Informatics methods as tools to solve industrial problems

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Spis treści

Artificial intelligence techniques in cloud manufacturing

dr inż. Grzegorz Kłosowski

Keywords
Artificial intelligence, fuzzy logic, cloud computing, cloud manufacturing, multi-criteria selection

Abstract
The paper presents the cloud manufacturing concept with multi-selection mechanism based on artificial intelligence. In particular, artificial intelligence techniques may be useful in the process of linking orders with contractors. This process is featured by a high degree of complexity due to the need for automated selection of potential contractors of previously placed orders based on various criteria (e.g. machine park, company location, etc.) for the defined production orders. The process of linking orders with suppliers is automatic and due to the fact that it is implemented in real time it should run quickly. The dynamics of the process is driven by changes that may occur at any time in companies considered as potential contractors. The changes referred to may include issues related to the quantity, quality, functionality and occupancy of available at the time CNC machine tools which are monitored in a system governed by the cloud manufacturing concept by intranet integration in LAN/WAN.

1. Introduction

The concept of cloud manufacturing involves the use of the idea of cloud computing in manufacturing processes. Cloud computing is a solution based on ubiquitous and cheap access to the Internet. Because of this, it would be possible to abandon the previously popular information model in which each computer should be equipped with its own storage medium (e.g. hard disk) for storing programs and information files. In enterprises, such concept forces maintaining their own, complex and expensive to maintenance the IT infrastructure, which includes servers, software, workstations, specialized IT personnel, etc.

The concept of cloud computing assumes that both data and computer programs are brought outside the company. The term "cloud computing" reflects important feature of this concept which is a topographical indeterminacy where the resources are stored. Actually, both the software and data are stored in so-called data centers which are managed by cloud services providers. The cloud computing general model includes three sub models: SaaS (Software as a Service), PaaS (Platform as a Service) and IaaS (Infrastructure as a Service) [1]. In short, both the software and data can be accessed via web browser in cloud computing concept. Therefore, there is no obligation to maintain an expensive whole-house IT infrastructure. Just fast, reliable Internet connection, and workstations for running a web browsers are enough. Given the service nature of the use of cloud-based technology, the entrepreneur simply enters into a contract with an appropriate provider of cloud services, and pay only for the use of resources (time of use, number of users, etc.). This cost scheme allows significant reduction of expenditure incurred by the company to maintain a sphere of information.

2. Cloud manufacturing concept

Since cloud computing is a concept for a strictly IT sphere, the question arises of the possible application of this idea in other areas of business. The concept of cloud manufacturing is a proposal to apply the idea of cloud on the basis of industrial applications. It assumes the development of an Internet portal, acting on a highly automated place to sell and buy manufacturing orders and production capacity. Portal would enable registration of principals who accurately and completely specify the subjects to perform. From the other side, the cloud manufacturing portal should allow to register the manufacturers having some CNC machines. This is important because the machines should be able to connect to the LAN or WAN. Thus, realizing the concept of cloud manufacturing portal would have a real-time access to all necessary information concerning the means of production. In addition, the portal would provide system MRP/ERP as a service (access in your web browser), whose main purpose would be to schedule in advance all the tasks of the machines registered in the cloud manufacturing. This is necessary in order to automate the planning of production orders.

The above-described concept of cloud manufacturing contains a number of problematic issues that require separate solutions. These include such issues as:

- Developing an interactive form through which customers could define the production orders
- Developing interfaces for real-time tracking the availability of manufacturing resources of registered production plants (potential contractors).
- Development of MRP/ERP systems offered as a service SaaS.
- Developing mechanisms for choosing the best performer for the defined production order.
- Adjusting the SaaS/PaaS portal to handle a variety of industries.
- Solving the problem of data security and protection, as well as the adjustments to the rules on the international nature of the projects (such as when the client and contractors are located in different countries), etc.

This paper focuses on the problems of the best performer selection mechanisms in relation to the described production order. One of the main tasks that should be realized by the described computer system is the analysis and evaluation of potential contractors in the context of different criteria. The above problem can be divided into multi-selection problem. There are many methods and algorithms that can be applied to solve the problem, but due to the simplicity and availability of ready-made tools fuzzy logic deserves special attention.

Applications of fuzzy logic in the multicriteria decision-making problems enjoys a growing interest in recent years. As previously mentioned, there are many methods and tools that can be applied to solve specific problems without the need for a detailed exploration of issues of fuzzy sets. In this paper an example of a fuzzy controller is presented, whose task is to select the best contractor for the specific contract manufacturing. The controller was developed using the popular Matlab Simulink simulation software.

As previously mentioned, the main goal of the portal functioning according to the idea of cloud manufacturing, is to associate the respective manufacturers to a particular order. It seems that the rational move is to look for providers who meet the criteria due to the nature of the order rather than seeking orders for specific performers. The first model gives much more possibilities than the second one since a single structurally and technologically complex order, can be performed by many manufacturers. In this case, the approach of seeking orders for the contractor would be inefficient.

So the starting point are the pieces of information about the subject that the client wants to manufacture. This information should be entered into the database using an interactive form. A sample form should include the following information about the object of the contract: the weight of parts, dimensions, type of material, complexity level (development of materials, structure of the product etc.), technical documentation (drawings), optional – process specifications (technology spec.), etc. Based on the mentioned above information that was entered into the system by the customer the automatic selection of appropriate suppliers (contractors) that were also previously registered was performed, and their resources are monitored in real time.

This choice is made based on specified criteria, for which the quantity and type, and weight should be established in advance. This issue belongs to the realm of multi-criteria analysis. Methods of multi-criteria analysis are known for many years, but with the rapid development of information technology and increased computational power of PCs, the development of tools that are used in the mentioned above issues is being made. As already noted, in recent years, fuzzy logic has been used very successfully.

3. Fuzzy controller for multicriteria selection

Described in this paper fuzzy logic controller was based on Mamdani model. The controller has the specific inputs corresponding to different criteria for suppliers selection. The following criteria for selecting suppliers to given order was defined:

1. Experience of the contractor regarding the type of production,
2. Ratio of manufacturing processes that can be done in the company to the amount of all manufacturing processes of given order,
3. Distance between the client and the contractor,
4. Order execution time,
5. Price.

Mentioned above inputs (selection criteria) are defined as linguistic variables, with values from the interval (0 1) (Fig. 1).

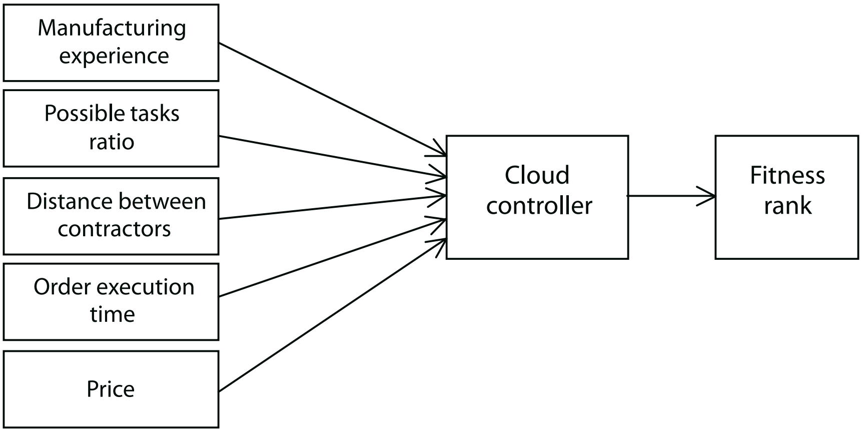

Fig. 1. Fuzzy controller: inputs and output

The first selection criterion (input variable) reflects the experience of the contractor regarding the proper type of production. This is the variable that can take real values between 0 and 1 In the next stage, the value of the variable is fuzzified with five triangular shape membership functions according to formula (1).

$$\mu_{\tilde{A}}(x,a,b,c)=\max(\min(\frac{x-a}{b-a},\frac{c-x}{c-b}),0) \quad (1)$$

where: a, b, c - parameters determining the location of the various angles of triangle, $x \in R$, $0 \leq x \leq 1$

The membership functions of the described input variable vary the parameters a, b, and c . The function shapes correspond to the five linguistic variables that determine the degree of membership of an input parameter to a particular fuzzy set (Table 1).

Tab. 1 Triangular fuzzy numbers associated to linguistic variables of Manufacturing-experience

Fuzzy variables	**Parameters**		
	a	*b*	*c*
Very Poor	0	0	0,3
Poor	0,1	0,3	0,5
Fair	0,3	0,5	0,7
Good	0,5	0,7	0,9
Very Good	0,7	1	1

Graphic waveforms of individual fuzzy variables used to fuzzification of input variable called Manufacturing-experience presents Fig. 2. It can be observed that used in each membership function parameter values cause the fuzzy variables are not identical.

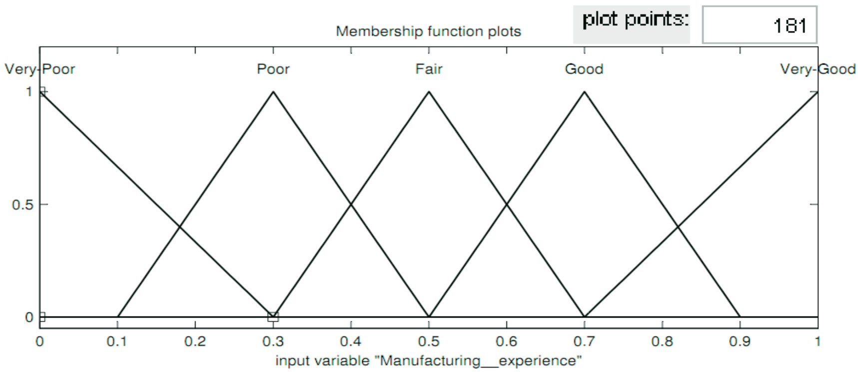

Fig. 2. The linguistic terms of fuzzy variable, Manufacturing-experience and membership functions

Parameters of membership function for the other four input variables shows Tab. 2, while the graphic waveforms were presented on the example of the input variable Possible-task-ratio in Fig. 3.

Tab. 2 Triangular fuzzy numbers associated to linguistic variables of other inputs

Fuzzy variables of different inputs				Parameters		
Possible-task-ratio	Distance-between-contractors	Order-execution-time	Price	a	b	c
Very Low	Very Far	Very Long	Very Expensive	0	0	0,25
Low	Far	Long	Expensive	0	0,25	0,5
Average	Average	Average	Average	0,25	0,5	0,75
High	Close	Short	Cheap	0,5	0,77	1
Very High	Very Close	Very Short	Very Cheap	0,75	1	1

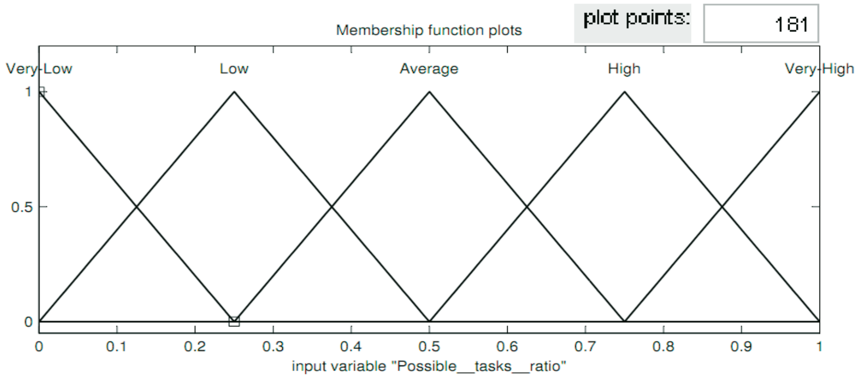

Fig. 3. The linguistic terms of fuzzy variable, Possible-task-ratio and membership functions

Another operation, following the fuzzification phase, is to carry out specific actions on fuzzy sets. The nature of these activities is determined by the fuzzy rules. In this case, the following five fuzzy rules was applied:

1. If (Manufacturing experience is Very-Poor) or (Possible tasks ratio is Very-Low) or (Distance between contractors is Very-Far) or (Order execution time is Very-Long) or (Price is Very-Expensive) then (Fitness rank is Very-Poor)
2. If (Manufacturing experience is Poor) or (Possible tasks ratio is Low) or (Distance_between_contractors is Far) or (Order execution time is Long) or (Price is Expensive) then (Fitness rank is Poor)
3. If (Manufacturing experience is Fair) or (Possible tasks ratio is Average) or (Distance_between_contractors is Average) or (Order execution time is Average) or (Price is Average) then (Fitness rank is Fair)
4. If (Manufacturing experience is Good) or (Possible tasks radio is High) or (Distance between contractors is Close) or (Order execution time is Short) or (Price is Cheap) then (Fitness rank is Good)
5. If (Manufacturing experience is Very-Good) or (Possible tasks ratio is Very-High) or (Distance_between_contractors is Very-Close) or (Order execution time is Very-Short) or (Price is Very-Cheap) then (Fitness rank is Excellent)

The described controller provides the ability to assign different weights to different fuzzy rules, but due to the nature of the problem it was decided that all the rules will be treated equally.

Both the quantity of the fuzzy rules as well as their design depends on many factors and there is no strict rule governing these issues. Certainly an important element is the number of input and output variables, and the number of the fuzzy variables for both the inputs fuzzification and the outputs defuzzification cases. Fig. 4 shows the surface of possible solutions which shows the values it can take the output variable (Fitness-rank) based on two selected input variables: Possible-task-ratio and Manufacturing-experience.

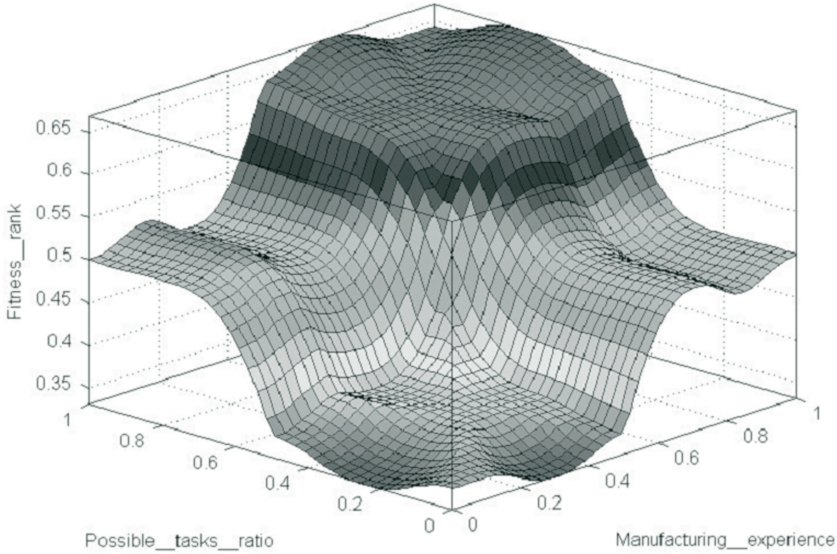

Fig. 4. The answer surface for 2 inputs: Possible-task-ratio and Manufacturing-experience

As can be seen, the surface shown in Fig. 4 is very irregular in shape, indicating a high complexity of implemented inference mechanism. In addition, it must be recognized that Fig. 4 shows only a fraction of the problem because the fuzzy controller takes into account as many as five criteria when selecting contractors for each job, and not just two of them.

In the picture mentioned above the major advantage of inference systems based on fuzzy sets - their effectiveness can be seen. The development of ICT technologies has made that with relatively little effort using ready-made software for modeling it is possible to develop a driver that takes into account a lot of subtle elements, not omitting even the minor issues that could have any influence on the decision on the selection of the contractor.

