the technologically proper sequence of various works.

Since time couplings can affect differently the earliest and latest work starting and completion dates, there exists a set variants of construction work organization methods which incorporates the above kinds of time couplings.

Variant 1 – For the earliest dates:

$$t_{ij}^{wr} = t_{i+1,\,j-1}^{wr}\,, \tag{5.3}$$

and:

$$t_{ij}^{wr} > t_{i-1,\,j-1}^{wr}\,. \tag{5.4}$$

Definition:

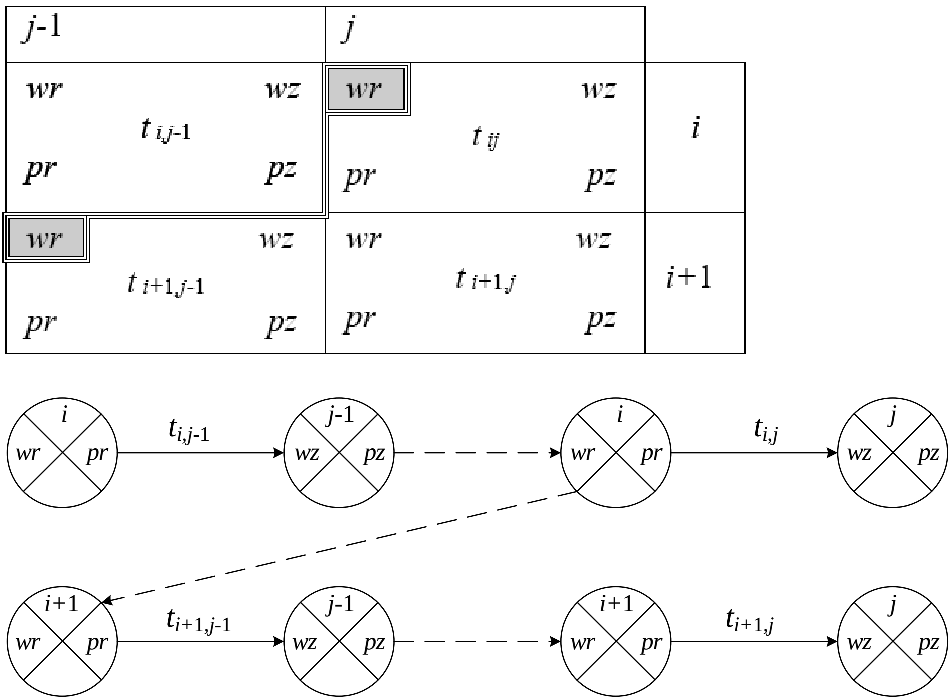
$$\bigwedge_{x \in W} \bigvee_{X \subset W} f(x_1) = a \vee f(x_2) = b \Leftrightarrow x_1 \geq x_2\,, \tag{5.5}$$

where: $X = \{x_1, x_2\}$.

Possibilities:

$$\begin{aligned} x_1 = x_2 \Rightarrow x_1 = a, &\quad \text{i.e.} \quad t_{ij}^{wr} = t_{i+1,\,j-1}^{wr},\, S^P = 0 \\ x_2 < x_1 \Rightarrow x_2 = b, &\quad \text{i.e.} \quad t_{ij}^{wr} > t_{i-1,\,j-1}^{wr},\, S^P > 0 \end{aligned}\,. \tag{5.6}$$

Formula (5.3) ensures the simultaneous commencement of successive (in the order determined by the technology) works *j*, *j*-1 on adjacent work fronts *i*, *i*+1. This relation secures the relationships between works of different kinds on work fronts.



Variant 2 – For the earliest dates.

$$t_{ij}^{wz} = t_{i+1,\,j-1}^{wz}, \tag{5.7}$$

and:

$$t_{ij}^{wz} > t_{i-1,\,j-1}^{wz}. \tag{5.8}$$

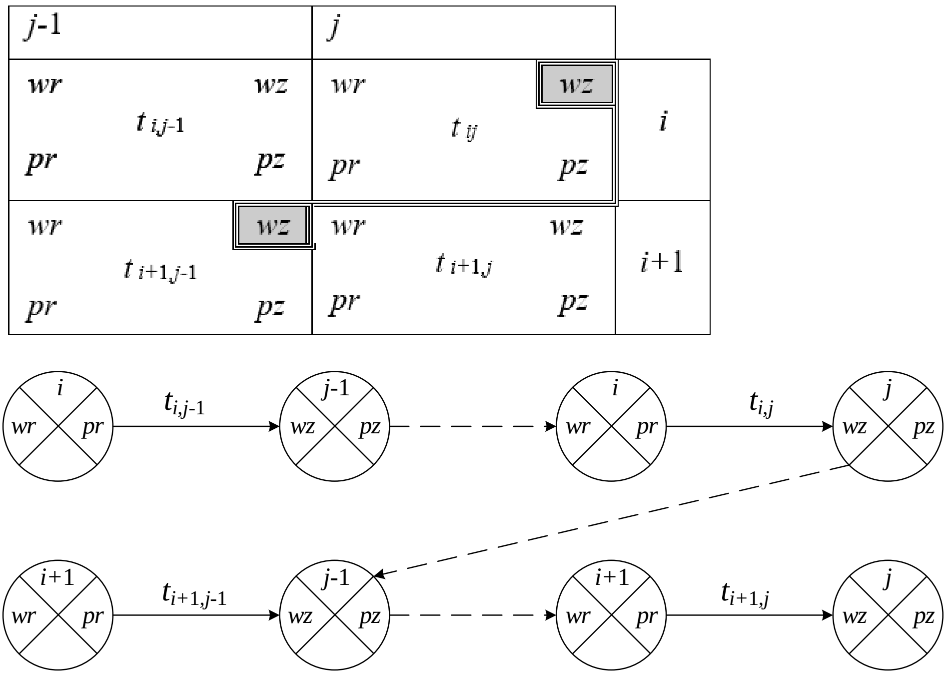
Definition:

$$\bigwedge_{y\in P}\ \bigvee_{Y\subset P} f(y_1) = a \vee f(y_2) = b \Leftrightarrow y_1 \geq y_2, \tag{5.9}$$

where: $Y = \{y_1, y_2\}$.
Possibilities:

$$\begin{aligned} &y_1 = y_2 \Rightarrow y_1 = a, \quad \text{i.e.} \quad t_{ij}^{wz} = t_{i+1,\,j-1}^{wz} \\ &y_2 < y_1 \Rightarrow y_2 = b, \quad \text{i.e.} \quad t_{ij}^{wz} > t_{i-1,\,j-1}^{wz} \end{aligned}. \tag{5.10}$$

Formula (5.7) ensures the simultaneous completion of works of different kinds on successive work fronts. Furthermore the technological dependence sequencing various works on a given work front is preserved.



Variant 3 – For the earliest dates.

$$t_{ij}^{wr} = t_{i+1,\,j-1}^{wr}, \tag{5.11}$$

and:

$$t_{ij}^{wr} > t_{i-1,\,j-1}^{wz}. \tag{5.12}$$

Definition:

$$\bigwedge_{u\in P}\bigvee_{R\subset P} f(y_1)=a\vee f(y_2)=b \Leftrightarrow y_1 \ge y_2\,, \tag{5.13}$$

where: $R = \{y_1, y_2\}$.
Possibilities:

$$\begin{array}{l} y_1 = y_2 \Rightarrow y_1 = a, \quad \text{i.e.} \quad t_{ij}^{wr} = t_{i+1,j-1}^{wz}, S^P = 0 \\ y_2 < y_1 \Rightarrow y_2 = b, \quad \text{i.e.} \quad t_{ij}^{wr} > t_{i-1,j-1}^{wz}, S^P > 0 \end{array}. \tag{5.14}$$

Formula (5.11) expresses the limitations presented above. The difference is that appropriate time characteristics of constructions works, linked together by zero-value diagonal couplings ensuring the realization (in a technological sequence) of works of different kinds on work fronts, are taken into account.

Variant 4 – For the earliest dates.

$$t_{ij}^{wr} = t_{i+1,j-1}^{wr}\,, \tag{5.15}$$

and:

$$t_{ij}^{wr} > t_{i-1,j-1}^{wz}\,. \tag{5.16}$$

Definition:

$$\bigwedge_{z\in W}\bigvee_{Z\subset W} f(z_1)=a\vee f(z_2)=b \Leftrightarrow z_1 \ge z_2\,, \tag{5.17}$$

where: $Z = \{z_1, z_2\}$.
Possibilities:

$$\begin{array}{l} z_1 = z_2 \Rightarrow z_1 = a, \quad \text{i.e.} \quad t_{ij}^{wr} = t_{i+1,j-1}^{wz}, S^D = 0 \\ z_2 < z_1 \Rightarrow z_2 = b, \quad \text{i.e.} \quad t_{ij}^{wr} > t_{i-1,j-1}^{wz}, S^P > 0 \end{array}. \tag{5.18}$$

In formula (5.15) the zero-value inversely diagonal couplings linking the earliest work completion dates on adjacent work fronts are preserved. The sequentially of various works on fronts is ensured through the use of diagonal couplings.

Variant 5 – For the latest dates.

$$t_{ij}^{pr} = t_{i+1,j-1}^{pr}\,, \tag{5.19}$$

and:

$$t_{ij}^{pr} > t_{i-1,j-1}^{pr}\,. \tag{5.20}$$

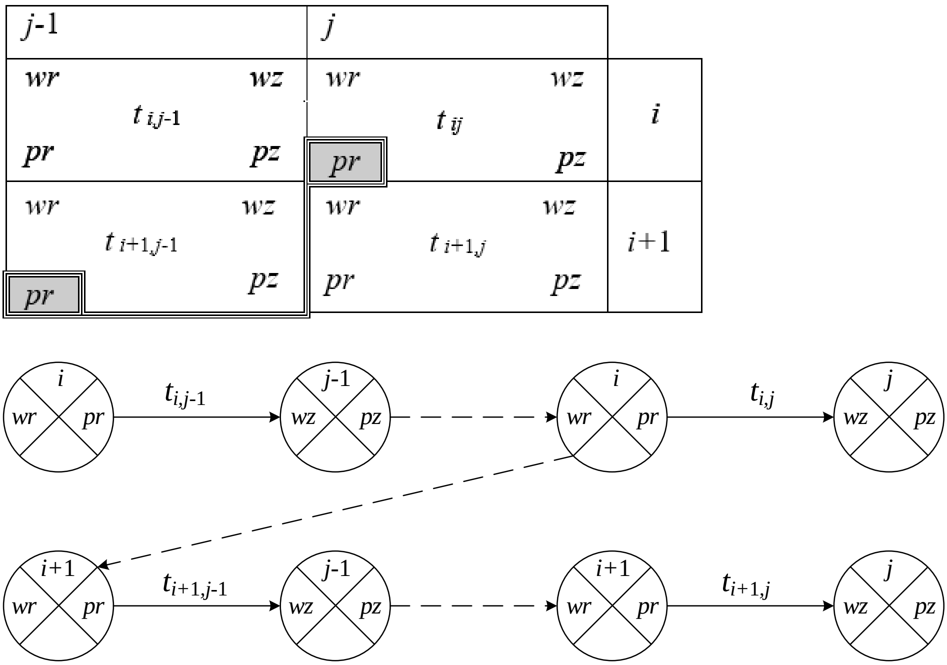
Definition:

$$\bigwedge_{u\in P}\bigvee_{V\subset P} f(u_1)=a\vee f(u_2)=b \Leftrightarrow u_1 \ge u_2\,, \tag{5.21}$$

where: $V = \{u_1, u_2\}$.
Possibilities:

$$\begin{array}{l} u_1 = u_2 \Rightarrow u_1 = a, \quad \text{i.e.} \quad t_{ij}^{pr} = t_{i+1,j-1}^{pr}, S^D = 0 \\ u_2 < u_1 \Rightarrow u_2 = b, \quad \text{i.e.} \quad t_{ij}^{pr} > t_{i-1,j-1}^{pr}, S^P > 0 \end{array}. \tag{5.22}$$

Formula (5.19) links together construction works on work fronts in the way specified above but it takes into account only appropriate time characteristics, i.e. the latest work starting dates. The basic condition having the form of zero inversely diagonal couplings has been imposed whereas the diagonal couplings impose the order in which works of different kinds are realized, but their value is greater than zero.



Variant 6 – For the latest dates.

$$t_{ij}^{pz} = t_{i+1,\,j-1}^{pz}\,, \tag{5.23}$$

and:

$$t_{ij}^{pz} > t_{i-1,\,j-1}^{pz}\,. \tag{5.24}$$

Definition:

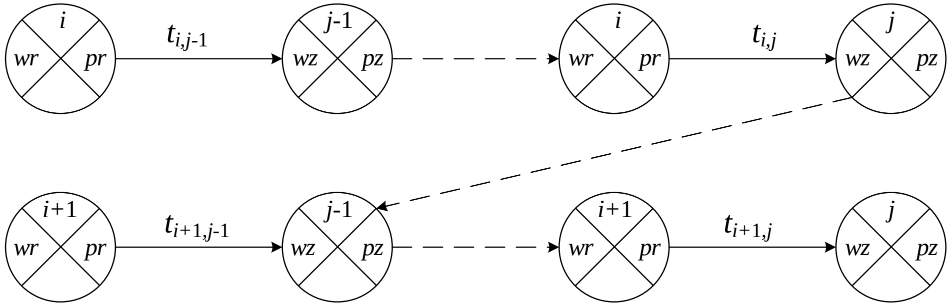
$$\bigwedge_{u \in P} \bigvee_{U \subset P} f(u_1) = a \vee f(u_2) = b \Leftrightarrow u_1 \geq u_2\,, \tag{5.25}$$

where: $U = \{u_1, u_2\}$.

Possibilities:

$$\begin{array}{l} u_1 = u_2 \Rightarrow u_1 = a, \quad \text{i.e.} \quad t_{ij}^{pz} = t_{i+1,\,j-1}^{pz},\, S^D = 0 \\ u_2 < u_1 \Rightarrow u_2 = b, \quad \text{i.e.} \quad t_{ij}^{pz} > t_{i-1,\,j-1}^{pz},\, S^P > 0 \end{array}. \tag{5.26}$$

Formula (5.23) represents technological limitations imposed on realized construction works, ensuring that the latest work completion dates will be taken into account.



5.2. Scheduling of construction works with diagonal couplings and zero inversely diagonal couplings

Basic assumptions

An investment task in the form of a set of structures (work fronts) $O = \{O_1, O_2, ..., O_n\}$, which are to be built by work gangs (construction firms) $B = \{B_1, B_2, ..., B_m\}$ is given. A sequence of processes: $P_i = \langle P_j, P_{i2}, ..., P_{im} \rangle$ is to be realized on each structure. The construction processes are to be realized in a specified technological order, i.e. each process P_{ik} can be realized after process $P_{i,k-1}$ has been completed but prior to process $P_{i,k+1}$. The following requirements have been imposed on the realization process. Work gangs (construction firms) move from one structure to another in a continuous way. Construction processes should be realized consecutively on sections O_i, i.e. inversely diagonal couplings will tie the date of completion of given process P_j on given structure O_i with the date of starting next process P_{j+1} on structure O_{i-1}. The value of the inversely diagonal couplings in a modelled stream should reach zero. Process P_{ik} on given structure O_i can be started after the completion of process $P_{i,k-1}$ on this structure.

The problem consists in establishing such a work realization sequence that:

$$L_{\min} = \max_i \left| T_{im} - D_i \right|, \tag{5.27}$$

where:
D_i – a directive realization date,
T_{im} – a date of completion of process P_m on structure i,
$L_{\min}$ – a minimum realization date.

Algorithm flow sheet

An organizational model, which incorporates the effect of diagonal couplings and zero inversely diagonal couplings, is constructed according to the flow sheet presented below. The calculation procedure is based on a construction work realization time matrix.

Phase 1

Determine work starting and completion dates in the first and second diagonal of matrix **T**.

Phase 2

Determine work starting and completion dates in the third diagonal. The date of starting the third work on the first structure (matrix indices 1, 3) is equal to the date of completion of the preceding work on this structure (indices 1, 2). The remaining works are determined from the following condition: $t_{1,2}^{wr} = t_{1,1}^{wz}; t_{1,2}^{wz} = t_{1,2}^{wr} t_{1,2}$, $t_{1,3}^{wr} = t_{1,2}^{wz}; t_{1,3}^{wz} = t_{1,3}^{wr} + t_{1,3}$.

The work starting dates in the third diagonal are: $t_{2,2}^{wr} = t_{1,3}^{wz}; t_{2,2}^{wz} = t_{2,2}^{wr} + t_{2,2}$.

Phase 3

Determine work starting and completion dates in the fourth diagonal. Start determining time characteristics from the second work (2, 2) on the second structure. Calculate $t_{1,4}^{wz}$ on the basis of the value of t_{22}^{wz} calculated in phase 2. Then subtract the numerical value of the time of duration of work $t_{1,4}$: $t_{1,4}^{wr} = t_{1,4}^{wz} - t_{1,4}$. If $t_{1,4}^{wr} \langle t_{1,3}^{wz}$ value $t_{1,4}^{wz}$ is corrected by the difference $t_{1,3}^{wz} - t_{1,4}^{wr} = a$ and then new values of the time characteristics of the works in the next diagonal are calculated. The correction is needed in order to satisfy the condition of collisionless work on structures by construction firms (work gangs). Proceed in the above way until the set of diagonals is exhausted.

The use of inversely diagonal couplings imposes the condition of consecutive commencement and completion of works in diagonals, i.e. $S^P = 0$, on works. Diagonal couplings ensure the shift of works in matrix rows and the fulfilment of the condition that works of one kind on adjacent fronts are carried out sequentially (indices in matrix i-1,j,j+1).

<u>Algorithm application example</u>

A work (renewal of paint coatings) duration matrix for four apartment buildings was prepared. A set of works, determined by a survey of the buildings, comprises the repair of interior plaster, exterior plaster, painting, and the repair of the facades. The durations written in matrix **T**:

$$\mathbf{T} = \begin{vmatrix} 9 & 6 & 7 & 4 \\ 7 & 4 & 10 & 4 \\ 12 & 9 & 1 & 6 \\ 6 & 4 & 5 & 3 \end{vmatrix}$$

The algorithm was applied to determine the time characteristics of the works.

Table 5.1. Time characteristics of the job

Project	Diagonals									
	1	2	3	4	5	6	7	8	9	10
	0	9	15	25						
I	9	6	7	4						
	9	15	22	29						
			22	29	33	52				
II			7	4	10	4				
			29	33	44	56				
					44	56	65	76		
III					12	9	11	6		
					56	65	76	82		
							76	82	86	91
IV							6	4	5	3
							82	86	91	94

The digits in the fields stand for in turn: work starting date, work duration, work completion date.

6. Methods of organizing construction work with parallel internal structure

According to the classification of construction work organization methods [1–3, 81], the parallel carrying out of jobs of one kind on different work fronts represents the parallel execution method. But considering that it is possible to form groups of jobs performed in parallel and independently, three versions of the method can be distinguished:

- with neglected simultaneity in the performance of jobs of different kinds,
- with the simultaneous performance of some jobs of different kinds,
- with the simultaneous performance of jobs of different kinds.

Through the action of time couplings (couplings between realization means, work fronts and in particular, zero diagonal couplings and reverse diagonal couplings) on construction jobs one can shape the internal structure of the latter. The linking of work complexes realized using various organizational methods is a basic practical problem.

As the realization of a complex of building structures is planned, problems with work scheduling under certain technological and organizational constraints crop up. A common constraint which affects the flow nature of construction processes is the continuity of operation of the leading machine (e.g. a tower crane).

In the modelling of a flow with parallel structure one can distinguish the following cases:

- the modelling of a flow with preserved continuity of the leading processes,
- the synchronization of organizationally interrelated work complexes.

The first of the above cases is described in [50] and the second is presented below.

The problem arose during the planning of the realization of complexes of industrial building structures. A practical example is the realization of three industrial building complexes by means of two tower cranes moving on one track. Two building complexes were to be realized in parallel but the second one was to be completed within a precisely fixed time contingent on the completion of assembly on the first building complex. This was so because the tower crane had to be moved from the second building complex to the third one due to the small size of the construction site (an operating plant) – there was space only for one track.

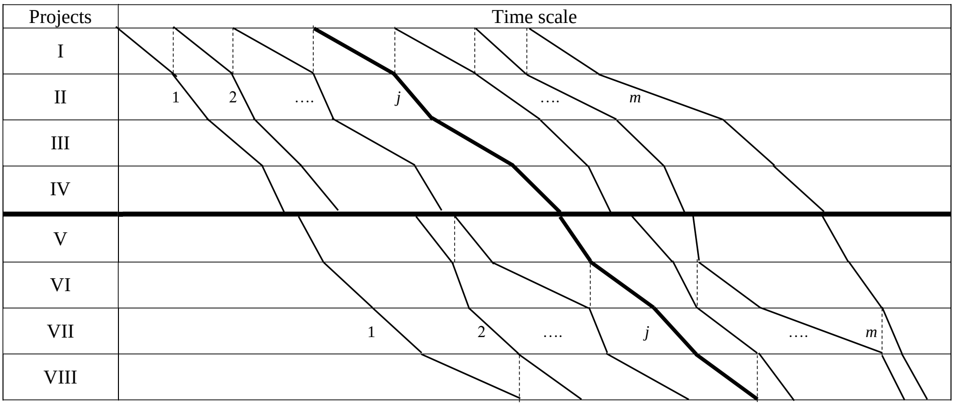


Figure 6.1. Work complex with parallel structure and preserved continuity of leading processes

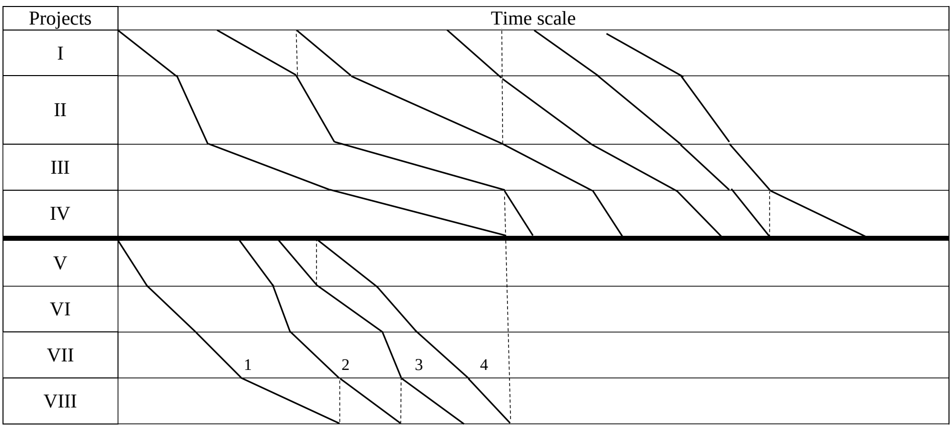


Figure 6.2. Work complex with parallel structure and organizationally interrelated groups of jobs

6.1. Scheduling of parallel work complexes

General formulation of problem

The problem of synchronization of organizationally interrelated complexes boils down to such allocation of work teams (firms) with their equipment which will minimize the total lead time under the following constraints:

- each process must be realized by a specialized gang (firm),
- no more than one process can be carried out simultaneously on a work front,
- the resource limitations must be taken into account.