Fig. 6 and 7 present how the driver works. Fig. 6 indicates the case in which all five input variables take the average values (0.5). The first five columns in the figure correspond to the five input variables while the sixth column corresponds to an output variable. The first five rows reflect five fuzzy rules while the bottom-right corner of the figure shows the step of defuzzification in order to obtain a scalar output variable value. It must be performed because output of each rule is fuzzy. This stage is realized using the Centroid Defuzzification Technique according to formula (2) and Fig. 5.

$$y^* = \frac{\int y \cdot \mu_{\tilde{c}}(y)dy}{\int \mu_{\tilde{c}}(y)dy} \qquad (2)$$

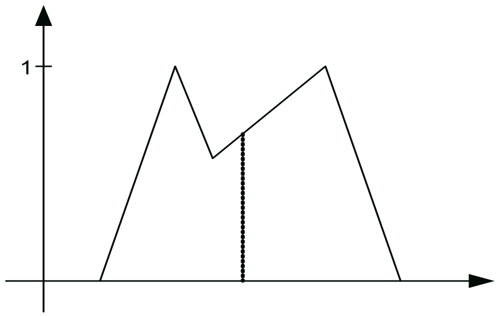

Fig. 5. Centroid Defuzzification Technique

As shown in Fig. 6 the average values of inputs cause that the average value of output variable is obtained.

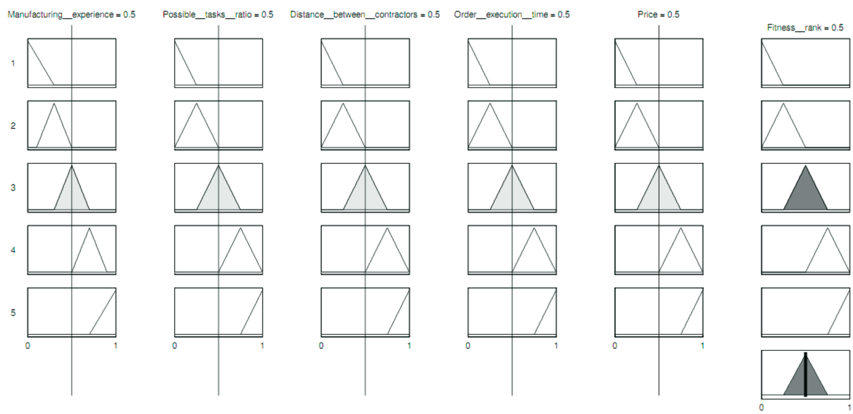

Fig. 6. Fuzzy reasoning: case "average"

Another case of the same application process conducted by the same rules shows Fig. 7. Here, by contrast, input variables take variety of values. As the result the value of output variable of 0.457 is obtained.

With the obtained results, a number between (0 1) can be assigned to any potential contractor which determine its suitability to perform a specific manufacturing order. Contractor that received the highest score should be preferred for the selection.

In practice, the selection mechanism should allow the human factor. It is hard to imagine the automation extent so wide that the financial decisions (including entering into an agreement with the contractor) would be made without the human (manager) participation. On the other hand one can imagine a situation where the machine would give the client a choice among a few selected suppliers with information on prices and lead times offered by each of them.

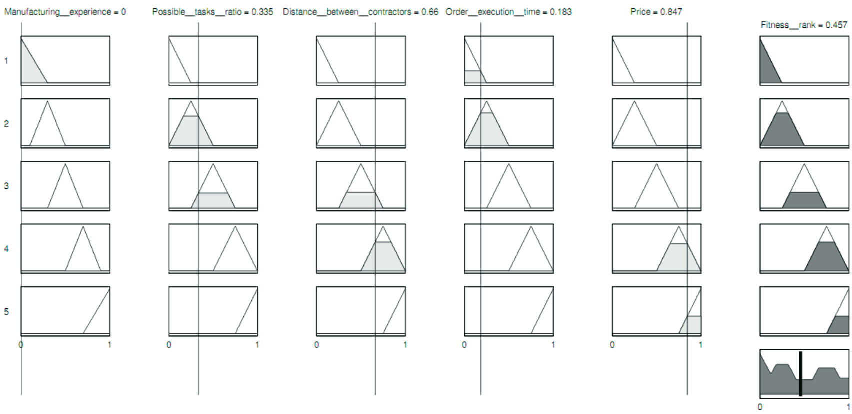

Fig. 7. Fuzzy reasoning: case "mixed"

The described above mechanism of contractors selection is suitable for simple jobs that can be performed entirely by a single contractor. In practice, there are orders placed, both in terms of design and technology which due to its complexity must be implemented in several stages by different manufacturers. In such a situation it arises the problem of allocation of orders among the various groups of activities and for individual activities and scheduling. In case of complex orders the criteria for selection of the contractor cannot remain the same as in case of simple orders or semi complex orders.

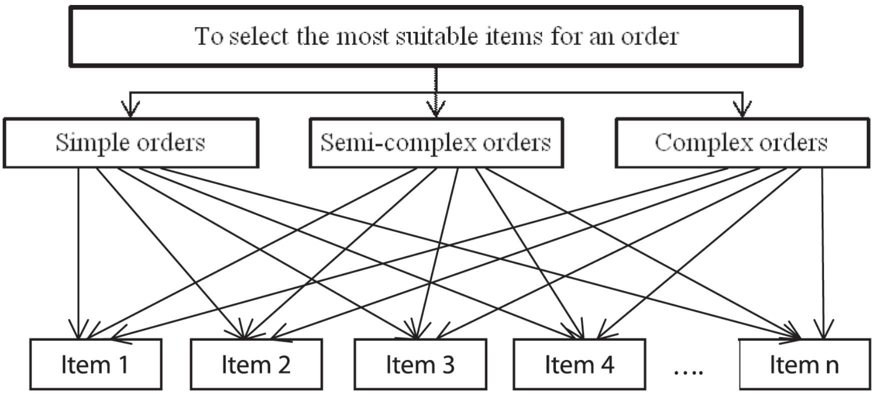

Fig. 8. Description of decision-making process (ranking) from n alternative items for an order under criteria [2]

In order to solve the problem of criteria diversity in relation to different types of orders it is possible to use the method of FMCDM (Fuzzy Multicriteria Decision Making) used for example in Recommender Systems. A simplified version of this method can assume three-step orders division: simple orders, semi-complex orders and complex orders. Each type of production order has assigned its own set of criteria for contractors selection (Fig. 8). Therefore, the whole multi-selection process can be divided into two similar stages.

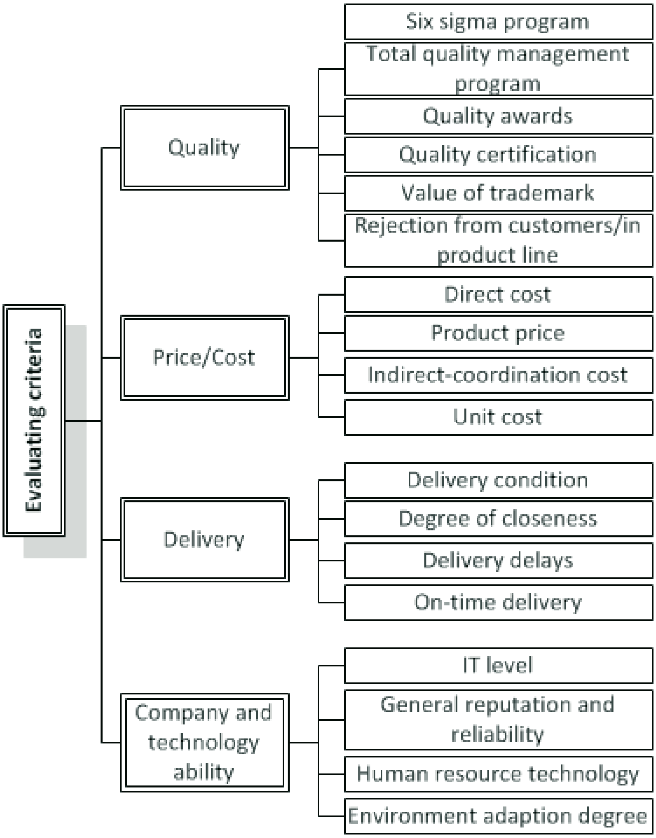

Fig. 9. Hierarchy of evaluating criteria and sub-criteria to soft computing process [4]

The first stage is classification of given order into one of three generic groups, which is synonymous with the assignment of a specific group of criteria for the contractor selection. The second stage takes into account already described in detail above the choice among a number of potential contractors then present them to client and give him a possibility for final choice. It should be noted that the first phase of this process is fully automatic and does not include human intervention. Besides, it is a singular choice what means that the specific order must be clearly classified into a particular generic group.

The classification can be arranged through the proper selection of the output variable membership function. If the output variable takes the real values from the interval (0 1) this interval can be divided into sub-bands to which correspond different types of orders, such as:

- (0 0,33) – simple orders,
- (0,34 0,66) – semi-complex orders
- (0,67 1) – complex orders

Identification of relevant selection criteria is an essential element of decision-making model, especially in organizing the supply chain. In such cases a number of aspects are considered mainly related with the profile of the company. In this case it is easy to see many parallels to the problem of supplier selection. The difference is that when looking for a contractor for a particular job the nature of cooperation is short-lived. In contrast, when looking for a supplier or company contractor the co-operation is usually long-term nature (for multiple orders). Criteria for the selection of suppliers can be divided into two groups: quantitative criteria and qualitative criteria. The choice of multi-criteria selection method depends on the kinds of criteria that are taken into account when choosing a supplier or contractor.

Depending on the type of contractor selection criteria many methods can be used in multi-criteria selection process, such as: Data Envelopment Analysis (DEA), Fuzzy Set Theory (FST), Analytic Hierarchy Process (AHP) [6], Decision Support System (DSS) based on the AHP model [7], Cluster Analysis (CA) [8], Case Based Reasoning (CBR) [9]. Basically, Mathematical Programming (MP) models take into account the quantitative criteria [10]. In practice the hybrid solutions can be used which are combining both methods that are adequate for the quantitative and qualitative criteria, as exemplified by the AHP method (Fig.9) [3][4].

4. Conclusions

In this paper the proposal of use of fuzzy multi-criteria selection in cloud manufacturing concept was presented. Cloud manufacturing concept is a complex idea that consists of many different units, processes, services and incorporated methods, such as cloud computing, smart cloud, internet of things etc. One of the key processes is manufacturer selection. Basing on the collected detailed information about the particular order, computer system have to make a choice of some potential manufacturers that best fit to the given order.

The process of assigning the appropriate contractors for the specific task is complex therefore soft computing methods seem to be suitable and particularly artificial intelligence. This article provides an example of the use of fuzzy logic to solve one of the key problems of cloud manufacturing concept. This paper also contains a brief analysis of currently available methods and trends in this area of science. Clearly, the rapid development of computer techniques has a significant impact on the way of processes realization within the logistics chain. Direction of ICT changes indicates a steady progress in the automation of decision processes which impacts on a tangible sphere that is known as the Internet of things.

Zastosowanie metod sztucznej inteligencji w technologii chmury produkcyjnej

Streszczenie

W artykule zaprezentowano sposób wykorzystania mechanizmu selekcji wielokryterialnej bazującego na sztucznej inteligencji, w metodologii chmury produkcyjnej. W szczególności, techniki sztucznej inteligencji mogą być użyteczne w procesie kojarzenia zleceń z wykonawcami. Proces ten charakteryzuje się dużym stopniem złożoności, z uwagi na konieczność przeprowadzenia zautomatyzowanego doboru potencjalnych wykonawców złożonych zamówień w oparciu o różnorodne kryteria (np. posiadany park maszynowy, lokalizacja przedsiębiorstwa itd.) dla zdefiniowanych zleceń produkcyjnych. Proces kojarzenia zleceń z dostawcami ma charakter automatyczny i z uwagi na to, iż realizowany jest w czasie rzeczywistym, powinien przebiegać szybko. Dynamika procesu powodowana jest zmianami, jakie mogą nastąpić w każdej chwili, w przedsiębiorstwach rozpatrywanych jako potencjalni wykonawcy. Zmiany, o których mowa mogą dotyczyć kwestii związanych z ilością, jakością, funkcjonalnością i zajętością dostępnych w danym momencie obrabiarek CNC, które dzięki intranetowej integracji w sieciach LAN/WAN, są na bieżąco monitorowane w systemie realizującym koncepcję chmury produkcyjnej.

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Cooperation between CRM and ERP systems in enterprise

dr inż. Agnieszka Bojanowska

Keywords
CRM, ERP, customer relationship management, enterprise resource planning, implementation, enterprise, organization

Abstract
CRM system is generally implemented in enterprise which has developed ERP structure. The most frequent technical problems connected with implementation are caused by these two systems. These systems should effectively exchange important data. This paper characterizes these two systems and is related to possibility of mutual adjustment of CRM and ERP, paper shows possibilities of cooperation between these two systems. In paper were pointed more important benefits from ERP and CRM implementation and also from cooperation between CRM and ERP. Author is focusing on problem through the prism of CRM systems and their requirements. In cooperation between CRM and ERP could help some other system implemented in organization. About such systems was also mentioned in this paper.

1. Characteristic of ERP and CRM

ERP (Enterprise Resource Planning) can be defined as a packaged business software system that lets an organization automate and integrate the majority of its business processes, share common data and practices across the enterprise and produce and access information in a real-time environment. The ultimate goal of an ERP system is that information must only be entered once.[14] Figure1 shows the concept of the ERP system. ERP system has influence indirectly on customers, employees and suppliers. It has data from Central Database and gives back some information there. The most important thing is to provide effective information flow through Sales and Distribution, Service, HR, Financial Department, Analytical Department, Production Department ect. The other system which largely basis on information is CRM – Customer Relationship Management. Such systems use information technology (IT) based applications to support and integrate various business functions so to achieve standardization, synchronization, coordination, improved information management and customer responsiveness.[21]

In some organizations, CRM is simply a technology solution that extends separate databases and sales force automation tools to bridge sales and marketing functions in order to improve targeting efforts. Other organizations consider CRM as a tool specifically designed for one-to-one customer communications, a sole responsibility of sales/service, call centers, or marketing departments.[16]

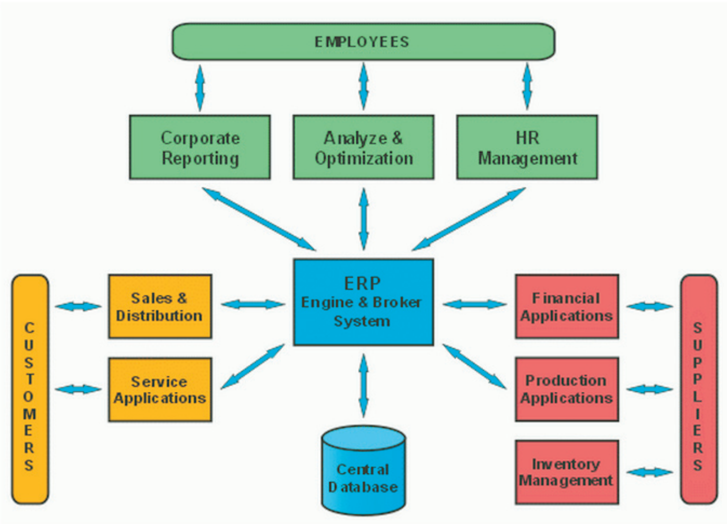

Figure 1. The concept of the ERP system [6]

This paper require such definition of CRM system which contains technical information about system. CRM is holistic management conception but it is based on IT technology (example of such solution was shown on Figure 2). An enterprise builds a database about its customers. This database describes relationships in sufficient detail so that management, salespeople and customer service representatives can access information, match customer needs with product plans and offerings, remind customers of service requirements and know what other products a customer had purchased.[14] Without database and IT solutions CRM won't work properly in enterprise. CRM systems collect information about customers, especially about:

- their behavior,
- history of contacts with enterprise,
- history of payment,
- history of loyalty,
- preferences about products and services,
- history of service requests.