One should calculate the values of the following variables: $t_z, t_j^x, t_j^y,\ j = N$ and $u_j^k,\ j \in N_k$ which minimize objective function $t_z \to \min$, where:

t_z – the time of completion of all the jobs;
t_j^x – the time of commencement of the *j*-th job, *jM*;
t_j^y – the time of completion of the *j*-th job, *jN*;
c_j – the time of performance of the *j*-th job $c_j > 0$;
u_j^k – the quantity of resources (the number of gangs) allocated to carry out job *j*.
The following constraints should be taken into account: $t_j^y - t_j^x \geq \sum c_j$, $t_j^x \geq t_j^y$, $t_j^x \geq t_0$, where t_0 – the time of commencement of the jobs ($t_0 = 0$), $t_z \geq t_i^y, i \in N$, $t_0, t_2, t_j^x, t_j^y \geq 0, i \in N$, $C_{jw} = f_j^k(u_j^k), j \in N_k$, where f_j^k – a function of the quantity of means, $\alpha_j^k \leq u_j^k \leq \beta_j^k, j = N_k, k \in Q_1$. To carry out a particular process one must have at one's disposal a work team consisting of α_j^k workers but no more than β_j^k.

Furthermore, the quantity of realization means in model parallel flow P_{k+1} must be adequate to carry out the task in the fixed time: $T_{k+1} \Pi T_k$, where T_{k+1} – the time of performance of the work complex in parallel flow $k + 1$, T_k – the time of performance of the group of jobs in flow *k*.

Scheme of problem solution

The problem is related to the control of the realization of building structures. Rational control here consists in the fixing of commencement dates t_j^x and completion dates t_j^y for the jobs. The control quantity is a vector whose components determine job durations c_{jw}, the sequence of the jobs and allocation of gangs u_j^k. Rational control in this case has a closed character since it is applied to the modelling of a complex of jobs prior to their commencement. It consists in the matching of realization means (gangs and their equipment) with the tasks to realize a complex of building structures in flow P_{k+1} depending on the lead time of the partial flow in complex P_k.

An outline of the procedure for controlling the realization process is presented below.

Step 1 – check if complex lead time t_z^{k+1} corresponds to planned k_j^k for the prescribed constraints. If $t_z^{k+1} \leq t_z^k$ go to step 4.

Step 2 – determine a control vector taking into account the new conditions of realization of the jobs (the durations of the remaining jobs) and the additionally needed realization means.

Step 3 – realize optimization algorithms of the minimum-time modelling of flows and check the condition of parallel interdependence of the flows, i.e. if $t_z^{k+1} \le t_{zj}^k$.

Step 4 – work out time characteristics for flow modelling.

Step 5 – end the process of control.

The above is a discrete optimization problem which boils down to the creation of a set of permissible controls. The latter follow from the size of set of resources *u*. Thus a set of models for the realization of the complex is formed by changing the components of the control vector $\underset{j \in M}{\wedge} \underset{i \in M}{\wedge} [X_{ij}(t) = f[U_{ij}^1(t), U_{ij}^2(t), \ldots, U_{ij}^q(t)]]$,

where X_{ij} (t) – the state of the *j*-th job on the *i*-th building structure at instant *t*, $U_{ij}(t)$ – the quantity of realization means allocated to carry out job *j* on building structure *i*.

The solution of the problem is quite complex because the flow must be harmonized each time, i.e. the technological and organizational constraints must be taken into account without changing the structure of the flow.

Harmonization algorithm

Step 1 – harmonize flow P_k, determine a rational order of realizing the building structures and a work organization method.

Step 2 – fix dates t_j^x of starting jobs in first partial flow P_k.

Step 3 – harmonize flow P_{k+1}, determine a rational order of realizing the building structures and a work organization method.

Step 4 – check if $t_{k+1}^y \le t_k^y$, if no, go to step 5, if yes, go to step 7.

Step 5 – adjust the input matrix elements by increasing the quantity of realization means (gangs) and reducing the lead times of the particular jobs on the building structures.

Step 6 – check the condition expressed by relation in step 4.

Step 7 – determine the time characteristics of the complex flows.

Under the imposed constraints, several iterations are needed to harmonize the parallel work complexes. Since the partition function is discontinuous (stepwise), the means must be chosen in a discrete way.

This allows us to test practically the entire set of controls. The effect is determined mainly be the quantity of means gathered on the critical path, i.e. min t_z should be found under these constraints: $\sum Y_j(t) f_j(U_j) \le t_z$, $U_j \le U^k$, $t_u \ge 0$, where y_j – a binary digital variable, 1 – if the arc's load is C_j, j – 1, 2, …, m; 0 – otherwise.

The problem of synchronization of parallel complexes of organizationally interrelated jobs often occurs in construction practice during the realization of, for example, assembly when there is a limiting factor (e.g. a tower crane).

6.2. Variants of construction work organization method taking into account diagonal and reverse diagonal couplings

By applying diagonal and reverse diagonal couplings to a group of technologically interrelated jobs one can synchronize the latter (the earliest and latest commencement and completion dates) and ensure their parallel execution.

In this method [8, 81, 83], jobs of one rank form diagonals whose duration is equal to the durations of critical jobs. Although a few critical jobs (of the same duration) may occur in a diagonal, only one of them is taken into account when determining the lead time for a work complex.

If the symbols shown in fig. 5.3 are used, the lead time for a work complex can be expressed by:

$$T = \sum_{q=1}^{n+m-1} t_{ijq} \ , \tag{6.1}$$

where:

T – the lead time for a complex incorporating critical jobs, with diagonal and reverse diagonal couplings taken into account;

t_{ijq} – the duration of the critical job of the j-th kind on the i-the work front, which determines the duration of the q-th diagonal;

$n+m-1$ – a number of diagonals.

Also small-scale problems can be harmonized by means of time couplings through which parallelism can be imposed on the particular construction jobs. Taking into account the interdependences between commencements and completions for the early and late dates one can distinguish a construction work organization method and its variants, characterized by the simultaneous action of diagonal couplings and reverse diagonal couplings.

Three main steps can be distinguished in the solution of such a problem. In the first step (tab. 6.1), the first series of intermediate matrices, in which all the rows of the initial matrix are placed in the first row, are constructed. Then the prospective matrices are determined by indicating the branches which should be developed in the dendrite that is being built. These are matrices with the lowest possible minimum (LPM) for a work complex lead time.

Table 6.1. Step I – matrix with successively determined rows placed in its first row

		Kinds of jobs							
		1	2	...	j	...	...	m-1	m
Projects	U	t_{k1}	t_{k2}	STEP I	row	determined	(U)	$t_{k,m-1}$	$t_{k,m}$
	2	t_{21}	t_{22}					$t_{2,m-1}$	$t_{2,m}$
	j								
	...								
	n	$t_{n,1}$	$t_{n,2}$					$t_{n,m-1}$	$t_{n,m}$

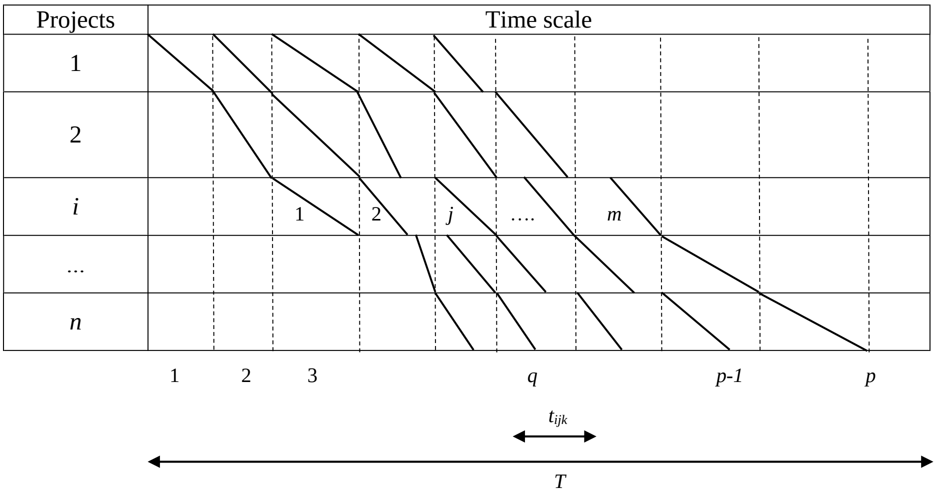


Figure 6.3. Cyclogram with critical jobs determined taking into account diagonal and reverse diagonal couplings

In the second step (tab. 6.2) we construct series of matrices in which depending on the number of rows in the initial matrix, two, three … or *n*–2 rows (work fronts) are determined. Thus the second step is divided into stages in which: the first rows are successively determined from matrices indicated as prospective in the first step and the next rows are obtained from the next rows of the initial matrices. One should note that if the lowest possible minima (LPM) determined in the second step become equal to or higher than the LPM obtained in step I, the branches from step I should be regarded as prospective since this moment. Only when all the competing matrices have been considered and the prospective branches of the variant tree have been indicated we can pass to the last step in the solution.

Table 6.2. Step II – matrix constructed in intermediate stages

		Kinds of jobs							
		1	2	...	*j*	...	...	*m*-1	*m*
Projects	i^p_{F1}								
	k^p_{F2}								
	U			STAGE II – row determined in stage *k*					
	...								
	n-1								
	n	$t_{n,1}$	$t_{n,2}$					$t_{n,m-1}$	$t_{n,m}$

i^p_{Fk} – prospective rows determined in intermediate stages.

In the third step no intermediate matrices are introduced and final matrices with all the rows, i.e. all the partial complexes (partial fronts), fixed are constructed. At the same time, the LPM indicator becomes actual work complex lead time *T*. Each of the final matrix has *n*–2 rows determined to be prospective in the preceding steps of the solution. In one fully developed branch of the dendrite there are only two possible combinations. The number of final matrices which should be expanded is a double of the prospective branches indicated in the preceding steps.

<u>Calculation of time characteristics</u>

The solution consists in the determination of the earliest and latest job commencement and completion dates, the critical path, the numerical values of the couplings in the earliest and latest dates, the total time reserves and the minimum lead time for the whole complex.

Using the symbols from table 6.2 we can calculate the above quantities from the relations given below:

$$t_{ij}^{wr} = \max(t_{i+1,j-1}^{wr}; t_{i-1,j+1}^{wr}) \ . \tag{6.2}$$

The earliest job commencement and completion dates are calculated from:

$$t_{ij}^{wz} = t_{ij}^{wr} + t_{ij} \ . \tag{6.3}$$

This means that when fixing the earliest dates of commencement of jobs *j* on front *i*, one should take into account the diagonal couplings from the preceding partial flow and from the next partial complex on front *i*+1 of jobs *j*–1 as well as the reverse diagonal couplings from the next partial flow and from the preceding partial complex of front *i*–1 of jobs *j* + 1. The earliest dates of completion of jobs *j* on front *i* are fixed by adding up the earliest commencement dates and lead times for these jobs.

The latest dates of completion of jobs *j* on front *i* are determined taking into account the diagonal couplings from the preceding complex and from the next partial flow of front *i*–1 of job *j* +1 as well as the reverse diagonal couplings from the next partial complex and from the preceding partial flow of front *i*+1 of jobs *j*–1:

$$t_{ij}^{pz} = \min(t_{i-1,j+1}^{pz}; t_{i+1,j-1}^{pz}) \ , \tag{6.4}$$

$$t_{ij}^{pr} = t_{ij}^{pz} - t_{ij} \ . \tag{6.5}$$

The couplings between the realization means for the earliest dates are determined as a difference between the earliest dates of completion of the jobs preceding given jobs *j* on front *i*–1 in the partial flow and the earliest dates of starting given jobs *j* on front *i*:

$$S_{ij}^{sw} = t_{i-1,j}^{wz} - t_{ij}^{wr} \ . \tag{6.6}$$

Similarly, the couplings between the fronts for the earliest dates are determined as a difference between the earliest dates of starting given jobs *j* on front *i* and the earliest dates of completing the preceding jobs in the partial complex of front *i* of jobs *j*–1:

$$S_{ij}^{fw} = t_{ij}^{wr} - t_{i,j-1}^{wz} . \tag{6.7}$$

For the latest dates, the couplings between the realization means are determined from this relation:

$$S_{ij}^{SP} = t_{i+1,j}^{pr} - t_{ij}^{pz} . \tag{6.8}$$

Similarly, the couplings between the work fronts for the latest job completion dates are determined from this relation:

$$S_{ij}^{fp} = t_{i,j+1}^{pr} - t_{ij}^{pz} . \tag{6.9}$$

The total time reserves are a difference between the latest and earliest job commencement dates and between the latest and earliest job completion dates:

$$R_{ij} = t_{ij}^{pz} - t_{ij}^{wr} - t_{ij} = t_{ij}^{pr} - t_{ij}^{wr} = t_{ij}^{pz} - t_{ij}^{wz} . \tag{6.10}$$

Variants of method

An analysis of the interactions between time couplings shows that they result in the parallel arrangement of the jobs within a complex along the time axis. This applies to four possible situations: the earliest and latest dates of commencement and completion of the jobs. The combinations of the characteristics represent combinations of consecutive and parallel methods.

Variant 1 – in which the earliest job commencement dates are linked:

$$t_{ij}^{wr} = t_{i+1,j-1}^{wr} = t_{i-1,j+1}^{wr} . \tag{6.11}$$

j-1	*j*	*j*+1	
		wr $t_{i-1,j+1}$	*i*-1
	wr t_{ij}		*i*
wr $t_{i+1,j-1}$			*i*+1

This is defined as follows:

$$\bigwedge_{x \in W} \bigvee_{N \subset W} f(x_1) = a \wedge f(x_2) = b \wedge f(x_3) = c \Rightarrow a = b = c , \tag{6.12}$$

where: $N = \{x_1, x_2, x_3\}$.

Formula (6.11) imposes a practical constraint – the successive kinds of jobs j–1, j and j+1 must be started simultaneously on neighbouring work fronts i–1, i and i+1. The jobs, despite their different durations, start simultaneously and are carried out in parallel. They are linked by zero couplings, forming a group of jobs performed in parallel.

Variant 2 – in which the latest job commencement dates are linked:

$$t_{ij}^{pr} = t_{i+1,j-1}^{pr} = t_{i-1,j+1}^{pr} . \qquad (6.13)$$

j-1	j	j+1	
		$t_{i-1,j+1}$ pr	i-1
	t_{ij} pr		i
$t_{i+1,j-1}$ pr			i+1

This is defined as follows:

$$\bigwedge_{y \in P} \bigvee_{Y \subset P} f(y_1) = a \wedge f(y_2) = b \wedge f(y_3) = c \Rightarrow a = b = c , \qquad (6.14)$$

where: $Y = \{y_1, y_2, y_3\}$.

Formula (6.13) guarantees the simultaneity of the "late commencement" of jobs j–1, j and jobs j+1. They are carried out sequentially according to a technological order. The jobs differ in their execution technologies. They are realized in parallel on independent work fronts. This situation occurs commonly in practice.

Variant 3 – in which the earliest job completion dates are linked:

$$t_{ij}^{wz} = t_{i+1,j-1}^{wz} = t_{i-1,j+1}^{wz} . \qquad (6.15)$$

j-1	j	j+1	
		wz $t_{i\text{-}1,j+1}$	i-1
	wz t_{ij}		i
wz $t_{i+1,j\text{-}1}$			i+1

This is defined as follows:

$$\bigwedge_{v \in P} \bigvee_{V \subset P} f(u_1) = a \wedge f(u_2) = b \wedge f(u_3) = c \Rightarrow a = b = c\,, \qquad (6.16)$$

where: $V = \{u_1, u_2, u_3\}$.

Formula (6.15) specifies that construction jobs must be completed at the same time. Though their durations may be different, the jobs will be performed in parallel in the final stage to be completed simultaneously. They may begin at different times. They occur on different work fronts which are organizationally independent. The technological processes are ordered sequentially, i.e. $t_{j-1} \prec t_j \prec t_{j+1}$. Such a situation occurs during the realization of technologically different jobs on different building structures or their parts.

Variant 4 – in which the latest job completion dates are linked:

$$t_{ij}^{pz} = t_{i-1,\,j+1}^{pz} = t_{i+1,\,j-1}^{pz}\,. \qquad (6.17)$$

j-1	j	j+1	
		$t_{i\text{-}1,j+1}$ pz	i-1
	t_{ij} pz		i
$t_{i+1,j\text{-}1}$ pz			i+1

This is defined as follows:

$$\bigwedge_{x\in W}\bigvee_{x\subset W} f(x_1)=a \wedge f(x_2)=b \wedge f(x_3)=c \Rightarrow a=b=c\ , \qquad (6.18)$$

where: $X = \{x_1, x_2, x_3\}$.

Formula (6.17) specifies that jobs of different kinds, j–1, j, j+1, must be completed simultaneously on different building structures or their parts. This applies to the latest job completion dates and results in practical situations when jobs are performed in parallel in the final stage. The dates of their commencement may be different because of their unequal labour demand but they must be completed at the same time.

The presented variants of construction work organization make possible the parallel execution of jobs in different technologies on different building structures or their parts (work fronts). This is owing to zero diagonal couplings and reverse diagonal couplings that simultaneously occur in the organizational models. The couplings make it possible to synchronize jobs in a complex (group) by ordering them according to the adopted technological sequence.

6.3. Scheduling of parallel groups of construction jobs in complex

Scheduling in production management consists in ordering tasks. The rudimentary models which are used in practice neglect this problem. This is often due to technological constraints (a typical sequence of jobs) or to arbitrary decisions of the investors.

The presented above algorithm for scheduling construction jobs by means of parallel-flow methods [40, 41, 51] can be applied to the parallel performance of a group of construction jobs in a complex for work fronts of different size. Work organization methods employing time couplings [1–3, 81] are used for the synchronization of the jobs.

<u>Basic assumptions</u>

Set F of building structures (work fronts) to be realized is given: $F = \{F_i\}$ where $i = 1, \ldots, N$. A sequence of technological processes forming set: $R = \{R_j\}$ where $j = 1, \ldots, M$, are to be carried out on the building structures. Processes from set R should be carried out in this fixed technological order: R_1 { R_2 { ... {R_M. The particular processes are to be carried out by specialized gangs forming set: $B = \{Bj\}$ where $j = 1, \ldots, M$. In set B one can distinguish processes which form subset R_1 for which (for design-technological-organizational reasons) the whole building structure is a work front and processes forming subset R_2 for which smaller lots on building structures F_i are work fronts.

When the processes in set R_2 are realized, each object F_i is a set of several work lots d_f: $R = R_1 + R_2$, $R_1 = \{R_j\}$ where $j = 1, \ldots, l$, $R_2 = \{R_j\}$ where: $j = l + 1, \ldots, M$, $F_i = \{d_f\}$ where: $f = 1, \ldots, g$. Taking into account the different (for the different processes) division of a complex of building structures into work fronts, a rational sequence (ensuring the collisionless work of the gangs) for carrying out the jobs must be established.

Algorithm for matrix determination of jobs realization sequence

The following solution of the above problem was proposed:

- Divide vertically complex of processes R into groups of partial subprocesses R_1 and R_2 carried out on heterogeneous work fronts (building structures and lots).
- Divide horizontally the processes in set R_2 into particular work fronts F_i.
- Establish a rational sequence of the realization of work fronts in partial subcomplexes K and P_i, where: $i = 1, \ldots, n$.
- Synchronize the partial subcomplexes.

A scheme of an algorithm used for dividing a work complex into partial subcomplexes K and P_i is shown in fig. 6.4. The design of an operation duration matrix, taking into account the division of a work complex into partial subcomplexes, is shown in fig. 6.5.

Having established a rational sequence in which the fronts in subcomplex K are to be realized and a rational sequence of realizing the work lots in subcomplexes P, one should synchronize the complexes. It is assumed that subcomplexes P_i will be realized in an order corresponding to that in which the fronts in subcomplex K are realized.

A cyclogram of two synchronized groups of jobs forming subset R_2, realized on neighbouring fronts F_i and F_{i-1} is shown in fig. 6.6. The duration of the expansion of front F_i in relation to front F_{i-1} was calculated on the basis of the durations of the expansion and folding of the successive jobs on neighbouring fronts:

$$\begin{gathered} t_{ij}^{z} = t_{ij}^{pz} - t_{i,j-1}^{pz} \\ t_{ij}^{r} = t_{ij}^{wr} - t_{i,j-1}^{wr} \end{gathered}, \tag{6.19}$$

where:

$t_{i,j}^{z}$ – the time in which process j folds in relation to process j–1 on front i,

$t_{i,j}^{r}$ – the time in which process j is expanded in relation to process j–1 on front i,

$t_{i,j}^{pz}$ – a late date of completion of job j on front i,

$t_{i,j}^{wr}$ – an early date of starting job j on front i.

$$T_i^r = \max\left[\sum_{j=1}^{d} t_{i-1,j}^{z} - \sum_{j=1}^{d} t_{ij}^{r} : d = 1, 2, ..., n\right], \tag{6.20}$$

where: T_i^r – the time in which front i is expanded in relation to front $i + 1$.

The obtained values T_i^r are used to update the time characteristics of the jobs from set R_2 carried out on fronts Fi (in the same order as for subcomplex *K*).

To link subcomplexes P_i consolidated with subcomplex *K* (fig. 6.4) one should calculate the duration of the expansion of flow *k*+1 in relation to flow *k*.