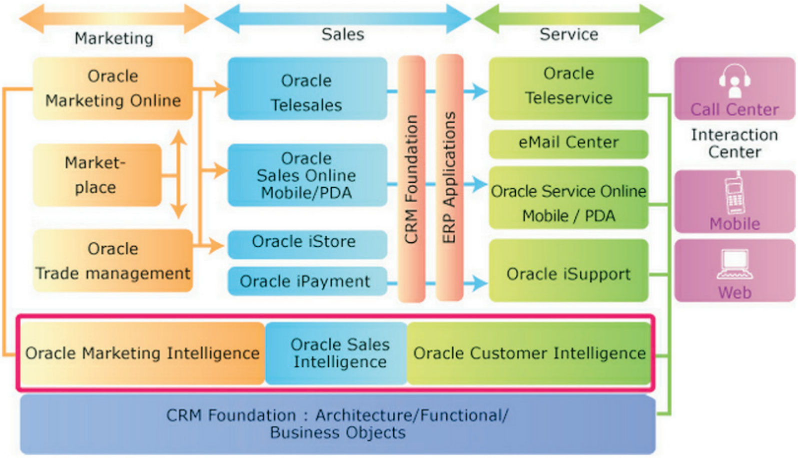

Figure 2. Oracle CRM solution – example of CRM system in enterprise [15]

As was shown on Figure 1 ERP system have connection witch customers by sales, distribution and service applications. It is important to provide high level data by this channels from CRM system. In CRM systems there are some parts responsible for marketing, sales, service. ERP application, as was shown on Figure 2, play an important role between service and sales fields. It is very important to connect market requirements with enterprise resources. CRM can help to identify requirements of customers at sale and service field. It is said that effectively work CRM system can lead to such relation with customers when some synergy occur. Enterprise Resource Planning can be easier because data from Customer Relationship Management systems can determine the amount of resources for production, creation new products and services or for after sale service. It is also easier to estimate future demand not only for products and services but also for complementary goods and for consumables. By skilful relation management, customers can be actively participated in product or service creation process. Information from customers can help to improve production process but this information should be provided at the right time and at the right place in ERP system. The same thing is with demand for service and for consumables. CRM gives us some tools like cross-selling and up-selling. Cross-selling is the action or practice of selling among or between established clients, markets, traders, etc. or the action or practice of selling an additional product or service to an existing customer.[11] Up-selling is a sales technique whereby a seller induces the customer to purchase more expensive items, upgrades, or other add-ons in an attempt to make a more profitable sale.[1] These tools can also have impact on work of ERP system.

2. Ways of CRM and ERP integration

ERP and CRM are both systems which requires a lot of data. This factor can help to designate some main ways to integrate these two systems:[Own work based on:19]

- *Full integration* – idea is that the ERP vendor delivers a product in which you configure specific CRM functionality just like current components. There is a single central database, on which the system operates. The borders between ERP and CRM become more and more vague toward this ideal. Example of such solution is Oracle solution for some organizations.
- *Modules* – Some ERP vendors offer a module for CRM next to their standard ERP packages. The m,odule is developed to work solely with the corresponding ERP. As a result, the integration level is quite high. Example of such solution is SAP.
- *Bolt-ons and interfaces for 3rd party product* – Best-of-breed CRM vendors are becoming more and more aware of the fact that their clients use ERP. For quite some time now, they design or tailor their products to connect with certain third-party ERP systems as bolt-ons. Tere are also cooperations between ERP and CRM vendors for developing an interface, which lead to a shorter implementation time and better integration. Example of such solution can be Siebel's CRM and Epicor ERP.
- *Existing ERP with CRM from ASP* – There are some ASPs who can handle the integration of hosted ERP into existing CRM system or conversely. But for the time being this is very costly and ineffective solution.

In CRM implementation process very important issue is to effectively share supply chain information. A supply chain is essentially a network of inter-connected and interdependent organizations mutually and cooperatively working together to control, manage and improve the flow of materials and information from the suppliers to the end users.[13] There is conception of SCM (Supply Chain Management) which is the management of a network of interconnected businesses involved in the provision of product and service packages required by the end customers in a supply chain.[12] These conception gives basis for CRM implementation. Building relations with suppliers is very similar to building relations with customers. CRM and ERP are very important systems in whole system connected with Collaborative Planning, Forecasting and Replenishment (CPFR) which is a concept that aims to enhance supply chain integration by supporting and assisting joint practices. CPFR seeks cooperative management of inventory through joint visibility and replenishment of products throughout the supply chain. Information shared between suppliers and retailers aids in planning and satisfying customer demands through a supportive system of shared information.[10] Figure 3 shows how CPFR connects manufacturer and retailer. When system works properly promotions and forecasts, which can be costly, can be shared and financed by both parties. CPFR has an influence on CRM and ERP and also on APS (Advanced Planning System) which is a type of system that tracks costs based on the activities that are responsible for driving costs in the production of manufactured goods. An APS allocates raw materials and production capacity optimally to balance demand and plant capacity.[18]

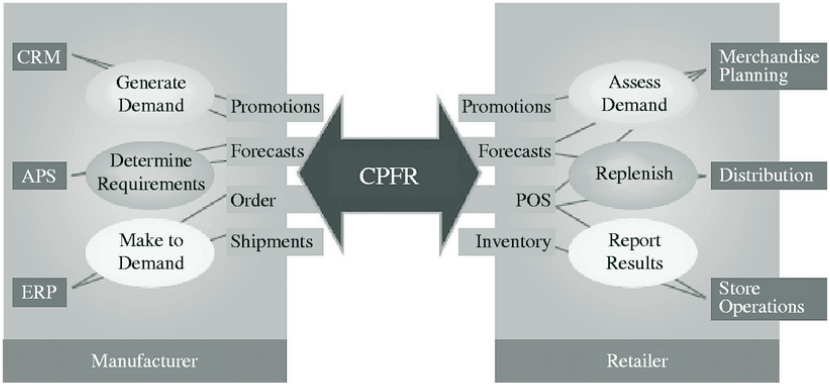

Figure 3. Collaborative Planning, Forecasting and Replenishment in Supply Chain Managment [8]

In CPFR model is very important to integrate analysis, strategy and planning, execution, demand and supply management. As we can see on Figure 4 there is one center of activities between manufactures and retailer – customer. These approach, which puts customer into center of interests, is base for customer relationship management philosophy. Nowadays, in global competition, entrepreneurs couldn't forget about customer. As we can see on Figure 4 there are some aspects of CPFR which are closely related to ERP systems such as order planning or production and supply planning. Theses aspects should be fields to integrate CRM and ERP because ERP without knowledge from CRM couldn't meet the demands of global market. For example from CRM system ERP can gain information about customers needs, preferences, complains, need of complementary goods or consumables. These knowledge force size of production and demand for materials. Numerous analyzes can help to provide demand and market preferences. In modern CRM system we have some tools as data mining which can help to estimate customer behavior which can help to effective enterprise resource planning.

Figure 4. The CPFR model with retailer and manufacturer tasks aligned with their corresponding Collaboration Tasks [2]

Enterprise Resource Planning systems tend to deploy Supply Chain Management and/or Customer Relationship Management techniques, in order to successfully fuse information to customers, suppliers, manufacturers and warehouses, and therefore minimize system-wide costs while satisfying service level requirements.[20] Systems implemented in organization take advantage of knowledge gained through the use of data mining techniques. Both CRM and ERP can be also supported by data mining. It is said that data mining can discover information hidden within valuable data assets. Knowledge discovery, using advanced information technologies, can uncover veins of surprising, golden insights in a mountain of factual data.[7] Particularly through data mining – the extraction of hidden predictive information from large databases – organizations can identify valuable customers, predict future behaviors, and enable firms to make proactive, knowledge-driven decisions.[17] Thanks to data mining systems such as CRM or ERP can create new knowledge from data. Figure 5 shows how simple but effective is process of data mining and how it works for management systems in organization. Also ERP systems can use data mining directly to creating new knowledge or indirectly by CRM which uses data mining.

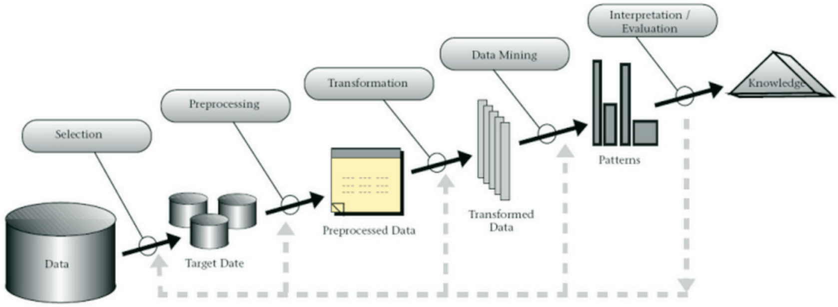

Figure 5. KDD process by Fayyad 1996 [3]

3. Profits from the integration of ERP and CRM solutions for the enterprise

When entrepreneurs decide about some implementation of system such CRM or ERP they expect some profits. The implementation should be economically justified. Some profits could be uncountable but most of profits should be quantifiable and significant. Such systems as CRM or ERP are expensive. Besides software entrepreneurs should invest in to stuff training, hardware and intranet. They can use existing solutions such as Oracle CRM or SAP ERP. But sometimes they need specially tailored solution for their organization. The important issue is to adjust implemented system to existing solutions. Such systems as CRM and ERP should cooperate effectively to bring adequate rate of return.

When ERP and CRM cooperate enterprise can achieve inter alia benefits from operational activity of CRM:[9]

- improved response time to customer requests for information,
- delivered product meeting customer requirements,
- reduced costs of buying the product/service,
- reduced costs of using the product/service,
- immediate access to order status,
- greater breadth of solution options,
- more responsive technical support.

When CRM system provides valuable data to ERP system, enterprise can achieve such profits as:[5]

- streamlining processes and workflows with a single integrated system;
- reduce redundant data entry and processes and in other hand it shares information across the department;
- establish uniform processes that are based on recognized best business practices;
- improved workflow and efficiency;

- improved customer satisfaction based on improved on-time delivery, increased quality, shortened delivery times;
- reduced inventory costs resulting from better planning, tracking and forecasting of requirements;
- turn collections faster based on better visibility into accounts and fewer billing and/or delivery errors;
- decrease in vendor pricing by taking better advantage of quantity breaks and tracking vendor performance;
- track actual costs of activities and perform activity based costing;
- provide a consolidated picture of sales, inventory and receivables.

Obviously these benefits are results from not only cooperation between CRM and ERP but also from efficiency of ERP system itself.

Integration between CRM and ERP solutions enables everyone in organization – from sales and marketing to accounting and finance to support and shipping – to work together, efficiently, in the business of building profitable customer relationships.[4] Such integration has high impact on employees. Integrating CRM with ERP gives employees greater customer insight by enabling them to:[4]

- quickly identify cross-sell and up-sell opportunities,
- easily access payment history information,
- identify additional purchasing power – or lack thereof,
- identify availability of products for Sales,
- quickly assist customers with order status,
- generate more accurate quotes and proposals.

All these profits show that there is need in organization to integrate CRM and ERP systems.

3. Conclusions

Nowadays when enterprises implement more and more new systems into their organization there is high need to provide effective cooperation between implemented systems. CRM and ERP are system which exchange important data. There are some different ways to integrate these systems: full integration, modules, bolt-ons and interfaces for 3rd party product and existing ERP with CRM from ASP. All of them have some advantages and disadvantages. Entrepreneurs should develop some solution which will be the best for their organization and will bring significant benefits. In these paper were listed some important benefits from CRM and ERP implementation and from cooperation between these systems. But everything is deepened on organization, stuff, training, quality of software, quality of hardware and obviously on management.

Współpraca modułów systemu CRM z modułami ERP

Streszczenie
System CRM jest najczęściej wdrażany w przedsiębiorstwie, które ma już rozbudowaną strukturę ERP. Najczęstsze problemy techniczne związane z wdrożeniem są powodowane współpracą pomiędzy tymi dwoma systemami. Muszą się one efektywnie wymieniać ważnymi danymi. Artykuł pokrótce charakteryzuje oba systemy i dotyczy możliwości wzajemnego dostosowania CRM do ERP, wskazuje na możliwe sposoby współpracy tych systemów. W artykule wymieniono ważniejsze korzyści jakie można uzyskać z wdrożenia systemu ERP oraz systemu CRM a także ze współpracy tych systemów w przedsiębiorstwie. Autorka skupia się na problemie przez pryzmat specyfiki systemów CRM i ich wymagań. We współpracy tych systemów mogą pomagać lub pośredniczyć inne systemy wdrażane w przedsiębiorstwie. O tego typu systemach również wspomniano w niniejszym artykule.

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Analysis of the effect of the parameters of drilling on the durability arms of drill bits

dr inż. Piotr Jaremek

Keywords
durability, wear, drilling bit, numerical solution

Summary
The article presents the results of impact assessments speed and axial load on the durability of tricone drill equipment. They are presented in graphical form and the analysis of fixed regularity.
The most important factors governing the wear of arms borer is the persistence of elements of its weapons and process parameters such as: drilling speed and axial load acting on drill bit. This pattern of the teeth is the phenomenon of their slip relative to the destroyed rocks. This is the wear of abrasive in the lubricant, can what is drilling. Mathematical modeling of the rock drilling process and the diagnosis (simulation) on this basis, the wear of drill bit arms, allows for optimized selection of tools.

On the basis of the developed [3-5] general mathematical model of kinetics wear material of drill bit arms in conditions of sliding friction in contact with the rock were considerations by two methods have two functions that describe the distribution of contact pressure. For each of the methods has been developed detailed models: the first, second and third degree approximations, taking into account the parameters of drilling process, the geometry of the tricone drill bit as well as physico-mechanical property of the material of tribological pairs. Developed models made it possible to carry out the analysis of wear of the drill bit teeth in function of the duration of the process of drilling, rotational speed and pressure axial operating at the borer according the following dependencies [6,7]:

The Method I

- model of a first degree (I.1)

$$T_i = \frac{AR_{oi}\tau_t^m h_{**}\left(e^{cm\frac{h_*}{h_{**}}} - 1\right)}{60u^2 n_1 R_i s_i \tau_o^m cm} \tag{1}$$

- model of the second degree (I.2)

$$T_i = \frac{AR_{oi}\tau_t^m h_{**}\left(e^{cm\frac{h_*}{h_{**}}} - 1\right)}{60u^2 n_1 (R_i - h_* \cos\delta) s_i \tau_o^m cm} \tag{2}$$

- model of the third degree (I.3)

$$T_i = \frac{AR_{oi}\tau_t^m}{60u^2 n_1 s_i \tau_o^m} \cdot \int_0^{h_*} \frac{e^{cm\frac{h}{h_{**}}}}{R_i - h\cos\delta} dh \tag{3}$$

where:

$$\tau_o = f \cdot \sqrt{\frac{E \cdot N \cdot [1 - \sin(0.5\gamma)]}{\pi(1-\nu^2) l_\Sigma \cdot [0.5 l \cos(0.5\gamma)]}} \tag{4}$$

The Method II

- model of a first degree (II.1)

$$T_i = \frac{A\tau_t^m R_{oi}}{60 R_i s_i f^m u^2 n_1}\left(\frac{\pi(1-\nu^2)_\Sigma}{EN\left(1 - \sin\frac{\gamma_i}{2}\right)}\right)^{\frac{m}{2}} \frac{\left[\left(h_* \sin\frac{\gamma_i}{2} + \frac{l}{2}\cos\frac{\gamma_i}{2}\right)^{\frac{m}{2}+1} - \left(\frac{l}{2}\cos\frac{\gamma_i}{2}\right)^{\frac{m}{2}+1}\right]}{\left(\frac{m}{2}+1\right)\sin\frac{\gamma_i}{2}} \tag{5}$$

where:

$$M_i = \frac{R_{oi} A\tau_t^m}{60u^2 f^m R_i n_1 s_i}\left[\frac{\pi(1-\nu^2)_\Sigma}{NE\left(1 - \sin\frac{\gamma_i}{2}\right)}\right]^{\frac{m}{2}} \left[\left(\frac{m}{2}+1\right)\sin\frac{\gamma_i}{2}\right]^{-1},$$

$$D_i = \left(\frac{l}{2}\cos\frac{\gamma_i}{2}\right)^{\frac{m}{2}+1}.$$

- model of the second degree (II.2)

$$T_i = \frac{A\tau_t^m R_{oi}}{60(R_i - h_* \cos\delta) s_i f^m u^2 n_1} \left(\frac{\pi(1-\nu^2)_\Sigma}{EN\left(1-\sin\frac{\gamma_i}{2}\right)} \right)^{\frac{m}{2}} \frac{\left[\left(h_* \sin\frac{\gamma_i}{2} + \frac{l}{2}\cos\frac{\gamma_i}{2} \right)^{\frac{m}{2}+1} - \left(\frac{l}{2}\cos\frac{\gamma_i}{2} \right)^{\frac{m}{2}+1} \right]}{\left(\frac{m}{2}+1\right)\sin\frac{\gamma_i}{2}} \tag{6}$$