The auxiliary quantities are: early and late dates of commencement and completion of jobs on the particular fronts in boundary flows *k* and *k*+1, durations of folding job *k* on the particular fronts and durations of expanding job *k*+1 on the particular fronts:

$$
\begin{aligned}
& t_{i,k}^{z} = t_{i,k}^{pz} - t_{i-1,k}^{pz} \\
& t_{i,k+1}^{r} = t_{i,k+1}^{wr} - t_{i-1,k+1}^{wr} \qquad , \\
& T_{k+1}^{r} = \max_{d} \left[\sum_{i=1}^{d} t_{i,k}^{z} - \sum_{i-1}^{d-1} t_{i,k+1}^{r} \right]
\end{aligned}
\tag{6.19}
$$

where:

$d = 1, \ldots, n$;

$t_{i,k}^{pz}$ – a late date of completion of job *k* on front *i*;

$t_{i,k+1}^{wr}$ – an early date of commencement of job *k*+1 on front *i*;

$t_{i,k}^{z}$ – the time in which job *k* folds on front *i*;

$t_{i,k+1}^{z}$ – the time in which job *k*+1 is expanded on front *i*;

T_{k+1}^{r} – the time in which job *k*+1 is expanded in relation to *k*.

The obtained value $T_{k+1,j}^{r}$ is used to update the time characteristics of the jobs carried out within set R_2.

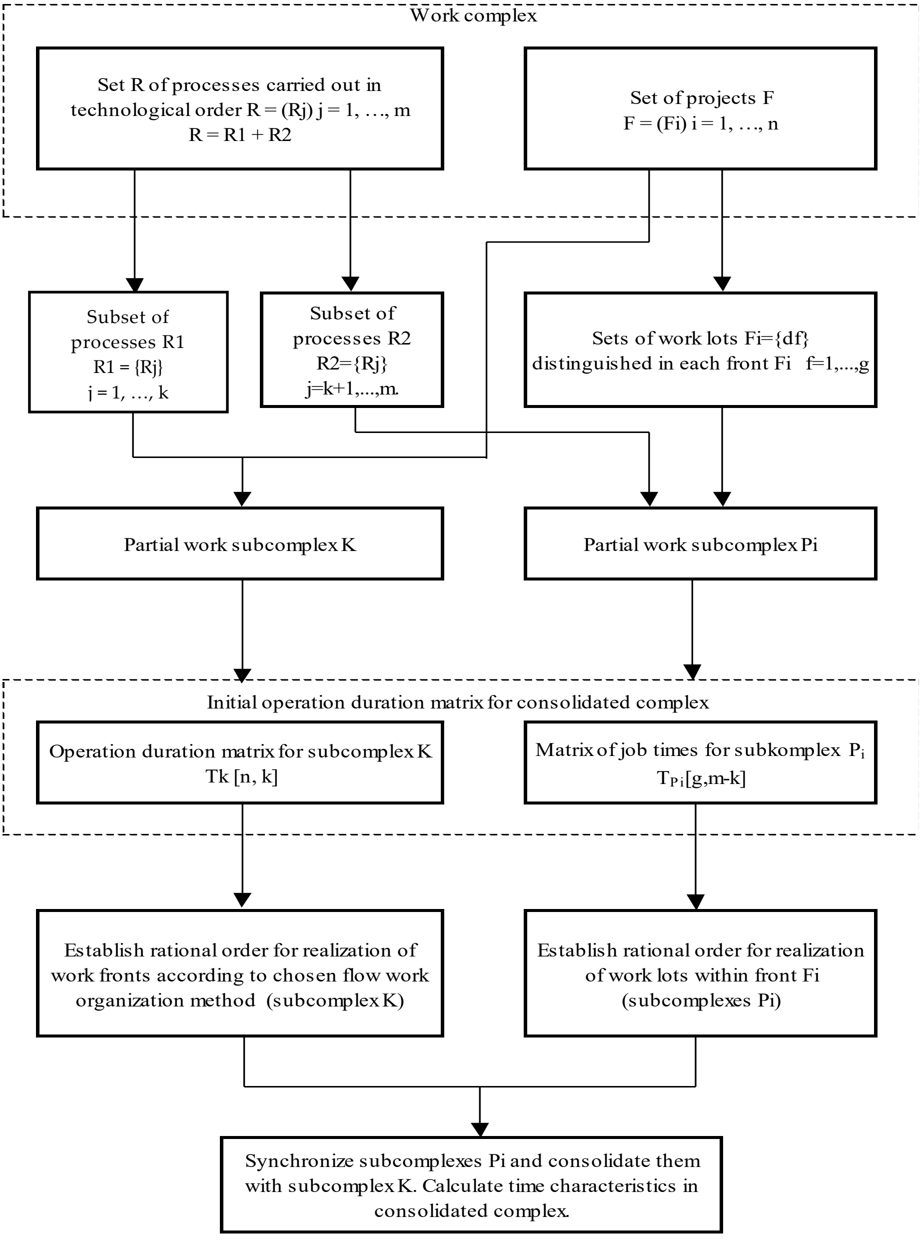


Figure 6.4. Scheme of algorithm for work complex division into partial subcomplexes

An algorithm for scheduling jobs, which takes into account practical technological and organizational constraints and ensures minimum-time optimization, is described below. The algorithm enables the synchronization of jobs of different kinds, which can be performed on a larger number of work lots created by dividing the work front into smaller tasks after the elimination of breaks [52].

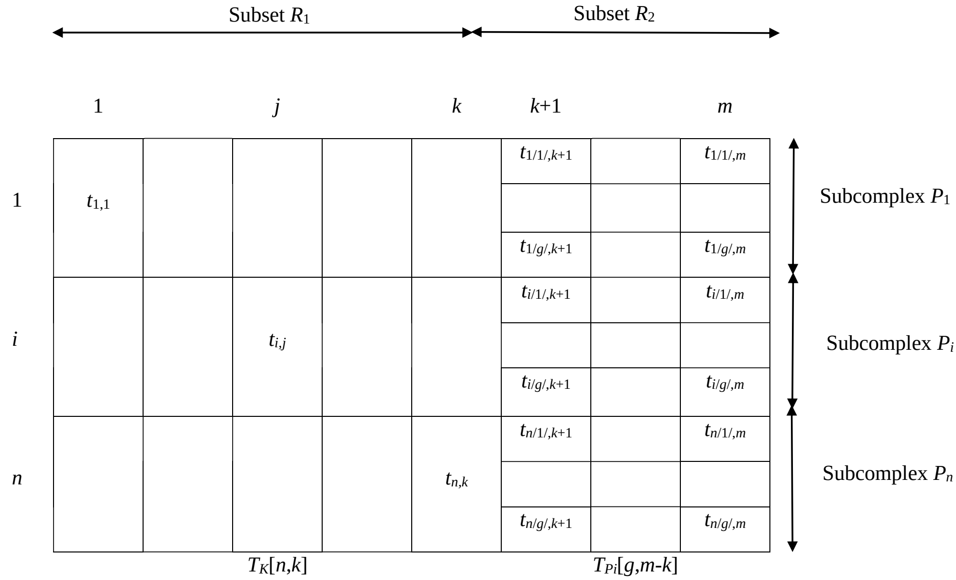


Figure 6.5. Construction of operation duration matrix

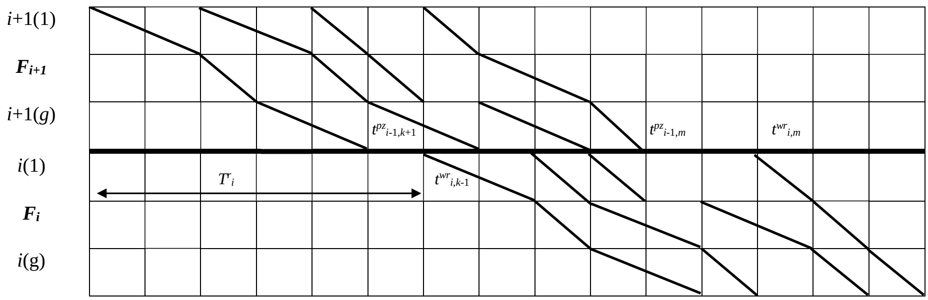


Figure 6.6. Synchronization of work groups R_2 realized on fronts F_{i-1} and F_i

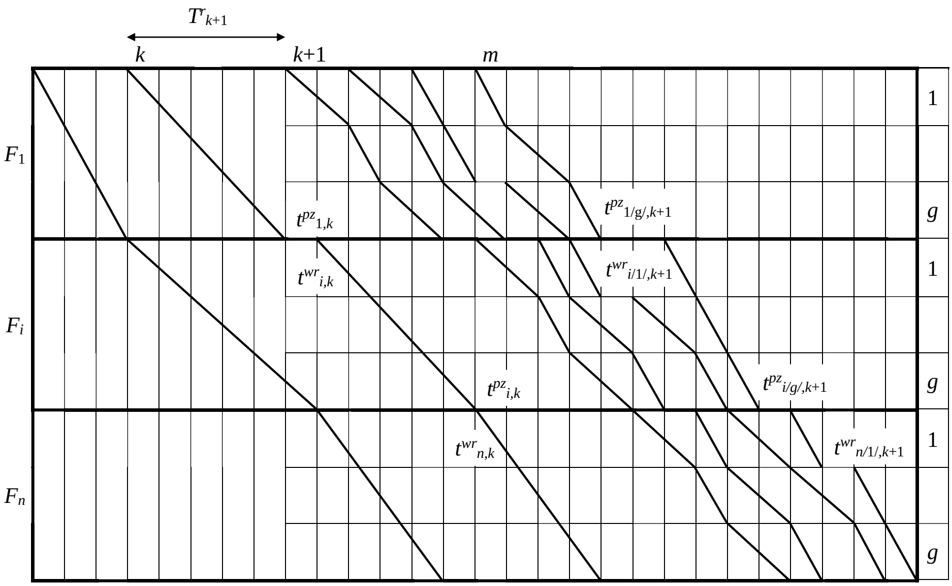


Figure 6.7. Consolidation of work subset R_2 with work subset R_1

7. Methods of organizing construction work with parallel-flow internal structure

According to the classification of work organization methods given in [1–3, 81, 83], one can distinguish a group of organization methods which make it possible to carry out jobs of one kind (parallel partial flows in a complex) in parallel. The parallel-flow methods are based on the principle that each of the firms (work groups) moves along an individual critical path (different from the paths along which the other firms move) on its work lots. This principle applies to both identical and different technological processes distinguished during the planning of a building structure. This problems was first addressed in [50] and further investigated in [80].

Parallel-flow methods are an expansion of the basic flow methods [1–3, 81, 83] used for planning the course of construction work. Taking into account the technological and organizational constraints that occur in practice, new methods of organizing construction jobs which make it possible to synchronize them and establish the order of carrying them out as well as algorithms for scheduling tasks at the minimum-time optimization criterion have been developed.

In the group of flow organization methods the following parallel-flow methods can be distinguished:

- with the continuous use of realization means [40];
- with the continuous performance of jobs on work fronts [41];
- taking into account both couplings between realization means and work fronts [51];
- taking into account simultaneously couplings between realization means, couplings between work fronts and diagonal couplings;
- taking into account simultaneously couplings between realization means, couplings between work fronts and reverse diagonal couplings.

The assumptions of the parallel-flow method with the continuous use of realization means and those of the method with the continuous performance of jobs on work fronts are discussed below. They were also discussed in [40, 41]. Since it is necessary to delineate the area of influence of time couplings on construction jobs and indicate the course of further studies, only a few methods are presented here.