- model of the third degree (II.3)

$$T_i = \frac{A\tau_t^m R_{oi}}{60 f^m s_i u^2 n_1} \cdot \left(\frac{\pi(1-\nu^2)_\Sigma}{EN\left(1-\sin\frac{\gamma_i}{2}\right)} \right)^{\frac{m}{2}} \cdot \int_0^{h_*} \frac{\left(\frac{l}{2}\cos\frac{\gamma_i}{2} + h\sin\frac{\gamma_i}{2} \right)^{\frac{m}{2}}}{R_i - h\cos\delta} dh \tag{7}$$

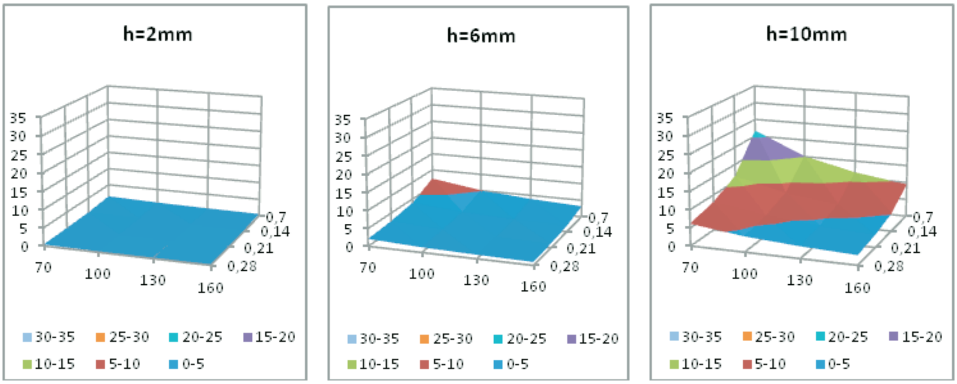
In the above equations assume the following indications: *A, m* - dimensionless ratios of frictional strength of the test material in the tribological association and accepted the conditions of the test, *c* - unknown constant determined by experimental, *f* - coefficient of sliding friction, $\tau_o = \tau_{max}$ for h=0 i x=0; $\tau_o = fp$ (0,0), $\tau_t = 0.5\sigma_{0.2}$ - yield stress of the material shear, $\sigma_{0.2}$ - the contractual yield of material tensile strength, $h_* = h \rightarrow h_{**}$ - discrete values of wear; h_{**}- permissible wear of teeth, γ_i- the vertex angle of the teeth, δ - angle of inclination of plane of the rims to the plane of the cross section of the hole, R_Φ - rolling radius of the drill rim relative to the rock hole, R_i - the radius of the rim, s_i- the jump between teeth, $u = \omega_2/\omega_1$ - number of gear, $n_1 = \frac{30\omega_1}{\pi}$ - rotational speed of the drill bit, ω_1 - angular speed of the borer, ω_2- angular velocity of bit, E,ν - Young's modulus and Poisson number rocks, N - load on the borer, $l_\Sigma = \sum_1^{\tilde{n}_i} l_{ni}$ - the length of the sum of the line of contact of the teeth borer with the rock, l_{ni}- the length of the line of contact of individual teeth, l - the width of the top of the teeth, $\tilde{n}_i$ - the number of teeth on each rim simultaneously in contact with the rock, L_i - sliding friction road of the teeth on the i-th rim per rotation of the borer, $n_i = \frac{2\pi R_i}{s_i}$ - number of teeth per i-th rim. Carried out an analysis of the impact of both the basic parameters of the process of drilling: axial load action on the drill bit and speed of borehole, on the durability of the work in the drilling hole. The analysis calculation assumes the following values for the parameters of drilling process: axial load N=0,07; 0,14; 0,21; 0,28 *MN*, rotational speed n_1=70; 100; 130; 160 *obr.*/min. The calculations were carried out for all combinations of values of parameters. The method first calculated on each rim the average longevity $\{Ti=f(h_*)\}$of the teeth borer in accordance with the accepted forms of models of second and third degree approximations for the developed methods and equations and described: (2), (3). The calculations were permissible wear of h_{**}=0,01m. Calculation of durability made for discrete values of wear of the teeth borer h_*=0,001; 0,002...0,01$m \rightarrow h_{**}$. Also according to the other methods is calculated on each rim the average longevity $\{Ti=f(h_*)\}$ of the teeth borer in accordance with the accepted forms of models of second and third degree approximations are designed for second and described equations: (6), (7). The calculations were permissible wear of teeth h_{**}=0,01m. Calculation of durability made for discrete values of wear of the teeth borerh_*=0,001; 0,002...0,01$m \rightarrow h_{**}$. Design parameters of the drill bit are provided in tab 1.

Table 1. Parameters of the drill bit (tricone)

bit/rim	R_{oi} [mm]	R_i [mm]	n_i [*pieces*]	γ_i [deg]	s_i [mm]	l_{ni} [mm]	L_i [mm]
I/1	143	92	20	42	29.7	18	67.42
I/2	103	73	17	44.9	28.3	10	57.24
I/3	65	45	12	43.6	25.7	10	47.30
I/4	17	13	5	41.4	25.4	27	29.86
II/1	145	92	19	42	31.3	13	68.11
II/2	110	80	19	43.8	27.9	12	58.79
II/3	78	55	14	43.7	26.2	10	50.85
II/4	40	28	7	43.3	27.3	10	39.21
III/1	140	90	21	42	28.3	27	66.80
III/2	90	64	16	44	26.5	10	53.98
III/3	55	38	10	44.3	25	10	44.28

Other data adopted for calculations: u=1.57, f=0.3, l_Σ =0.175m, l=0.002m, rock-granite, parameter values: τ_t=385MPa, E=2·10^4MPa, ν=0,25, A=1,27·10^5, c=1,2, m=1,8.The results of numerical solutions of durability of exter bit rims: I/1 and I/4, carried out in accordance with the first method based on the model I.2 are given in figure 1 (a), (b). While the analogue solutions carried out on the basis of the model and the I.3 is shown in figure 2 (a), (b). The results of the calculated durability T [hrs] (vertical axis) is shown in the following charts for discrete values of wear with 4 mm. For values h=0m not drawn chart, because the durability, so the time necessary to achieve the desired wear is constant for all possible combinations of parameters of drilling and is T=0 what confirms the carried out calculations.

a)



b)

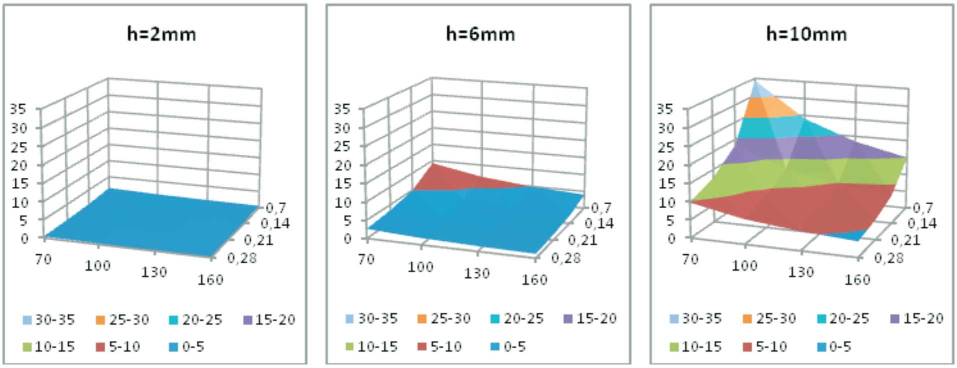
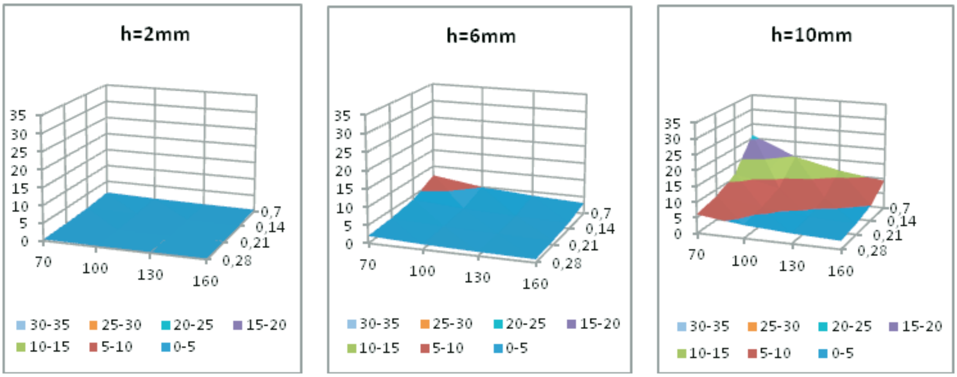


Fig 1 Durability calculated in accordance with a model 1.2 for equipment of bit number 1 and held a) to 1 rim (I/1) b) to 4 rim (I/4)

a)



b)

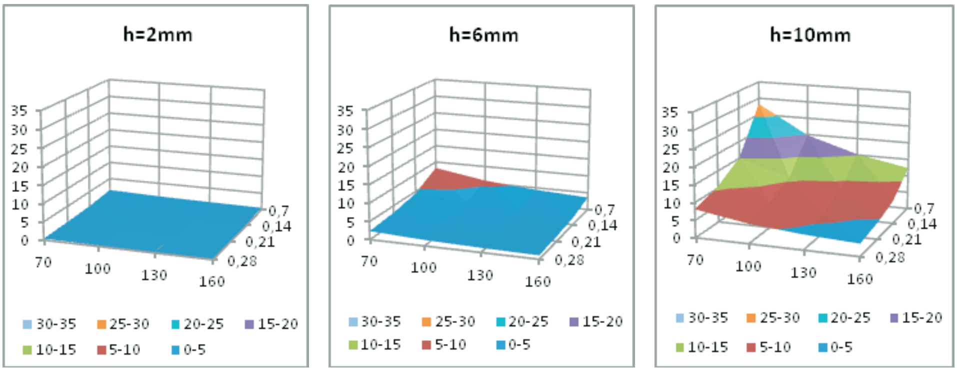
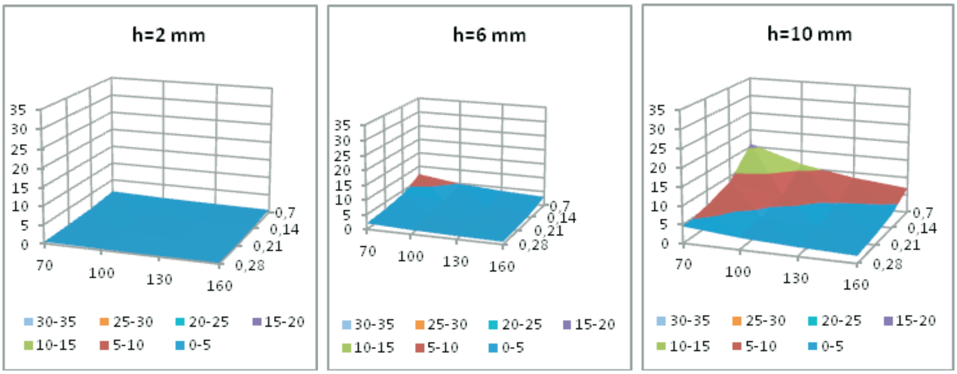


Fig 2 Durability calculated in accordance with a model 1.3 for equipment of bit number I and held a) to 1 rim (I/1) b) to 4 rim (I/4)

As for the calculations carried out in accordance with the use of the method of the first, were made the same calculation using the method second. The results of numerical solutions for model II.2 is given in figure 3 (a), (b) and for the model II.3 in figure 4 (a), (b).

a)



b)

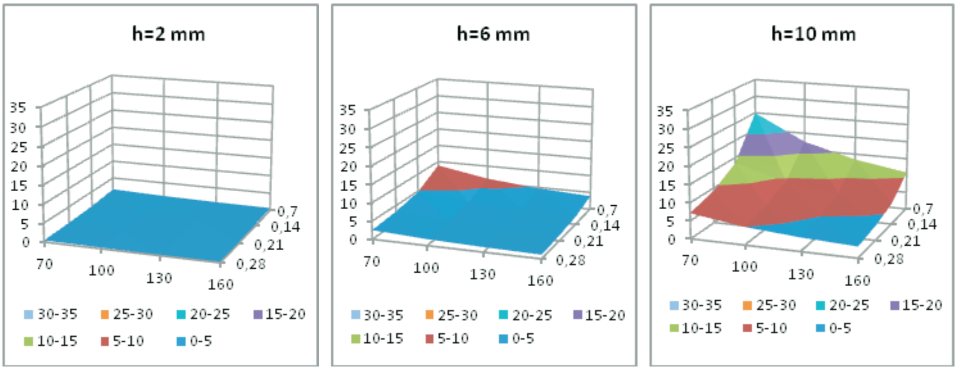
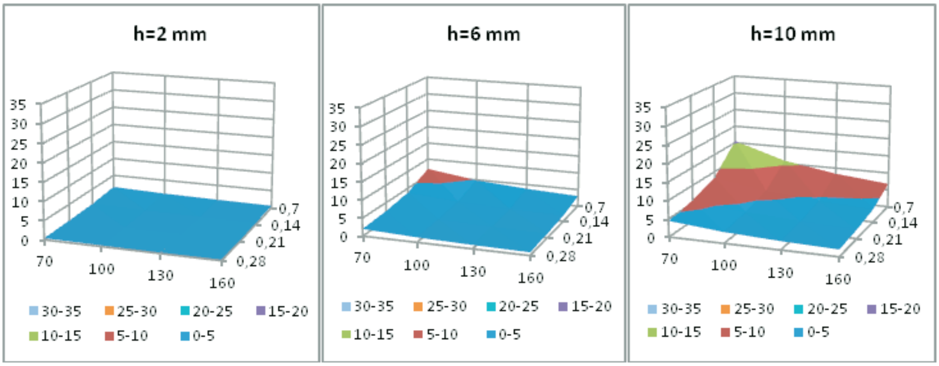


Fig 3 Durability calculated in accordance with a model 2.2 for equipment of bit number I and held a) to 1 rim (I/1) b) to 4 rim (I/4)

a)



b)

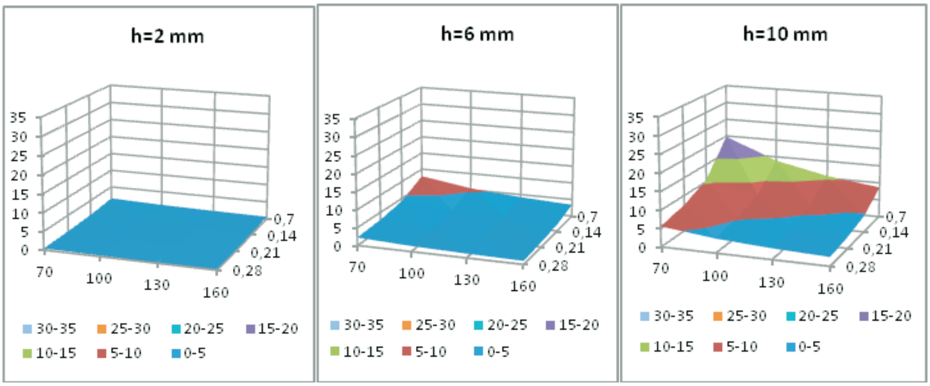


Fig 4 Durability calculated in accordance with a model 2.3 for equipment of bit number I and held a) to 1 rim (I/1) b) to 4 rim (I/4)

Conclusions

Analysis of the results of numerical solution, as shown in Figure 1, 2, 3, 4 allows you to draw the following conclusions:

For the rims external (I/1) the durability of the calculated by models 1.2 and 1.3 are close to each other for the entire range of the rotational speed and pressure axial during the process of drilling to achieve wear permissible ± 10 mm. The same fact can be noted in the case of the application in the analysis of models of 2.2 and 2.3.