7.1. Method of organizing parallel jobs of one kind

In the search for a solution to the problem of allocation of work fronts to construction firms[8], iterative procedures which allow one to investigate the possible

[8] On the basis of practical experience it is assumed that specialized or general-construction firms or gangs (teams) take part in the investment process. The terms: “gangs”, “construction firms”, “work teams” are used interchangeably in the description.

front-gang combinations through the successive algorithm steps and choose the configuration which will result in the lowest lead time value for the jobs on the front were used. A change in the rate of performance of jobs on work fronts caused by random factors lowers the reliability of a realization model. This aspect has been taken into account by incorporating a distribution function of job lead times obtained from statistical studies into the model.

Lead time *T* in which jobs on a given building structure or complex are carried out by constructions firms (one-speciality gangs) depends on the lead times in which the jobs on the work fronts on the building structure are carried out by the particular gangs, assuming that one gang (firm) works on one front:

$$T = \max \left\langle t_{nj}^{pz(B_1)} t_{nj}^{pz(B_2)};..., t_{nj}^{pz(B_k)};..., t_{nj}^{pz(B_s)} \right\rangle, \tag{7.1}$$

where: $t_{nj}^{pz(B_k)}$ – the latest date of completion of process *j* on front *n* by gang (firm) B_k.

$$p_{nj}^{pz(B_k)} = \sum_{i=1}^{n} t_{ik}^{B_k} + t_{ij}^{wr(B_k)}, \tag{7.2}$$

where:

$\sum_{i=1}^{n} t_{ij}^{(B_k)}$ – the total lead time for the jobs in process *j* performed by gang B_k on *n* fronts,

$t_{ij}^{wr(B_k)}$ – the earliest date of commencement of process *j* by gang B_k on the first $i = 1$ front.

The simulation of front-gang configurations is based on an initial matrix of job durations.

Step 1 – take the smallest valuefrom the set of elements of the first row of a submatrix comprising the lead times for the jobs in the first process.

$$\varphi_{\min} = \min_{i=1} \left\langle t_{ij}^{wz(B_1)}, t_{ij}^{wz(S_1)},..., t_{ij}^{wz(B_k)},..., t_{ij}^{wz(B_s)} \right\rangle. \tag{7.3}$$

The number of the gang carrying out process j and the number of the work front correspond to this value:

$$\varphi_{\min} = t_{ij}^{wz(B_k)} \rightarrow \langle 1, k \rangle. \tag{7.4}$$

[1] Set of labour demand submatrices
[2] *n*-th labour demand submatrix
[3] Divide submatrices into column vectors
[4] Matrix of gangs
[5] Divide matrices into submatrices and column vectors comprising groups of one-speciality gangs

[6] Assign appropriate gang matrix submatrices and column vectors to column vectors of labour demand submatrices
[7] Construct submatrices of expected lead times for each process and gang
[8] Check if one gang matrix column corresponds to one column of lead time matrix
YES
[9] Input values into final matrix
NO
[10] Create set of submatrices of expected lead times for each process and gang
[11] *n*-th submatrix
[12] Select minimum value from elements of first row of submatrix
[13] Assign number of front and gang to minimum value
[14] Fix commencement and completion dates for jobs on next front, i.e. for next row in process submatrix
[15] Compare expected job commencement dates and select minimum value, form pairs of numbers: front number and gang number
NO
[16] Is set of process submatrix fronts exhausted?
YES
[17] Is set of processes exhausted?
YES [9]
NO [11]
At the same time, create a set of pairs (front number–gang number) needed to make a schedule.

Step 2 – fix commencement and completion dates for the jobs belonging to a given process on the second work front, taking into account the time in which specified gang (firm) will carry out the jobs on the first front:

$$\begin{aligned} & t_{2j}^{wr} = t_{ij}^{wz} \\ & t_{2j}^{wr(B_1)} = t_{2j}^{wr(B_2)} = ,..., t_{2j}^{wr(S_s)} = 0\,. \\ & t_{wj}^{wr(B_k)} = t_{1j}^{B_k} \end{aligned} \tag{7.5}$$

Compare the expected commencement dates for the jobs on the second front and select the lowest value to which the date of commencing work on this front by the gang corresponds:

$$\begin{aligned} & \varphi_{\min} = \min_{i=2} \left\langle t_{2j}^{wr(B_1)}, t_{2j}^{wr(B_2)}, ..., t_{2j}^{wr(B_k)}, ..., t_{2j}^{wr(B_s)} \right\rangle \\ & \varphi_{\min} = t_{2j}^{wr(B_k)} \rightarrow \langle 2, k \rangle \end{aligned}\,. \tag{7.6}$$

Steps 3, 4 …, n – repeat the procedure as in step 2 until the set of work fronts is exhausted, assigning the elements of the set of fronts to the elements of the set of gangs to ensure the shortest lead time for a given kind of jobs.

All the above algorithm operations should be repeated for the remaining sets of work teams (firms) which are to realize the next processes and sets of work fronts. The algorithm enables the planning of work of gangs in a flow formed by the work organization method with zero couplings between realization means but without sequencing. By transposing the initial matrix and applying the above algorithm one can allocate work fronts to gangs using the work organization method with zero couplings between work fronts.

7.2. Planning continuous work of teams in work complex realization

Prior to fixing the dates of commencement and completion of jobs by gangs (construction firms) one should fix the expansion dates for partial flows $t_{rj}^{B_k}$ or $t_{ri}^{B_k}$ and then determine the values of the other time characteristics of the jobs in partial flows or complexes.

Values $t_{rj}^{B_k}$ for the particular processes and gangs can be calculated by applying the algorithm allocating work fronts to gangs.

To determine the expansion time for a specified partial flow using the flow work organization method with zero couplings between realization means one should create a set of work fronts on which jobs of given kind are performed by a given gang. Then in a fronts allocation table one should find the completion dates for the jobs on the fronts in the preceding process in the technological sequence.

Having these data, one can calculate the expansion time for the partial flows:

$$t_{rj}^{B_k} = \max_i \left[t_{i,j-1}^{pz(B_{k-1})} - \sum_{i=2}^{n-1} t_{ij}^{B_k} \right], \tag{7.7}$$

where:

$t_{i,j-1}^{pz(B_{k-1})}$ – the date of completion of a job by gang *k*–1 on front *i* in preceding process *j*–1,

$\sum_{i=2}^{n-1} t_{ij}^{B_k}$ – the sum of the lead times of jobs carried out by gang k on the allocated fronts.

This procedure should be repeated until the set of gangs (construction firms) is exhausted. The commencement and completion dates for the jobs are determined from this relation:

$$t_{ij}^{wr} = t_{ij}^{B_k} + \sum_{i=1}^{n} t_{i-1,j}^{B_k} , \tag{7.8}$$

in which: $\sum_{i=1}^{n} t_{i-1,j}^{B_k}$ – the sum of the lead times of the jobs in process *j* carried out by gang *k* on fronts *i*–1.

$$t_{ij}^{wz} = t_{ij}^{wr} + t_{ij}^{B_k}, \tag{7.9}$$

where: $t_{ij}^{B_k}$ – the lead time of a job in process j on front i carried out by a work team (construction firm).

To determine the time characteristics of a flow modelled by means of the flow work organization method with zero couplings between work fronts one can apply the above algorithm, but prior to this the initial table-matrix must be transposed. The algorithm enables the planning of engagement of contractors (work teams) without incorporating the sequencing problem into the organizational model.

<u>Application of algorithm to planning construction work complex realization</u>

Four building structures are to be realized. The construction jobs have been consolidated into four organizational sequences considering the homogeneity of the technological processes. The jobs of the first and fourth kind will be carried out in two partial flows. The planning data are given in the table-matrix below. The matrix elements are the lead times of the jobs performed on the building structures.

Table 7.1. Matrix of job lead times

PROJECTS	PARTIAL FLOWS					
	1_1	1_2	2	3	4_1	4_2
1		0 20 20	20 32 52	52 9 61		61 11 72
2	42 10 52		52 20 72	72 7 79	79 12 91	
3		57 15 72	72 25 07	97 4 101		101 18 119
4	89 8 97		97 18 115	115 12 127	127 20 147	

For the fixed sequence in which the jobs are to be realized the lead time is 147 units.

Step I

	1_1	1_2	2	3	4_1	4_2
1		0 20 20	20 32 52	52 9 61		61 11 72
	42 10 52		51 20 72	72 7 79	79 12 91	
						72 18 90
					91 20 105	

LPM = 105

	1_1	1_2	2	3	4_1	4_2
1		0 20 20	20 32 52	52 9 61		61 11 72
	44 8 52		52 18 70	70 12 82	82 20 102	
						72 18 90
					102 12 114	

LPM = 114

	1_1	1_2	2	3	4_1	4_2
2	0 10 10		10 20 30	30 7 37	37 12 49	
		15 15 30	30 25 55	55 4 59		59 18 77
					49 20 69	
						77 11 88

LPM = 88

	1_1	1_2	2	3	4_1	4_2
3	0 10 10		10 20 30	30 7 37	37 12 49	
		10 20 30	30 32 62	62 9 71		71 11 82
					49 20 69	
						82 18 100

LPM = 100

	1_1	1_2	2	3	4_1	4_2
3		0 15 15	15 25 40	40 4 44		44 18 62
	32 8 40		40 18 58	58 12 70	70 20 162	
						62 11 73
					90 12 102	

LPM = 102

	1_1	1_2	2	3	4_1	4_2
3		0 15 15	15 25 40	40 4 44		44 18 62
	30 10 40		40 20 60	60 7 67	67 12 79	
						62 11 73
					79 20 99	

LPM = 99

	1_1	1_2	2	3	4_1	4_2
4	0 8 8		8 18 26	26 12 38	38 20 58	
		6 20 26	26 32 58	58 9 67		67 11 78
					58 12 70	
						78 18 96

LPM = 96

	1_1	1_2	2	3	4_1	4_2
4	0 8 8		8 18 26	26 12 38	38 20 58	
		11 15 26	26 25 51	51 4 55		55 18 73
					58 12 70	
						73 11 84

LPM = 84

By calculating in the same way the tree node values at the lower levels we find the sequence of the building structures, i.e. 4–3–2–1. The lead time of the complex has been reduced from 147 units to 123.

8. Computer program for determining optimum sequence of jobs on work fronts and computing work organization model characteristics

8.1. Basic information on application software to drafting of work organization project

Introduction

The software makes it possible to use five construction work organization methods which greatly increase the decision manoeuvrability of a building production organizer. The methods take into account combinations of couplings between particular jobs performed on successive work fronts. The kinds of the incorporated couplings are the basis for the classification of the methods. The latter are applied to determine the optimum sequence of jobs on particular work fronts and then to calculate the work complex realization characteristics for this sequence. The characteristics are used to determine the effectiveness indices of a given realization whose course is planned according to the adopted work organization method. By comparing the effectiveness indices of the particular methods one can select the most effective method. The software is used to determine the optimum sequence of jobs and the basic characteristics of modelled processes for all the five methods.

For each work complex the following are determined:

- **The earliest job commencement and completion dates.**
- **The latest job commencement and completion dates.**
- **The coupling between work fronts**, i.e. the time between the completion of a current job and the commencement of the next job on a given work front. Such couplings determine the degree of continuity in the occupation of a front. If there are no downtimes on a front the couplings are equal to zero. They contribute significantly to the reduction of the lead time. They are minimized if the main criterion is to put the realized structures into service in the shortest possible time.
- **The coupling between realization means**, i.e. the time between the completion of a current job of a given kind and the commencement of a job of this kind on the next front. Such couplings determine the degree of continuity in the use of resources. If after completing a job on one front the gangs move without downtimes to the next front, the couplings are equal to zero. They are very important in building practice since they show the degree to which the gangs and the equipment are used.
- **The diagonal coupling**, i.e. the time between the commencement of a current job and the commencement of a job in the next partial flow on the preceding front.