For the rims of the internal (I/4) over time of drilling, durability calculated by the model 1.2 is more than durability calculated on the basis of the model of 1.3. These differences are not noticeable when wear is 2mm but grow with the wear of

6 mm. This fact apparently emphasizes the moment achieve by wear its limit, the limit values. The same can be observed in the case of the use of models in 2.2 and 2.3

Durability of the rims external I/1 is less than the durability of rims of the internal I/4. External rim needs less time to achieve acceptable wear and as shows the analysis is much more exposed.

Disparities arising from the analysis of the durability of the rims of the internal I/4 using models 1.2 and 1.3 and 2.2 and 2.3 tend to apply with detailed analysis of durability, models of third degree approximations 1.3 or 2.3.

Together with the increase of rotation speed the durability of bit rims, calculated by these methods, decreases in both analysed rims.

Increasing the rotation speed of the drill bit with a constant load is similar to a linear reduction of the durability of the teeth on the analysis's bit. In the case of smaller values of axial pressure durability of the teeth on the individual rim will be quite diverse. With the increasing pressure axial guidance these differences fall.

The increase in pressure causes a nonlinear axial depression of the durability of the teeth. On each rims of different bits I somewhat different nature of this nonlinear.

Developed models allow the qualitative and quantitative determination of the effect of the parameters of the drill parameters on drill bits wear.

Analiza wpływu parametrów wiercenia na trwałość uzbrojenia świdrów gryzowych

Streszczenie

W artykule podano wyniki oceny wpływu prędkości obrotowej oraz obciążenia osiowego na trwałość uzbrojenia świdra trójgryzowego. Przedstawiono je w postaci graficznej oraz przeprowadzono analizę ustalonych prawidłowości.

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Correction of the cutting tool path for milling operations with circular interpolation

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Keywords
Correction, tool path, high speed milling

Summary
The achievement of the assumed dimensional accuracy of workpieces together with maintenance of the highest possible efficiency of the process is an important issue during the high performance machining by numerically controlled machine tools. One way to achieve this goal is to reduce the dimension errors by using the cutting tool path correction. The article presents the cutting tool path correction by the value of the identified dimension error for milling operations with circular interpolation. Stages of the correction implementation are specified. The results of testing and verification of the method effectiveness are presented.

1. Introduction

Machining is a technology that produces about 50- 60% of the machines' parts. The alternatives are: plastic forming or moulding that in general avoid the loss of material which is their great advantage over machining. For many years the elimination of machining processes was anticipated. However, that did not happen up to the present day. In fact it is the only method of metal machining which gives the required surface roughness and dimensional accuracy at relatively low cost. Moreover, it is possible to manufacture a wide range of shapes, often unachievable by other techniques.

Machining elements of machines and devices using high performance technology is becoming widely used in production. The achievement of the assumed geometrical accuracy after machining is an important issue in relation to the developing high performance machining. So far, emerging solutions were working at reduced processing parameters [1,6] and the kinematic machine tool error compensation [2].

During high performance machining there can be identified a recurring problem which is inability to use the machine performance capability due to shape dimensional constraints of products. Modern machines have large power capacity allowing for machining with high parameters, but it significantly affects in reduction of the dimensional accuracy of the workpiece, therefore it is necessary to reduce the parameters, and this is associated with a lack of force utilization of installed machine. The solution to this problem can be machining with the tool path correction.

For the milling process there can be distinguished two strategies for tool path correction. They are dependent on the tolerance imposed by the manufacturer.

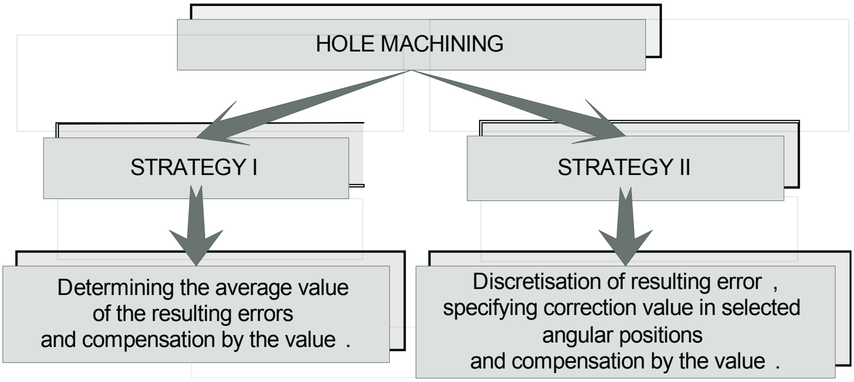


Fig. 1 Tool path correction strategies for milling machining
Source: self-elaboration

Strategy I is designed for machining holes with high tolerance range 6σ. The strategy is based on specifying an average value of the resulting errors and compensating by the value [4]. In this case, the correction can be made directly from the machine control panel achieving satisfactory results.

Strategy II is designed for machining holes with narrow tolerance range 1σ or 3σ. The strategy is based on the discretisation of resulting errors by specifying correction value in the selected angular positions and compensation by the value. The strategy is presented and reviewed in this article.

2. The process of milling

A characteristic feature of milling operations is that during the process cutting edges do not work simultaneously but periodically one after another. In connection this characteristic of the cutting process, the force is a pulse in nature and is closely linked to the number of blades, rotational speed and the value of the cutter feed. During milling operations cutting edge point trajectory is the result of feed and rotational motion of the tool.

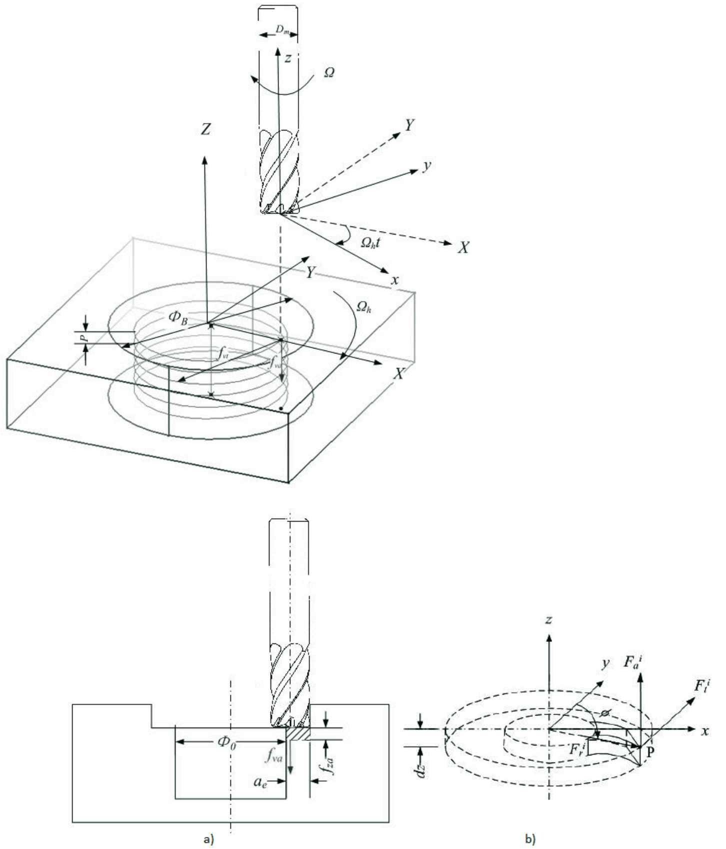


Fig. 2 Kinematics of the milling process with circular interpolation
Source: Liu C., Wang G., Dargusch M., Analytical Cutting Forces Model of Helical Milling Operations, World Academy of Science, Engineering and Technology, 59 (2011), p. 1082

Kinematics of the process is presented in Fig 2a, which presents: hole diameter after machining Φ_B, hole diameter before machining Φ_0, cutter diameter D_m, width of the machined layer a_e, thickness of the machined layer f_{za}, cutter rotational speed Ω, cutter feed speed Ω_h, cutter axial feed f_{va}, cutter tangential feed f_{vt}.

Based on the kinematic model the cutting forces model was designed [5]. In the references the model was determined analytically. The model is as follows (1):

$$\begin{Bmatrix} Fx \\ Fy \\ Fz \end{Bmatrix} = \sum_{j=1}^{Nm} \begin{Bmatrix} F_{X,j}\left(\Omega t + (j-1)\frac{2\pi}{N}\right) \\ F_{Y,j}\left(\Omega t + (j-1)\frac{2\pi}{N}\right) \\ F_{Z,j}\left(\Omega t + (j-1)\frac{2\pi}{N}\right) \end{Bmatrix} \tag{1}$$

where *Fx* – force directed toward the x axis as shown in Figure 2b , *Fy* – force directed toward the y axis as shown in Figure 2b , *Fz* – force directed toward the z axis as shown in Figure 2b , *Ωt* –rotation angle, *N* – number of cutter teeth [5].

This model defines the cutting forces including the interaction between the workpiece and the cutter, and the effect of spindle rotation, tangential and axial feed.

3. The method of cutting tool path correction

The method of cutting tool path correction by the value of the identified dimension error for milling operations can be presented in the following stages:

Conducting test process using the hole milling with high performance parameters.

Measurement of diameter deviation of the hole obtained during milling.

Determining correction value using the adopted strategy.

Including correction value in the geometric model of a machined hole.

Developing control program including the modified geometric model.

Experimental verification of the strategy.

3.1. The research methodology

For machining solid carbide cutter with a diameter of 16 mm with two blades and a dedicated geometry for machining metal alloys ISO group N (aluminum, magnesium, copper alloys etc.) - 16x25 - 82 - 30°W – Z2 was used.

Table 1 The geometry of the tools used for the research

Designation	Number of blade	Diameter of the working part	Diameter grip part	Overall length	The inclination angle of the secondary cutting edge
	z_n	D_c [mm]	dm_m [mm]	l_2 [mm]	λ_c' [□]
16x25-82-30W-Z2	2	16	25	82	30

Source: Polish Norms: PN-EN 573:3 2005

For the test sample there was used rolled rod with squared cross section 60x60mm made of AlCu4Mg1 alloy. Chemical composition and properties of the alloy are shown in Table 2. The alloy has high strength and yield strength, and has good anti corrosive properties. It is used on heavily loaded elements of aircraft constructions, vehicles and machines [3].

Table 2 Designation, chemical composition and properties of the alloy AlCu4Mg1

Mark by PN-EN 573-3: **EN AW-AlCu4Mg1**					Mark by ASTM: **2024**			
The concentration of elements %								
Si	Fe	Mg	Cu	Mn	Zn	Ti	Cr	Another
≤0,2	≤0,5	1,5	4,2	0,6	≤0,25	≤0,15	≤0,1	Zr + Ti ≤0,2
Mechanical on				**Physical on**				
Rm [MPa]	$Rp_{0,2}$ [MPa]	A %	HB	α [1/K°]	Temp. top.[C°]	P [g/cm³]		
185-470	75-325	20	55-138	23,2	500-638	2,78		

Source: Dobrzański L.A., Podstawy nauki o materiałach i metaloznawstwo. WNT 2007

The study used a sample: cubical samples of squared cross section 60x60 mm, 40 mm high with a pre-made hole Ø30 (located symmetrically) with a depth of 14 mm. Samples were machined to a depth of 14mm with machined layer width of 1 mm. An example of the machined samples shows Photo 1.

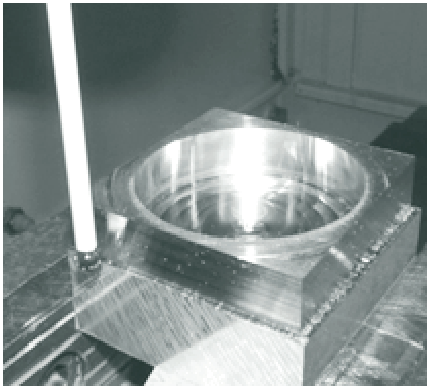


Photo 1 Example of machined sample

Source: self-elaboration

Measurements were made directly on the machine using a tactile probe OMP60 technical specifications of which are shown in Photo 2.

Courses of action	±*X*, ±*Y*, ±*Z*	
Transmission types	optical: infrared 360^0	
Transmission range	6m	
Unidirectional repeatability (2 σ with a typical shank)	± 1μm	
Measuring force *XY* minimum surface XY maximum surface in +*Z* axial direction	 0,75 N 1,4 N 5,3 N	
OMP60 probe measurement uncertainty		
Courses of measurement	axle X	axle Y
Dimension of the plate, mm	**100**	**100**
Average, mm	100,008	100,006
Standard uncertainty u_A, mm	0,00037	0,00037
Complex standard uncertainty u, mm	0,00053	0,00053
Expanded uncertainty U, mm	0,0011	0,0011
Adjustment P_{Ex}, mm	-0,008	-0,006

Photo 2 Technical Specifications of OMP 60 probe
Source: self-elaboration based on information from Renishaw company: www.renishaw.com.pl

The first measurement was performed at 36 points around the diameter circumference of the sample. In connection with the observed symmetry of the sample the measurement for other samples was carried out on half the diameter circumference at 18 points (Fig. 3).

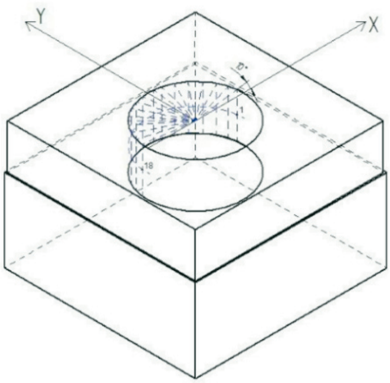


Fig.3 Scheme of the sample.
Source: self-elaboration

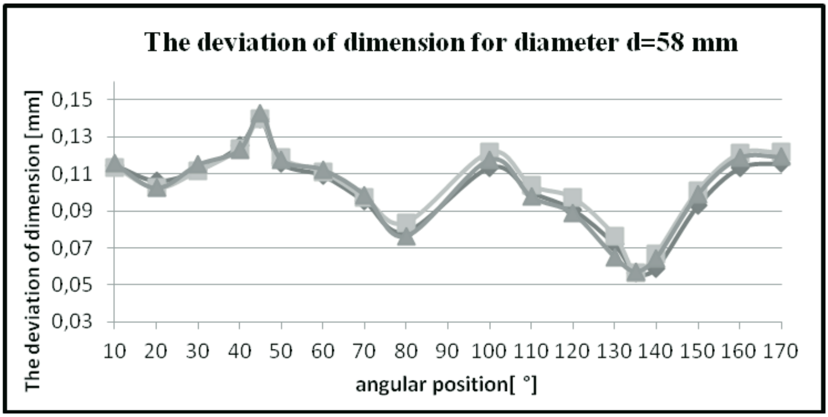
The machining process was performed on the machine: FV-580A with Fanuc control 0i-MC, producer: MOC Pruszków Mechanics. The machine had Kistler 9257B dynamometer.

In the course of the tests there were used constant machining parameters [3]:

- machining speed v_c= 400 m/min,
- rotational speed n = 7958 rev/ min,
- feed on the blade f_z = 0,1 mm/blade
- depth of machining a_p = 14 mm.

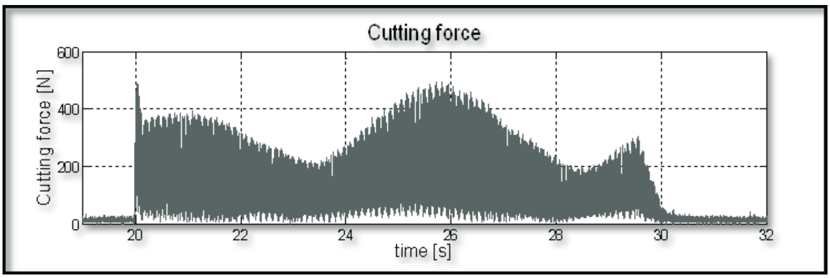
3.2. Test results and conducting correction

Dimension deviation for the next five samples from the experiment is shown in the graph (Graph 1). The resulting hole was smaller than the nominal. Lines on the graph with different markers represent more samples from the experiment.