- **The reverse diagonal coupling**, the time between the commencement of a current job and the commencement of a job in the next partial flow on the preceding front.
- **The total reserve time**, i.e. the difference between the latest and earliest date of commencement (completion) of a given job.

The software makes it possible to represent graphically the results of the computations:

a) for methods I and II to generate cyclograms of jobs with employment schedules,

b) for methods III, IV and V to generate cyclograms for early or late dates and draw a network of relationships.

The input data are prepared in the form of a matrix formed from the lead times of the particular kinds of jobs (determined according the output standards specified in job standard catalogues or other catalogues) on the particular work fronts (lots).

8.2. Illustrative application of the software

The capabilities of the software are demonstrated for a work organization case in an existing industrial plant.

Description of projects:

Truck unloading facility (structure 67a)

The truck unloading facility consists of two parts:

- an upper tower-like part,
- a lower ground-floor part.

The 6.0x13.8m (in plan) upper part consists of three above-ground storeys and one underground storey. The design provides for a platform for technical equipment at a level of +14.2m. The lower, ground-floor part constitutes a roofing for two truck unloading bays. The structure's in-plane dimensions are 10.8x20.0m and its roof-ridge height 5.30m. The truck unloading facility has been designed as a steel framework made of hot-rolled sections and vertical braces. The foundations have the form of reinforced concrete box which also functions as a basement.

Seed transport gallery (structure 67b)

The gallery consists of 9 bays and 3 seed transfer-support towers located at the places of seed transfer between floor levels. The gallery is a steel construction.

Railway seed unloading + cleaning facility (structure 16)

The railway seed unloading + cleaning facility was modernized by pulling down two of the three storeys and adding three new steel-construction storeys.

The modernized facility consists of:

- one underground storey and
- four overground storeys.

The building's dimensions in plane are: 12.0x36.0m. The height is 9.5m. The main structural components of the building are two-bay, two-storey, monolithic reinforced concrete frames in which the top storey was pulled down as part of modernization, a floor-roof with spandrel beams (the spandrel beams and the columns to the level of the floor over the unloading facility), a part of the roof in at level +5.45m, chutes and subtrack walls.

Railway seed unloading facility (structure 16a)

The structure consists of two parts: an underground part and an overground part.

The underground part has the form of a hopper for unloading railway cars.

The overground part is an enclosure for the railway car unloading station, which also serves as the structure supporting a gravity tank and platforms for transport equipment operation.

The underground part has the form of a reinforced concrete box with a width of 6.50m (outline) and an overall length of 28.2m. The overground part has the form of a steel construction constituting the load-bearing structure for the gravity tank and the equipment operation platforms and an enclosure for the seed unloading facility.

Overhead gallery (structure no. 4)

The gallery is made up of 8 spans and 2 transfer-support towers situated at the places where grain is transferred between floor levels.

The gallery route consists of two sections of about 60m total length. The gallery is a steel construction.

Structure no. 2 – complex consisting of: transformer station, electric switching station and control room (one building denoted as OST-3 in design), fan and cyclone rooms

The structure denoted by number 2 in the plans of the plant consists of the following structures: a transformer station, an electric switching station and a control room (one building denoted as OST-3 in the design), fan and cyclone rooms (denoted as 2a, 2b, 2c, 2d and 2e in the design).

<u>Division of structures into work lots and basic assumptions</u>

Transport gallery 67b is made up of bays of different length, assembled from shipped steel components on the ground and then fitted together on the support towers. Therefore the gallery was divided into four assembly lots (projects): 67b-1, 67b-2, 67b-3 and 67b-4. Other projects are: 67a, 16, 16a, 2, OST-3, 4, which are described earlier.

Then the jobs were placed into appropriate groups of technological processes, linked together into organizational sequences comprising jobs of one kind performed by specialist gangs. The gangs work without collisions on successive work fronts. Ten kinds of jobs were distinguished (described in detail in the project). During the grouping of the jobs it became necessary to take into account the organizational and technological factors affecting the lead times of the jobs, which until then had not been included in the cost estimate (e.g. technological and organizational downtimes, etc.).

One should bear in mind that the commencement of jobs of one kind on the next lot is contingent on the completion of the jobs of the same kind on the preceding lot. This is a basic condition for the application of flow organization methods to the scheduling of work complex 67a-67b.

In none of version the composition of the systems of technological processes were changed.

The lead times of the jobs and the composition of the gangs were determined on the basis of cost estimates, technical designs of the structures, practical experience and standards (Job Standards (JS) and Cost Estimate Standards (CES)). The labour demand and the machine demand for the distinguished kinds of jobs were determined.

Taking all this into account and fulfilling the overriding criterion of the shortest lead time, the **parallel-flow organization method** was chosen.

The technological and organizational constraints were taken into account in the calculations on the basis of which tables (matrices) of lead times for the jobs were drawn up.

If the realization of a complex of building structures is to be optimized, scheduling problems (determining the sequence of operations) must be solved. A change in the order in which jobs are performed on the particular projects has a significant effect on the technical-economic indices, particularly on the lead time for a complex of building structures. Therefore it is essential to determine the most advantageous (rational) sequence, but this is a highly laborious task requiring the testing of all the possible variants ($n!$). In presented example, there are 3628800 variants. Thus a controlled survey (directional search) is needed to find quickly an optimum sequence. This problem is solved by applying a modified branch-and-bound method, which is implemented in program *Organizator*.

Five flow methods were applied to work organization versions. Each of the options takes into account the technological and organizational constraints and so ensures collisionless work.

Method I ensures the continuous performance of jobs by gangs on a complex of structures. The continuity of work of one-speciality gangs in the realization of such a complex entails longer deployment of kinds of jobs, which affects the lead time of the whole complex.

Method II ensures the continuous performance of jobs on the particular structures but this results in gang work downtimes (if the job lead times differ between the structures).

Method III is a combination of the two above methods. As a rule it ensures the shortest lead time for the work complex but at the expense of gang-work and front downtimes. It is possible to find a critical path to determine the earliest and latest dates.

Methods IV and V take into account constraints expressed by diagonal and reverse diagonal couplings and they may contribute to a substantial reduction of the lead time for the whole complex of structures.

The *Organizator* software developed in the Building Design and Realization Division of the Institute of Building Engineering, Wrocław University of Technology was used for the calculations. The results obtained for optimal versions are given in table 8.1. They indicate that a reduction in the lead time has been achieved without any investments.

Table 8.1. Lead times for complex of structures depending on work organization methods

Method	Characteristic (Project system)	Lead time for complex [in days]
I	OST-3, 67a, 67b-1, 16a, 16, 4, 67b-2, 67b-4, 2, 67b-3	938
II	67b-3, 4, OST-3, 2, 16, 67b-1, 67a, 16a, 67b-2, 67b-4	472
III	2, 67b-2, 4, 67b-3, 67b-4, 67a, OST-3, 16, 67b-1, 16a	429
IV	2, 67b-2, 4, 67b-3, 67b-4, 67a, OST-3, 16 67b-1, 16a	599
V	2, 67b-2, 4, 67b-3, 67b-4, 67a, OST-3, 16, 67b-1, 16a	452

For example cyclograms of the unrhythmic work complex for methods III, networks of relationships for method III were drawn up.

The choice of a realization version and method was left with the investor and the contractor.

Each of the projects demonstrates that the work organization methods made it possible (through a combination of couplings) to analyze many versions of the realization process and to take the most proper decision about the performance of jobs on rationally ordered work fronts and as a result the work complex lead time was reduced without any additional material or capital expenditures.

9. Development of the Time Couplings Methods with the use of artificial intelligence methods

9.1. Introduction

Time Couplings Method (TCM) allows performing schedules considering technological and organisational limitations – TCM I (continuity of work on the working plots of land), TCM II (continuity of work of the construction brigades), TCM III (minimising the duration of the project implementation), TCM IV, V, VI (continuity of the construction processes with the use of catty-cornered couplings, reverse catty-cornered couplings, sites of works and implementation resources). TCM is based on the algorithmic notation, which enables the automatization of calculations and introduction of numerous limitations. Traditional methods of scheduling do not take into account the possibility to automatically prioritize tasks. In these methods, tasks are prioritised intuitively, basing on the engineering knowledge. The research is focused on modelling the construction projects, taking into account the tools of artificial intelligence as an optimisation calculation device. The research includes the issues, such as perfecting time couplings methods using metaheuristic algorithms, optimisation of the duration and cost correlation with the use of artificial intelligence methods (evolutionary algorithms, Tabu Search etc.), scheduling of construction projects with fuzzy duration of completing the tasks, planning construction projects with the dependencies between the duration, cost and resources, using genetic algorithms and a hybrid evolutionary algorithm. TCM was devised and developed by Professors V. Afanasjew (1980, 1985, 1988, 1994), J. Mrozowicz (1982,1997), Z. Hejducki (2000, 2004, 2004) and others. Specific methods of flow nature are involved in TCM application. The continuation of developing these methods has been undertaken. The issues pertaining to scheduling of construction processes with the use of artificial intelligence methods are displayed in (fig. 9.1).

This chapter is based on the materials of the authors presented in the work by Rogalska, Hejducki "The application of time coupling methods in the engineering of construction projects", Technical Transactions 9/2017.

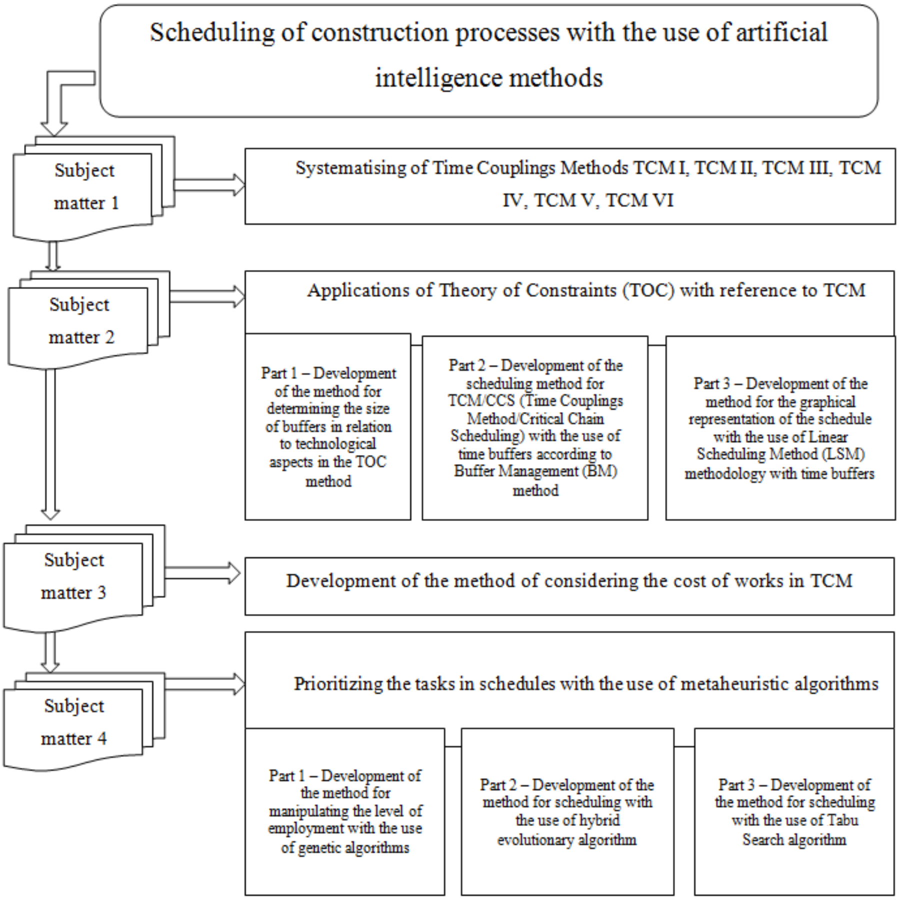


Figure 9.1. Scheduling construction processes with the use of artificial intelligence methods