Graph 1. Dimension deviation for five consecutive samples
Source: self-elaboration based on experimental grounds

During the study cutting forces were recorded. Cutting force was presented on the Graph 2.



Graph 2. Cutting force
Source: self-elaboration based on experimental grounds

In view of the repeatability of samples (standard deviation from the average at the level about 3 microns) it was possible to determine the experimental model. For a given constant machining parameters, parameters of the output were specified – dimension deviation of the entire diameter circumference at the angular positions every 10 °.

Based on this model it was possible to determine the radius value of the adjusted hole in successive angular positions. Adjusted value of the radius is presented in Table 3.

Table 3 The adjusted value of the radius

Angle α[□]	Adjusted radius R [mm]	Angle α[□]	Adjusted radius R [mm]	Angle α[□]	Adjusted radius R [mm]	Angle α[□]	Adjusted radius R [mm]
10	29,058	100	29,059	190	29,058	280	29,059
20	29,052	110	29,050	200	29,052	290	29,050
30	29,056	120	29,046	210	29,056	300	29,046
40	29,062	130	29,035	220	29,062	310	29,035
45	29,070	135	29,028	225	29,070	315	29,028
50	29,059	140	29,032	230	29,059	320	29,032
60	29,055	150	29,049	240	29,055	330	29,049
70	29,048	160	29,059	250	29,048	340	29,059
80	29,040	170	29,060	260	29,040	350	29,060

Source: self-elaboration

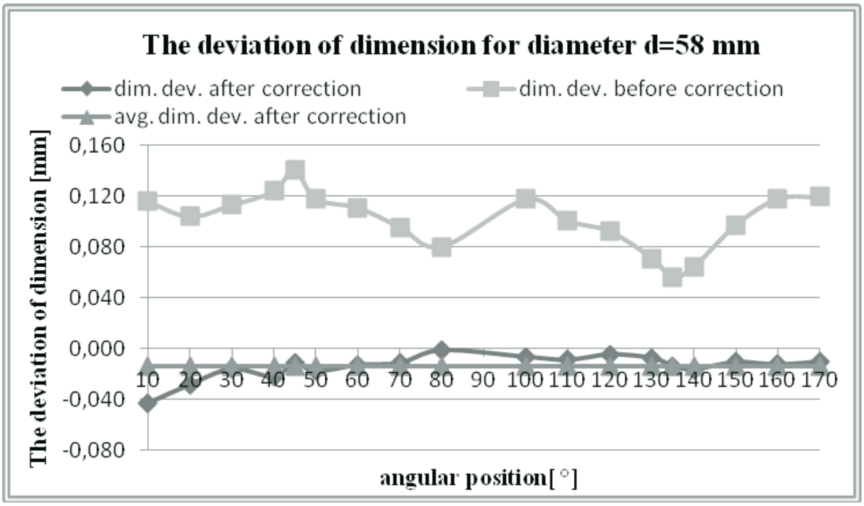
The next step is to take into account the correction value in geometric model of the machined hole. For this purpose, CAD Solid Edge software was used. In the software the tool path with specified correction was designed.

Then the data were imported into the program Edge CAM where on the basis of the new model the modified program control on numerically controlled machine tool was generated.

3.3. Verification of the method

In order to verify the method the milling process using a cutting tool path correction was carried out. The machining was carried out according to the modified tool control program of the machine under the same conditions as before the correction.

Dimension deviation after machining with the tool path correction is shown in Graph 3.



Graph 3. Dimension deviation before and after correction
Source: self-elaboration based on experimental grounds

After correction the average value of the dimension deviation was obtained at the level of - 0.014 mm, standard deviation from the average was 0.009 mm, and the average percent reduction in dimension deviation was achieved at the level of 85.72%.

After the correction of the maximum deviation scattering decreased from 0.08 mm to 0.04 mm.

From the research the following conclusion can be drawn: A strategy to improve the dimensional accuracy of objects with narrow tolerance range such as 1σ or 3σ is effective because the errors decreased by nearly 86%, and the standard deviation was reduced by half.

4. Conclusions

The article presented the concept of cutting tool path correction method by the value of the identified dimension error for high performance milling operations. The correction was carried out using the experimental model. It is a solution that can be implemented in an industrial environment for machining objects with narrow tolerance range such as 1σ or 3σ. In view of the repeatability of tests, with a small standard deviation from the average it will be possible for industrial purposes to build a geometric model using the tool path from one sample. By using this method the dimension errors of the machined workpiece can be greatly reduced even by about 80% - 90%.

For objects with a broad tolerance (6σ) another tool path correction strategy can be adopted. In such cases it is sufficient to carry out the tool path correction by the average dimension deviation. This is a strategy in which one can determine the average value of the error and enter it into the program directly from the machine tool control panel.

Korekcja toru narzędzia skrawającego dla obróbki frezowaniem z interpolacją kołową

Streszczenie

Osiągnięcie założonej dokładności wymiarowej przedmiotów obrabianych przy zachowaniu możliwie najwyższej wydajności procesu jest istotnym problemem podczas obróbki wysokowydajnościowej na obrabiarkach sterowanych numerycznie. Jedną z metod uzyskania tego celu jest zmniejszenie błędów wymiaru poprzez zastosowanie korekcji toru narzędzia skrawającego. W artykule przedstawiono korekcję toru narzędzia skrawającego o wartość zidentyfikowanego błędu wymiaru dla obróbki frezowaniem z interpolacją kołową. Określono etapy jej przeprowadzania. Zaprezentowano wyniki badań i weryfikację skuteczności wskazanej metody.

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Research of the surface roughness of polymer matrix composites by spherical and end cutters milling

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Keywords
machining, milling, GFRP composites, surface roughness

Abstract
Machining of polymer composites is quite difficult due to the anisotropy and heterogeneity of the structure of these materials. These materials belong to a group of difficult materials that require special tools. In this work the surface roughness was studied and chips was observed after milling glass fiber-reinforced polymer composite. To process uses two types of mills (mill cutters - front and spherical). The research program included seven different sets of cutting process parameters, varying in depth of cut, feed rate and cutting speed. Milling was carried out on a vertical machining center
Avila-VMC800HS, and the study of surface roughness tester Mitutoyo SJ - 301. Based on the survey were determined according to the form of graphs showing the effect of changing processing parameters on the machined surface roughness. This paper describes the type of materials used in the study, the type of tools, how to perform sampling and carry out the machining.

1. Introduction

The composite material is formed from at least two components with different properties. After combining the components into a new material, the composite has better properties than either of the components used separately. The composite material is a monolithic externally, but with visible borders between the components. The composite consists mainly of the matrix and fibers. Auxiliary materials and modifiers are additionally added during production.

Composites can be divided by type of matrix [6]. This could be a polymer matrix composites, ceramic matrix composites or metal matrix composites. Due to the nature of reinforcing phase there are fiber reinforced composites (continuous, short, articles of fibers) and reinforced particles.

Metals, ceramics and plastics can be used as matrix for composite materials. They fulfill the function of maintaining a reinforcing phase in the appropriate place in the composite. They are supposed to carry the stress on the reinforcing phase.

Metal matrix composites consisting of iron and its alloys, nickel alloys, metals and other non-ferrous alloys. Non-ferrous alloys can be aluminum, magnesium, silver copper, tin, lead and titanium. Ceramic matrix is mainly technical ceramics. Polymer matrix composites are composed of thermosetting plastics, thermoplastics and sometimes elastomers.

Reinforcing phase composites is mainly composed of reinforcing fibers and particles [7]. We use fiber glass, carbon, boron fibers, silicon carbide, aluminum oxide, kevlar, polyethylene, tungsten, titanium and many others.

Modifiers and additives may include release agents, hardeners, thickeners chemical streamlining and lubricants, antistatic agents, antifriction additives, dyes and pigments, antispasmodic and thinners.

1.1. Polymer composites

Polymer composites are materials consisting of a matrix in the form of thermosets, thermoplastics and elastomers and glass fibers, carbon fibers and kevlar fibers.

The use of polymers as a matrix composites has added features such as lightness, corrosion resistance, vibration damping capacity, good electrical and thermal insulation and ease of shaping.

The polymers used in the matrix have certain functions in the composite [8]:

- give the products their specific shape,
- allow composites to transfer the load to the fibers,
- have a decisive influence on the method of fabrication of composites,
- determine the thermal and chemical properties,
- determine the flammability of the composites.

The polymers have the properties of relaxation. This means that in time macromolecules return to a state of equilibrium. Return to the equilibrium structure eliminates interference caused by external factors. Fiber strength is much greater than the strength of polymer matrix. Polymer matrix has a low modulus of elasticity. Good adhesion to fiber ensures the proper transfer of load on the fiber. Good adhesion is essential for the operation of mechanical or thermal loads, which ensures the correct behavior of the composite.

The fibers used to form polymer composites determine the shape. The fibers carry the load, depending on the fibers in the material. They increase most of the properties of composites, such as tensile strength, but tensile strength also increases the surface roughness. This depends largely on the contribution of each component in the composite.

These characteristics of polymer composites decide to use these materials for highly loaded structures, while light, such as gliders, airplanes and construction teams missiles. In addition, polymer composites are used widely for items not working in conditions of heavy mechanical loads, such as housing and machine guards, skeletons, electrical and electronic equipment and production tooling. They are also used in the automotive industry, mainly for automotive body parts, oil pans, spring. They are made of them both fast racing boats as well as vaulting poles, tennis rackets, bicycle frames, and skiing.

1.2. Milling of polymer composites

Machining of polymer composites is difficult to determine due to the anisotropic and inhomogeneous composites. Machining depends on the properties, the percentage of matrix and fiber orientation of the fibers in the composite, and thus the type of treatment, expectations for the final space and cost. In addition to the tool selection process depends on the machining area, the number of operations, mainly in the nature of the work material.

The indicators characterizing the machining of polymer composites are the shape of the chip, surface roughness after treatment, temperature and cutting forces, wear and hardness of the blade material after milling.

Milling of polymer composites in contrast to the machining of steel and aluminum for different terms of use. This type of polymer composite processing refers to a correction of the surface and achieve the expected, the quality of pre-molded plastic surfaces. For this reason, one of the decisive parameter is the quality of surface roughness.

The tools for these materials are usually selected mills with indexable carbide when machining is short-lived. In addition, mills are relatively cheap compared to diamond coated cutters or diamond cutters, when scheduled to perform a large number of elements or the area treated is large, which in turn is associated with greater exposure to the blunt cutter, and in the worst case of destruction.

For machining of steel and aluminum tools can be used for ceramics, but in the case of composites are not used. These are the tools very hard but very brittle and not receiving heat from the cutting area. Heterogeneity of composite structure causes changes in the cutting load, which in turn can cause vibrations. Vibration fragile tools may lead to their destruction.

In the case of milling of polymer composites with carbon fiber (CFRP) and polymer composites with glass fibers (GFRP) determines the choice of tool blade material in the case of milling of polymer composites with natural fibers (AFRP) determines the geometry of the blade. Behavior of the material during milling depends mainly on the properties of reinforcing fibers. These properties are decisive for the selection tool. The fibers are characterized by high tensile strength and low Young's modulus. In addition, fibers have different thermal properties depend on the type of fiber.

Research in polymer composites machining is carried out for decades. Cross section of the existing knowledge was made by R. Teti in [1]. Composites are described and their distribution. Presents general characteristics of each of the three types namely composites of polymer matrix composites, metal matrix composites and ceramic matrix composites. Based on the available literature describes the phenomena associated with processing these materials and the problems encountered when machining. Described turning, cutting, drilling and milling for each material.

Effect of tool wear on cutting forces GFRP has been described in [2]. Angle of attack were investigated and clearance angle. Proved that the tool wear is mainly composed quickly rounding the tip. Wear rate and the lateral face decreases with increasing clearance angle. Horizontal and vertical forces increase with tool wear.

Article on the surface roughness after turning the work [3] under the authorship Palanikumar K., L. Karunamoorthy and R. Karthikeyan. This article assesses the effect of cutting parameters on surface roughness during machining turning GFRP material. Were taken into account the cutting speed, the angle of the fibers, depth of cut and feed rate. The study aimed at selecting the appropriate parameters to minimize the surface roughness. Investigations on the tube formed in the process of filament winding. The research found that the surface roughness is greater for the fibers at 90 ° compared to the alignment angle of 30 °. Furthermore, it was examined that increased cutting speed decreases the surface roughness, higher feed rate increases the roughness, and depth of cut has no significant effect on surface roughness.

Article [4] was devoted to examining the effects of cutting parameters on surface roughness of metal matrix composites. In this work takes into account the cutting speed, the percentage of individual components, depth of cut and feed rate. The material tested was a composite of aluminum alloy matrix reinforced with silicon carbide SiC rolling treated with carbide tools. The studies demonstrated that increasing the cutting speed, increasing the percentage of fibers to 25%, reducing the depth of cut and feed rate reduction reduces the roughness.

In the next article [5] on the surface roughness, considerations related to surface tension. Surface roughness was investigated by means of wetting. With the droplet contact with the surface roughness was studied, which is also a determinant of the surface tension. An examination of CFRP and GFRP-reinforced epoxy resin, where the average value of fiber in the structure is over 60%. The research determined that with the increase of surface roughness increases the surface tension in the case of CFRP and GFRP materials.

In [10] under the authorship of Wei Song, Aijuan Gu, Guozheng Liang and Li Yuan examined how the addition of aqueous ammonia to affect the structure of the composite surface roughness.

The final surface can assess the quality of treatment. It is very important in terms of its further use. In the later use of the machined surface element is important when moving loads, fitting parts and the formation of microcracks. In addition, tolerances and surface roughness is a measure of the use of a machine for processing. Previous studies conducted to assess the quality of surface roughness shows different results. Polymer composite material is difficult to determine the parameters due to various forms of fibers, how they are arranged and the way of making the composite. In the present paper is devoted to the specific material and processing was carried out and tested the quality of the surface.

2. The research methodology

The aim of the study was to assess the impact of face milling parameters of polymer matrix composites with glass fibers in the machined surface roughness.

At the start of the study sample was prepared. The sample was performed in a controlled room, that is, subject to appropriate conditions as temperature, humidity and amount of impurities. Then they were put into the autoclave. Once removed from the autoclave, the samples were subjected to machining to remove excess material.

After preparing the samples have been subjected to machining and machinability index has been appointed the materials in the form of surface roughness and shape of the chip.

For studies of roughness and obtained by cutting chips were established seven different sets of parameters with different values [9], such as depth of cut, feed rate and rotational speed of the cutter using two types of cutters ø10.

Before the start of milling operations must set the tool relative to the workpiece, that is, find out the origin, introducing him to the memory of the machine.

2.1. Preparation of samples

The sample consisted of the creation of supersaturation mat of fibers with epoxy resin. On each plate with a thickness of 10 mm accounted for 30 to 50 sheets. Each of the fabrics was laid alternately in the system that is 0-90. This system preserves the strength of the material in each direction. Temperature, humidity and amount of impurities in a controlled room should be maintained at a preset level namely at 18 - 30 ° C, humidity up to 60% and the amount not greater than 10,000 particles of $1m^3$.

After placing each fabric sample was inserted into the autoclave at 120 minutes at 177 ° C at 0.3 MPa. After removing from the oven the sample was left to cool.

The study was used shallow 20cm x 20cm x 1 cm glass fiber with epoxy resin saturated with the trade name EGL / EP 3200-120 lies in the 0-90 system.

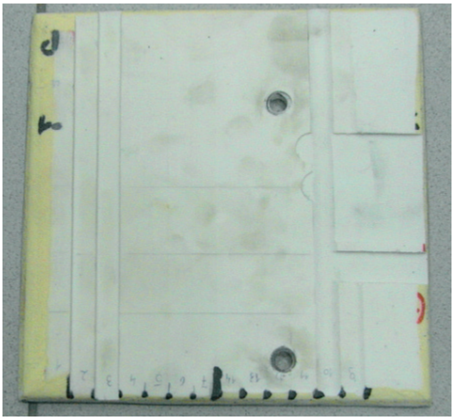

Fig. 1. Picture samples

2.2. Description of the technological equipment

Milling process of polymer composites with glass fibers was carried out on a vertical machining center Avila - VMC 800 HS.

Fig. 2. Vertical Machining Center

Table with an area of 1000 x 540 mm with the possibility of travel in three axes X (800 mm), Y (540 mm), Z (620 mm) provides space for mounting the vise.

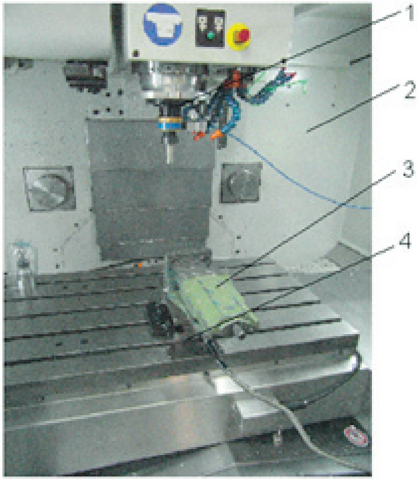

Fig. 3. The test stand: 1 - spindle, 2 - working space, 3 - vise, 4 - working table

2.3. Device for measuring surface roughness

The device for measuring the roughness Tester Mitutoyo SJ-301.

It is a device with a maximum surface roughness measurement from - to + 200 microns 150 microns or 350 microns span. When measuring the length was determined measuring 4mm segment.

Fig. 4. Device for measuring surface roughness

The device is equipped with a diamond-tip measurement. Measurement is about placing the tip on the surface. After starting the measurement probe moves across a surface and measure the roughness, as a mean value read in five points.

2.4. Tools

We used a spherical cutter and end cutter [9]. Milling width was equal to 70% of the diameter of the cutter.

a) b)

Fig. 5. Cutter used in the study: a) spherical, b) spindle

2.5. The technological parameters

We changed our depth of cut from 1 mm to 3 mm at constant values of the feed rate of 600 mm / min and rotational speed of the cutter equal to 4000 rev / min. Then, at constant values of depth of cut equal to 2 mm and equal to the cutter speed 4000 rev / min feed rate was varied in the range from 400 to 600 mm / min. The last parameter was changing the speed of the cutter. At constant values of the depth of cut (2 mm) and feed rate (600 mm / min) was changed cutter speed range from 3000 to 6000 rev / min.

3. Results of measurement of surface roughness and chip observation

The composite based on glass fiber is a good material for processing. While cutting is fast breaking up the chip, making it impossible to curl the chip on the tool, and good heat dissipation from the chips. During the machining there is no dust up.

a) b)

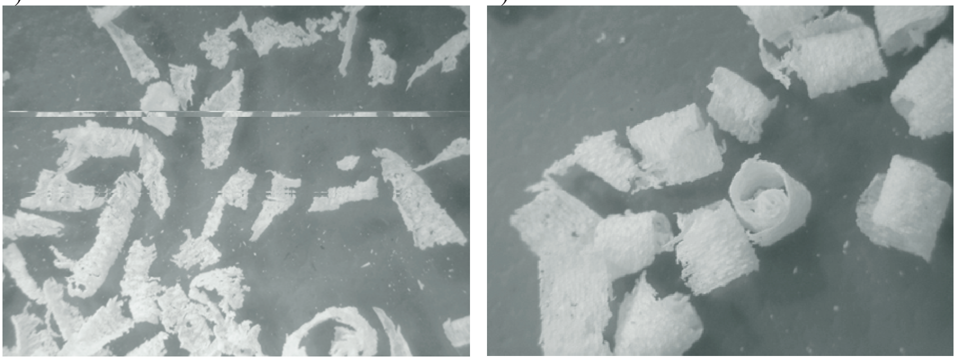

Fig. 6. The shape of the chip at 15x magnification after treatment with mill: a) spherical, b) finger

With increasing depth of cut increases the length of the chip. This applies to both the finger and spherical mill. Changing the feed rate does not give significant differences chips, the chips are similar form. Increasing speed cutting results in faster chip breaking, and thus the chips are shorter. Changing the cutter with a finger on the spherical causes fragmentation of chips.

For glass fiber with epoxy resin-saturated operation milling cutter finger is less parameters Ra, indicating a better quality of the surface after machining in comparison with a spherical cutter. This is due to dependence of surface roughness after milling spherical cutter on the number of transitions (the more you go the more accurate processing and less parameter Ra).

The results show that the increase in depth of cut with constant feed rate and speed cutting increases the roughness of both mills. When cutting with a greater depth of the cutter removes a greater amount of material which is associated with deterioration of machined surface.

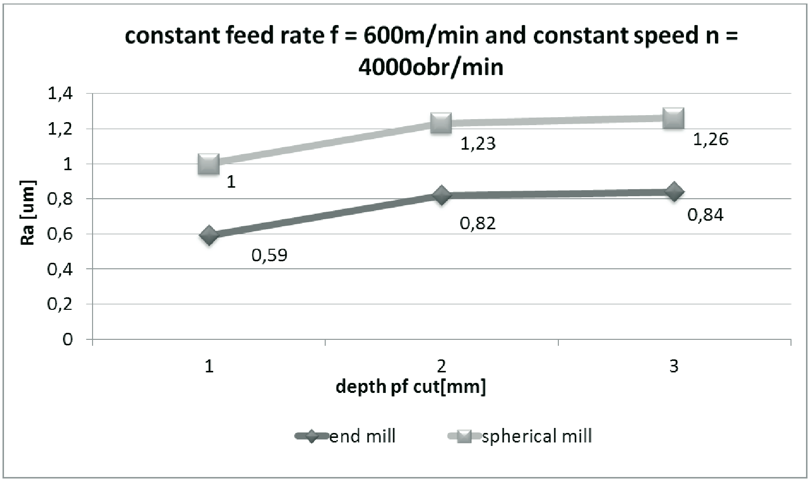

Fig. 7. Ra parameter depending on the depth of cut

In the case of milling with a constant cutting depth and the speed cutting of the visible increase in the value of the parameter Ra with increasing feed rate, in the case of the both mills. The increase in surface roughness is due to the shorter duration of the machining surface.

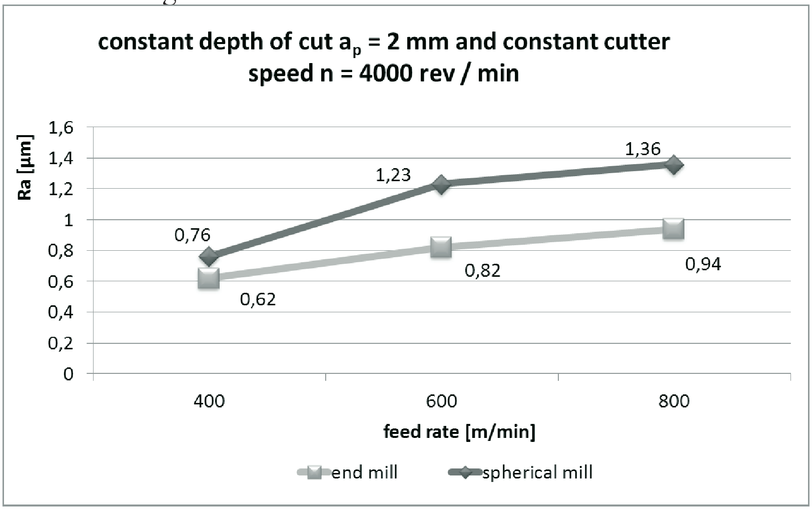

Fig. 8. Ra parameter depending on the speed rate

Increase speed cutter with a constant depth of cut and constant feed rate will improve the quality of the surface. As a result of increased speed of the cutter blade with a greater frequency of cutting the area. Results in improving the quality of the surface, thus reducing the roughness parameter Ra.

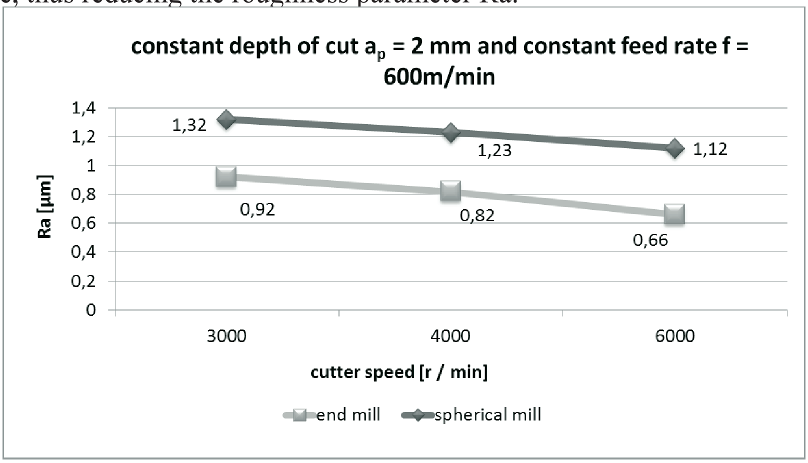

Fig. 9. The parameter Ra, depending on the cutter speed

4. Conclusions

The work shows the influence of changes in cutting parameters on machinability indicators such as surface roughness and shape of the chip. The chips reach a length of 2mm. Heat discharged by means of chips does not cause overheating of the tool and workpiece. Increasing the depth of cut results in the formation of long chips. Changing the speed of feed does not significantly affect the shape of the chips, but a change in mill speed increases the rate of breakage. In this way the chips are shorter for higher cutting speeds. Polymer composites based on glass fiber is characterized by small values of the parameter Ra. These are the values ranging from 0.59 microns to 1.36 microns. Due to the quality of the chips of glass fiber composite is a very good material for processing because it does not cause dust.

Badania chropowatości powierzchni kompozytów polimerowych po frezowaniu frezami kulistymi i trzpieniowymi

Streszczenie

Obróbka skrawaniem kompozytów polimerowych jest dość trudna ze względu na anizotropię i niejednorodność struktury tych materiałów. Są to tworzywa należące do grupy materiałów trudnoobrabialnych, wymagających użycia specjalnych narzędzi. W pracy przeprowadzono badania chropowatości powierzchni i obserwację wiórów po obróbce frezowaniem kompozytu polimerowego z włóknem szklanych przesyconym żywicą epoksydową. Do obróbki wykorzystano dwa rodzaje frezów (trzpieniowe walcowo – czołowe i kuliste). Program badań obejmował siedem różnych zestawów parametrów technologicznych frezowania, różniących się głębokością skrawania, prędkością posuwu i prędkością skrawania. Obróbka frezowaniem przeprowadzona została na pionowym centrum obróbkowym Avila – VMC800HS, a badania chropowatości powierzchni na profilografometrze Tester Mitutoyo SJ - 301. Na podstawie przeprowadzonych badań wyznaczone zostały zależności w postaci wykresów pokazujących wpływ zmieniających się parametrów obróbki na chropowatość obrobionej powierzchni. W pracy opisano rodzaj użytych do badań materiałów, rodzaj narzędzi, sposób wykonania próbek i przeprowadzenia obróbki skrawaniem.

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Differences between LQC with deterministic and random horizons

dr inż. Edward Kozłowski

Keywords
Linear quadratic control, adaptive control, stochastic system, dynamic programming, random horizon

Abstract
In this paper the adaptive control problem with random horizon is investigated, where the aim of control consists in the optimization of performance criterion, which has a quadratic form. The random horizon is modeled by the random variable with finite number of events. This problem is substituting by the optimal control problem with established horizon. The laws of optimal control of linear system with unknown parameters and simple example are given. Moreover the differences between the tasks with random and deterministic horizons are presented.

1. Introduction

The optimal control of stochastic systems with unknown parameters, which are evolved according to an Ito stochastic state system, is well known (see e.g. [1], [5], [9], [10], [18], [21]). The aim of control consists in the optimization a performance criterion, which are depended on the states of system and controls. On the other hand the optimal control process must increase the information about system parameters. In literature this problem is known as an adaptive control (see e.g. [2], [3], [9], [11], [12], [17], [19], [21]).

If the time horizon is established, then the desining of optimal controls for linear systems with quadratic cost LQC (Linear Quadratic control) is well known in the literature for both discrete and continuous time. However, how to make the control law when the horizon is not fixed? The horizon of control may depend on the system states as well as can be independent (completely external). In the first case the problem of optimal stopping of random processes is additionally considered (see e.g. [6], [7], [13], [14], [20]). The solution consists in the Snell envelope construction and definition of stopping moment. This problem may be solved by changing the stopping into control (see [4]). The second case concerns a situation, where the decision maker take the control at time zero and does not know the moment, which he can control an object (economic, technical, social etc. system). He has only a'priori knowledge about the distribution of random variable describing a control horizon (see [15]). Thus the random horizon doesn't depend on the behaviors of the system (the state of the system does not influence the horizon). We have got such a situation when we give the definition of the control horizon as e.g. the number of losses, the number of requests. In this case we model the horizon by a random variable state independent. The solution of this problem is based on the reduction to substitute task with established horizon.

The paper presents the LQC problem with random horizon which is modeled by binomial distribution. The work gives how to make a substituting task and determine the optimal control. For the primary and auxiliary tasks the aim of control is the same but has different forms. The paper shows additionally losses (costs) and the differences of control arising from ignorance of the control horizon. In the original task the performance criterion is defined as a sum of costs and heredity. However, the performance criterion of substituting task is defined as a scalar product between vector presenting loss and heredity and vector of probabilities of stopping in later moments and stopping at the moment.

The organization of the paper is as follows. The section 2 introduces the optimal control task with random horizon. The section 3 presents the modification of considered task to the task with an established horizon. The section 4 gives the optimal control of linear system for auxiliary task and differences between controls with unknown and established horizon. The simple applications of proposed approach are illustrated on two examples.

2. Problem formulation

Let (Ω, F, P) be a complete probability space. Suppose that m-dimensional random vectors w_1, $w2$, ... are independent and have a normal $N(0,I_m)$ distribution, ζ be a k-dimensional vector with a priori distribution $P(d\zeta)$, y_0 be an initial state with distribution $P(y_0)$ and a random horizon τ has the same discrete distribution $P(d\tau)$. We assume that all the above mentioned objects are stochastically independent.

Let the system be described by the state equation

$$y_{i+1} = y_i - Bu_i + C\xi + \sigma w_{i+1} \tag{1}$$

where i=0, 1, 2,..., $B \in R^{nxl}$, $C \in R^{nxk}$ and $\sigma \in R^{nxm}$. On (Ω, F, P) we define the family of σ-fields $Y_j = \sigma\ \{y_i : i = 0, 1,..., j\}$. A Y_j-measurable vector $u_j \in R^l$ will be called a control action, and $u = (u_0, u_1, u_2,...,u_N\ 1)$ an admissible control. The class of admissible controls is denoted by U. To specify the aim of control, we introduce a cost of control at time i as $u_i^T R u_i$ and a heredity function $y_\tau^T Q y_\tau$ (e.g. losses associated with system instability, no hit to the target). The random variable $\tau : \Omega \to \{0, 1,..., N-1\}$ presents horizon of control and has discrete distribution $0 \le p_i \le 1$ for $i \in \{0, 1,..., N-1\}$ and $\sum_{i=0}^{N-1} p_i = 1$. The objective function has a form

$$J(u) = E\left[\sum_{i=-1}^{\tau-1} u_i^T R u_i + y_\tau^T Q y_\tau\right] \tag{2}$$

where $u_{-1} = col(0,...,0)$ and represents the composite costs and losses function (CCLF). At any time $0 \leq j < \tau$, which is not a horizon of control, we take the control u_j, and at the time τ we do not take the control action but only calculate the value of heredity function. The aim of optimal control is to minimize CCLF

$$\inf_{u \in U} J(u) \tag{3}$$

and to determine a sequence of admissible control $u^* = (u_0^*, u_1^*, ... u_{\tau-1}^*)$ for which the infimum is attained.