9.2. Systematizing Time Couplings Methods TCM I, TCM II, TCM III, TCM IV, TCM V, TCM VI

While covering Subject matter 1, TCM methods from I to VI were systematised. Various applications of each of the six Time Couplings Methods were explained in a simple way in the following publications: Hejducki, Rogalska (2005, 2005, 2011, 2011), Rogalska, Hejducki, Łodożyński (2011), Rogalska, Hejducki, Wodecki (2014). Previous scientific on TCM were denied wide recognition because of extremely complicated mathematical notation. The methods were described according to the nomenclature and symbols used in the international literature. Priority premises in particular methods were presented, also for the comparative purposes, and schedules for every method were developed using the same database. It made presenting the research results in foreign journals possible. The most remarkable achievement of this stage was publishing a monograph Hejducki, Rogalska (2011) entitled "Time Couplings Methods: Construction Scheduling and Time/Cost Optimization". The effects of covering Subject matter 1 include systematising the TCM methods, introducing international symbols and indicating directions for further development of TCM.

9.3. Applications of Theory of Constrains (TOC) with reference to TCM

Subject matter 2 consists in introducing the premises of Goldratt's Theory of Constraints (TOC) into Time Couplings Method (TCM) – the research results were presented in publications: Bożejko, Hejducki, Rogalska, Wodecki (2011), Hejducki, Rogalska (2004, 2004, 2004, 2005, 2005, 2007, 2011, 2011, 2011). In TOC, projects buffers (PB) and feeding buffers (FB) are introduced into the schedule in order to secure a contractor and project manager from the delay in the date of completing the works. In theory, the size of time buffers constitutes 25% of an allotted critical path. In papers Rogalska, Hejducki (2005, 2005), the influence of choosing a critical path and, at the same time, the influence of the system of project and feeding buffers on the overall duration of completing the project was analysed. The analysis performed and the calculations done indicate that shortening the duration of particular processes by 25% does not result in shortening the overall duration of the whole project by the same value. The duration of completing the whole project depends on the choice of the particular elements in the critical chain. While analysing graphical models – cycle graphs built in accordance with the methodology of Linear Scheduling Model (LSM) – it can be noticed that adopting the critical chain which is the one closest to the right edge of the graph results in the largest shortening the duration of completing the project. What is more, in cycle graphs it is necessary to treat feeding buffers (FB) as separate processes, due to the condition of preserving the continuity of work of the construction brigades and avoiding financial losses connected with unnecessary stoppages. The method of calculating the size of time buffers, different from the value proposed by Goldratt (25%), was introduced. The newly proposed solution takes into consideration aspects, such as the level of risk in completing a particular process, as well as technological limitations, resulting from the manufacturing techniques, opportunities to accelerate the process completion and the necessity to preserve technological breaks. The elements of scientific novelty in Subject matter 2 are as follows:

- introducing TOC into the TCM method,
- determining the way of producing cycle graphs according to LSM in the TOC/TCM method and introducing international symbols,
- developing the methodology of calculations in TOC/TCM in accordance with Goldratt's premises and considering risk factors and technological fac-

tors of the influence, such as a change in the size of feeding buffers and a project buffer,
- performing the analysis of choosing the critical path from the available critical chains in the aspect of minimizing the duration of completing the project.

9.3. Development of the method of considering the cost of works in TCM

In Subject matter 3, the second criterion of optimisation, namely the cost minimization, was introduced into TCM Rogalska, Hejducki (2005). The methodology of calculating the correlation between the cost and duration in the TOC/TCM method while planning and completing construction projects was proposed. The method developed assumes three stages of creating schedules. The first stage is the output schedule, formed in a traditional way. The schedule of the second stage considers estimation of the risk connected with the duration and cost of completing the works. It is assumed that the risk can be estimated using all the available methods, including the Delphi method, fuzzy sets, automatic neural networks, Monte Carlo method (Risk 4.0), Method of Construction Risk Assessment (MOCRA) etc. As the result of the carried out analysis, the level of risk is estimated for particular processes and the second schedule is created, usually with the longer duration of completing the works, considering the risk connected with the duration and cost of completing the project. The third stage assumes the risk reduction by introducing levelling factors, such as insurances, changes in the logistics system, in the involvement of resources, work organization etc., as well as introducing feeding buffers and a project buffer, adequate for the opportunity to reduce the works. As the result of the changes made individually for every process, the third schedule is created and modified by reducing the influence of risk factors and by introducing time buffers. The correlation between the duration and cost of the project is estimated again, whereas the extra time and financial resources are left as security for completing the task. Schedules of the particular stages are presented in the following way – the 1st stage for the investor, the 2nd stage for the contractor and the 3rd stage for project manager.

The elements of scientific novelty in Subject matter 3 are as follows:

- developing the concept of introducing two criteria for optimization of the duration and cost into the TCM method,
- developing the methodology for the 3-stage scheduling, considering risk factors and reducing them by technological and organizational changes and by introducing TOC/TCM method,
- displaying the functional correlation between the size of buffers and the value of variance of the process duration, with the value of variance resulting from on the risk analysis.

9.4. Prioritizing the tasks in schedules with the use of metaheuristic algorithms

In Subject matter 4, resources were introduced into the TCM method as the third criterion of optimization. The methodology was developed to apply metaheuristic algorithms to prioritize the tasks in TCM, considering the duration, cost and using the resources Bożejko, Hejducki, Rogalska, Wodecki (2011), Hejducki, Rogalska (2011), Rogalska, Bożejko, Hejducki (2005). There was a premise adopted, according to which a schedule is an equivalent of the chromosome. New modified schedules are obtained by mutation, crossing and selection. Limitations, which ensure the logical construction of the estimated schedule and correct decoding of chromosomes, were introduced. Hard limitations, namely the ones which can never be omitted, are connected with the order of completing the tasks. They are taken into account in the objective function. For instance, the objective function was determined so as to get a regular level of employment of the construction site employees. Genetic algorithms, hybrid evolutionary algorithms and the Tabu Search algorithm were used for calculations.

In Part 1 of Subject matter 4, traditional evolutionary algorithms were used for calculations, because of their high usefulness for finding the range containing the optimal goal function value. The calculations result in an approximation of the value (a so-called suboptimal solution).

In Part 2 of Subject matter 4, hybrid evolutionary algorithms (memetic algorithms) were used for calculations, Bożejko, Hejducki, Rogalska, Wodecki (2008), Rogalska, Bożejko, Hejducki (2006, 2007, 2008), Rogalska, Hejducki, Wodecki (2008). The activity of an algorithm starts with creating the initial population, which might be created in a random way. The best member of the initial population is assumed to be the suboptimal solution. A new population is generated in the following way – a set of local minima is established and certain elements are determined, considering these population members which appear on the same positions in local minima, forming a set of determined elements and positions. Every permutation of a new population has certain elements on the determined positions. Permutations of finite sets can be identified with set population members of a set in a certain order. The algorithms finishes after generating a number of generations determined in advance – usually it was a number of generations from 10 000 to 100 000. The calculations were considered to be correct when, after increasing the number of generations by for instance 1000, a better result was not obtained. The results of the numerical experiments connected with using a hybrid evolutionary algorithm for scheduling the construction project were presented in the papers. The case of optimal planning of the course of construction works, while adopting a criterion of measuring the regularity in the demand for resources (the level of employment of the workers), was taken into account. Moreover, the limitations connected with applying the methodology of the critical chain (CSS/BM) were accepted. A classical genetic algorithm and a modified hybrid evolutionary algorithm were used for optimisation calculations.

Applying the Tabu Search (TS) procedure constitutes Part 3 of Subject matter 4, Rogalska, Hejducki (2010). This procedure was used for obtaining optimal solutions or solutions not far from the optimal ones. The basic idea of the algorithm is searching the space, including all the possible solutions, using a sequence of movements. In the sequence of movements, there are movements which are not allowed – called tabu movements. The algorithm avoids oscillating around the local minimum, as it is storing the information about the already checked solutions in the form of the tabu list (TL). Fuzzy numbers, used for the calculations, were generated from the average time in such a way that the time shorter than the average time by 16.6% was adopted as the minimum, whereas the time longer than the average by 33.3% was adopted as the maximum. The aim of the calculations did not consist in determining the value of fuzzy numbers of the duration of processes, but checking the possibility of using the Tabu Search algorithm for searching for the optimal solution – the minimum duration of the construction project. The results obtained were compared with the calculations made using the TS method and deterministic data. The average approximation errors of algorithms [%] were used for measuring the correctness of calculations. The average approximation errors were as follows – 18.4% for the deterministic TS and 7.5% for the fuzzy TS. Thus, it might be assumed that when the priority of tasks is determined by using the algorithm with fuzzy parameters, the value of the objective function is changed to a much smaller extent while the duration of completing the tasks is disturbed.

The elements of scientific novelty in Subject matter 4 are as follows:

- development of the method of scheduling with the use of ***Tabu Search algorithm***,
- replacing deterministic data with fuzzy data, according to the accepted rule, with the aim of obtaining the result with a smaller approximation error.

9.5. Summary and Conclusion

The research topic discussed in the paper has a specific character as it relates to the unique accomplishments of the authors. There is no possibility of citing other researchers' work. The work schedule is an essential part of the construction process. It is important to take into account the constraints of resources and working methods. Due to algorithmic nature of the calculations, there is a possible application of many new solutions to the TCM as: application of controlled buffers depending on technological and organizational conditions or using of probabilistic values of execution times of construction processes.

The thesis addresses the problems of modelling construction works which have not been solved to date, with time couplings being taken into account. These refer to methods of accomplishing consecutive works without considering their concurrence, as well as parallel performance methods with works of one kind or various works being carried out in parallel.

There are presented some issues which arise when one considers diagonal and inversely diagonal couplings, oblique couplings and their combinations, making up new models of work construction organization, as well as algorithms for work synchronization and practical numerical examples are given. Methods for organizing works of parallel structure have been developed, together with the algorithms for work scheduling, taking time couplings into account, which constitute parallel relationships between works and their complexes. The various effects of time couplings on works and their complexes ensuring a series structure are given. Time couplings constituting series relationships between works and groups of works have been identified. In order to get a more thorough picture of organizational possibilities expressed by means of time couplings, variants of both parallel and series stream methods have been presented. Each of the new methodologies presented has been verified by numerical examples relating to realization of construction works. They verify the new organizational methods incorporating time couplings.

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