3. Decomposition LQC with random horizon at finite number events to LQC with deterministic horizon.

This section presents a transformation of task with random horizon to task with deterministic horizon. Using the definitions of conditional probability and condition expectation the composite costs functional (2) can be presented as

$$\begin{aligned} J(u) &= E\left[\sum_{i=-1}^{\tau-1} u_i^T R u_i + y_\tau^T Q y_\tau\right] = E y_0^T Q y_0 P(\tau = 0) + \\ &E\left(u_0^T R u_0 + y_1^T Q y_1\right) P(\tau = 1) + \ldots + E\left[\sum_{i=0}^{N-1} u_i^T R u_i + y_N^T Q y_N\right] P(\tau = N) \\ &= E\left[\sum_{i=0}^{N-1} u_i^T R u_i \sum_{j=i+1}^{N} P(\tau = j) + \sum_{i=0}^{N} y_i^T Q y_i P(\tau = i)\right] \\ &= E\left[\sum_{i=0}^{N-1} u_i^T R u_i P(\tau > i) + \sum_{i=0}^{N} y_i^T Q y_i P(\tau = i)\right] \end{aligned} \tag{4}$$

Finally, the above functional can be presented as

$$J(u) = E\sum_{i=0}^{N}\left(u_i^T R_i u_i + y_i^T Q_i y_i\right) \tag{5}$$

where

$$R_j = P(\tau > j)R \quad \text{and} \quad Q_j = P(\tau = j)Q \tag{6}$$

for $j = 0,..., N$. From distribution of random horizon τ we see, that $P(\tau > N)=0$ thus $R_N=[0]$ (zeros' matrix). Therefore, we substitute the task of optimal control with random horizon at finite number of events (3) by the task of optimal control with finite horizon

$$\inf_{u\in U} E\left(\sum_{i=0}^{N-1}\left(u_i^T R_i u_i + y_i^T Q_i y_i\right) + y_N^T Q_N y_N\right) \tag{7}$$

The CCLF value is the same but the designing of optimal control for task with establish horizon is easer. Below we consider the auxiliary (replacement) task (7) to design the optimal control of system (1) with random horizon τ.

If the number of infinitesimal events of random variable τ is infinity, it would be necessary to check whether the value of the functional (5) is bounded (see [15]). In the present case we do not need this check because the functionals of losses and heredities are bounded and the number of infinitesimal events of random horizon τ is finite, therefore the performance criterion is bounded. Thus, we have not a problem of convergence for the task with finite horizon.

4. The determining of optimal control

To determine the optimal control of system (1) for the problem (7) it is necessary to prove the lemmas below.

Lemma 1. Let ξ be a k-dimensional random vector, $\Psi \in R^{kxk}$ - deterministic matrix and $\{Y_i\}_{i\geq 0}$ - non decreasing σ family on the probability space (*Ω, F, P*). If Σ_i presents a second central moment of random vector ξ conditioned on σ - fields Y_i

$$\Sigma_i = E\left(\left(\xi - E(\xi|Y_i)\right)^T\left(\xi - E(\xi|Y_i)\right)\middle|Y_i\right)$$

then

$$E\left(\xi^T\Psi\xi\middle|Y_i\right) = E\left(\xi^T\middle|Y_i\right)\Psi E\left(\xi|Y_i\right) + tr\left(\Psi\Sigma_i\right) \tag{8}$$

and for any $0 \leq i < j$

$$\begin{aligned} E\left(\xi^T\Psi E\left(\xi|Y_j\right)\middle|Y_i\right) &= E\left(E\left(\xi^T|Y_j\right)\Psi E\left(\xi|Y_j\right)\middle|Y_i\right) \\ &= E\left(\xi^T|Y_i\right)\Psi E\left(\xi|Y_i\right) + tr\left(\Psi\left(\Sigma_i - \Sigma_j\right)\right) \end{aligned} \tag{9}$$

Proof.
By the conditional expectation properties for any $0 \le i < j$ we have

$$\begin{aligned}
&E\left(\xi^T \Psi E\left(\xi \middle| Y_j\right) \middle| Y_i\right) = E\left(E\left(\xi^T \Psi E\left(\xi \middle| Y_j\right) \middle| Y_j\right) \middle| Y_i\right) \\
&= E\left(E\left(\xi^T \middle| Y_j\right) \Psi E\left(\xi \middle| Y_j\right) \middle| Y_i\right) = tr\left(\Psi E\left(E\left(\xi \middle| Y_j\right) E\left(\xi^T \middle| Y_j\right) \middle| Y_i\right)\right) \\
&= tr\left\{\Psi E\left(E\left(\xi\xi^T \middle| Y_j\right) - \Sigma_j \middle| Y_i\right)\right\} = tr\left\{\Psi E\left(\xi\xi^T \middle| Y_i\right)\right\} - tr\left\{\Psi \Sigma_j\right\} \\
&= E\left(\xi^T \middle| Y_i\right) \Psi E\left(\xi \middle| Y_i\right) + tr\left(\Psi\left(\Sigma_i - \Sigma_j\right)\right)
\end{aligned}$$

whereas

$$E\left(\xi^T \Psi \xi \middle| Y_i\right) = tr\left\{\Psi E\left(\xi\xi^T \middle| Y_i\right)\right\} = E\left(\xi^T \middle| Y_i\right) \Psi E\left(\xi \middle| Y_i\right) + tr\left(\Psi \Sigma_i\right).$$

Lemma 2 (the optimal filtration of conditionally normal sequences)
If the random vector ξ has a priori normal distribution $N(m_0, \Sigma_0)$ and the system is described by linear state equation

$$y_i + 1 = f_1(y_i) + f_2(y_i)\xi + \sigma(y_i) w_{i+1}$$

then we have:
1. the conditional distribution $P(\xi \mid Y_i)$ is normal $N(m_i, \Sigma_i)$;
2. the best estimator of random vector ξ $m_i = P(\xi \mid Y_i)$ and the conditional covariance matrix $\Sigma_i = E\left(\left(\xi - E\left(\xi \middle| Y_i\right)\right)^T \left(\xi - E\left(\xi \middle| Y_i\right)\right) \middle| Y_i\right)$ are given by

$$\begin{aligned}
m_i = &\left[I + \Sigma_0 \sum_{j=0}^{i-1} f_2^T(y_j)\left(\sigma(y_j)\sigma^T(y_j)\right)^{-1} f_2(y_j)\right]^{-1} \\
&\times \left[m_0 + \Sigma_0 \sum_{j=0}^{i-1} f_2^T(y_j)\left(\sigma(y_j)\sigma^T(y_j)\right)^{-1}\left(y_{j+1} - f_1(y_j)\right)\right]
\end{aligned} \tag{10}$$

and

$$\Sigma_i = \left[I + \Sigma_0 \sum_{j=0}^{i-1} f_2^T(y_j)\left(\sigma(y_j)\sigma^T(y_j)\right)^{-1} f_2(y_j)\right]^{-1} \Sigma_0 \tag{11}$$

The proof of the above lemma and more information of optimal filtration of conditionally normal sequences can be found in [16].

Corollary 1. If the stochastic linear system is represented by state equation (1) then the conditional expected value of random vector ξ and the conditional covariance matrix are described by

$$m_j = \left[I + j\Sigma_0 C^T \left(\sigma\sigma^T\right)^{-1} C \right]^{-1} \left[m_0 + \Sigma_0 C^T \left(\sigma\sigma^T\right)^{-1} \sum_{i=0}^{j-1} \left(y_{i+1} - y_i + bu_i \right) \right] \tag{12}$$

and

$$\Sigma_j = \left[I + j\Sigma_0 C^T \left(\sigma\sigma^T\right)^{-1} C \right]^{-1} \Sigma_0 \tag{13}$$

The optimal control of linear system (1) for the auxiliary task (7) contains in follow

Theorem 1. If $\det\left(R_i + C^T K_{i+1} C\right) \neq 0$ where

$$K_i = Q_i + K_{i+1} - K_{i+1}^T A_i K_{i+1}, \tag{14}$$

$$L_i = 2K_{i+1}B + L_{i+1} - K_{i+1}^T A_i \left(2K_{i+1}B + L_{i+1}\right), \tag{15}$$

$$H_i = B^T K_{i+1} B + B^T L_{i+1} + H_{i+1} - \left(K_{i+1}B + \frac{1}{2} L_{i+1} \right)^T A_i \left(K_{i+1}B + \frac{1}{2} L_{i+1} \right) \tag{16}$$

$$Z_i = tr\left(\left(B^T L_{i+1} + H_{i+1}\right)\left(\Sigma_i - \Sigma_{i+1}\right)\right) + tr\left(B^T K_{i+1} B \Sigma_i\right) + tr\left(\sigma^T K_{i+1} \sigma\right) + Z_{i+1} \tag{17}$$

$$A_i = C\left(R_i + C^T K_{i+1} C\right)^{-1} C^T \tag{18}$$

for $i = 0,\dots,N$-1 and $K_N = Q_N$, while L_N, H are zero matrices, $Z_N = 0$ then the optimal control is

$$u_i^* = -\left(R_i + C^T K_{i+1} C\right)^{-1} C^T \left(K_{i+1} y_i + \left(K_{i+1}B + \frac{1}{2} L_{i+1} \right) E\left(\xi | Y_i\right) \right) \tag{19}$$

and

$$\inf_{u \in U} E\left(\sum_{i=0}^{N-1} \left(u_i^T R_i u_i + y_i^T Q_i y_i \right) + y_N^T Q_N y_N \right) = W_0\left(y_0\right)$$

where

$$W_i\left(y_i\right) = y_i^T K_i y_i + 2y_i^T L_i E\left(\xi|Y_i\right) + E\left(\xi^T|Y_i\right) H_i E\left(\xi|Y_i\right) + Z_i \tag{20}$$

Proof. First we define the Bellman function as

$$W_i(y_i) = \inf_{u_i} E\left(u_i^T R_i u_i + y_i^T Q_i y_i + W_{i+1}(y_{i+1}) \middle| Y_i\right) \tag{21}$$

for $j = 0, 1, \ldots, N - 1$ and $WN(yN) = y_N^T Q_N y_N$. At time $N - 1$ we have

$$\begin{aligned}
&E\left(u_{N-1}^T R_{N-1} u_{N-1} + y_{N-1}^T Q_{N-1} y_{N-1} + y_N^T Q_N y_N \middle| Y_{N-1}\right) = E\left(u_{N-1}^T R_{N-1} u_{N-1} + y_{N-1}^T Q_{N-1} y_{N-1}\right. \\
&\left. + (y_{N-1} + B\xi + Cu_{N-1} + \sigma w_N)^T Q_N (y_{N-1} + B\xi + Cu_{N-1} + \sigma w_N) \middle| Y_{N-1}\right) \\
&= u_{N-1}^T \left(R_{N-1} + C^T Q_N C\right) u_{N-1} + 2u_{N-1}^T C^T Q_N \left(y_{N-1} + BE\left(\xi | Y_{N-1}\right)\right) + y_{N-1}^T Q_{N-1} y_{N-1} \\
&+ \left(y_{N-1} + BE\left(\xi | Y_{N-1}\right)\right)^T Q_N \left(y_{N-1} + BE\left(\xi | Y_{N-1}\right)\right) + tr\left(B^T Q_N B \Sigma_{N-1}\right) + tr\left(\sigma^T Q_N \sigma\right)
\end{aligned}$$

Thus the optimal control at time $N - 1$ is

$$u_{N-1} = -\left(R_{N-1} + C^T Q_N C\right)^{-1} C^T Q_N \left(y_{N-1} + BE\left(\xi | Y_{N-1}\right)\right)$$

and

$$\begin{aligned}
&W_{N-1} = y_{N-1}^T Q_{N-1} y_{N-1} + tr\left(B^T Q_N B \Sigma_{N-1}\right) + tr\left(\sigma^T Q_N \sigma\right) \\
&+ \left(y_{N-1} + BE\left(\xi | Y_{N-1}\right)\right)^T \left(Q_N - Q_N C\left(R_{N-1} + C^T Q_N C\right)^{-1} C^T Q_N\right) \left(y_{N-1} + BE\left(\xi | Y_{N-1}\right)\right) \\
&= y_{N-1}^T \left(Q_{N-1} + Q_N - Q_N A_{N-1} Q_N\right) y_{N-1} + tr\left(B^T Q_N B \Sigma_{N-1}\right) \\
&+ 2y_{N-1}^T \left(Q_N - Q_N A_{N-1} Q_N\right) BE\left(\xi | Y_{N-1}\right) + tr\left(\sigma^T Q_N \sigma\right) \\
&+ E\left(\xi^T | Y_{N-1}\right) B^T \left(Q_N - Q_N A_{N-1} Q_N\right) BE\left(\xi | Y_{N-1}\right)
\end{aligned}$$

Finally

$$W_{N-1} = y_{N-1}^T K_{N-1} y_{N-1} + 2y_{N-1}^T L_{N-1} E\left(\xi | Y_{N-1}\right) + E\left(\xi^T | Y_{N-1}\right) H_{N-1} E\left(\xi | Y_{N-1}\right) + Z_{N-1}$$

We assume that equation (20) is true for i+1. From (21) and the properties of condition expectation we have

$$\begin{aligned}
&E\left(u_i^T R_i u_i + (y_i - a)^T Q_i (y_i - a) + W_{i+1}(y_{i+1}) \middle| Y_i\right) = u_i^T R_i u_i + (y_i - a)^T Q_i (y_i - a) \\
&+ E\left((y_i + B\xi + Cu_i + \sigma w_{i+1})^T K_{i+1} (y_i + B\xi + Cu_i + \sigma w_{i+1})\right. \\
&\left. + (y_i + B\xi + Cu_i + \sigma w_{i+1})^T L_{i+1} E\left(\xi | Y_{i+1}\right) + E\left(\xi^T | Y_{i+1}\right) H_{i+1} E\left(\xi | Y_{i+1}\right) \middle| Y_i\right) + Z_{i+1}
\end{aligned}$$

$$= u_i^T \left(R_i + C^T K_{i+1} C\right) u_i + 2u_i^T C^T \left(K_{i+1} y_i + \left(K_{i+1} B + \frac{1}{2} L_{i+1} \right) E\left(\xi | Y_i\right)\right)$$
$$+ y_i^T \left(Q_i + K_{i+1}\right) y_i + y_i^T \left(2K_{i+1} B + L_{i+1}\right) E\left(\xi | Y_i\right) + tr\left(\left(B^T L_{i+1} + H_{i+1}\right)\left(\Sigma_i - \Sigma_{i+1}\right)\right)$$
$$+ E\left(\xi^T | Y_i\right)\left(B^T K_{i+1} B + B^T L_{i+1} + H_{i+1}\right) E\left(\xi | Y_i\right) + tr\left(B^T K_{i+1} B \Sigma_i\right) + tr\left(\sigma^T K_{i+1} \sigma\right) + Z_{i+1}$$

Thus the optimal control at time *i* is

$$u_i^* = -\left(R_i + C^T K_{i+1} C\right)^{-1} C^T \left(K_{i+1} y_i + \left(K_{i+1} B + \frac{1}{2} L_{i+1} \right) E\left(\xi | Y_i\right)\right)$$

and

$$W_i = y_i^T \left(Q_i + K_{i+1} - K_{i+1}^T A_i K_{i+1}\right) y_i + tr\left(\left(B^T L_{i+1} + H_{i+1}\right)\left(\Sigma_i - \Sigma_{i+1}\right)\right) + Z_{i+1}$$
$$+ y_i^T \left(2K_{i+1} B + L_{i+1} - K_{i+1}^T A_i \left(2K_{i+1} B + L_{i+1}\right)\right) E\left(\xi | Y_i\right) + tr\left(B^T K_{i+1} B \Sigma_i\right) + tr\left(\sigma^T K_{i+1} \sigma\right)$$
$$+ E\left(\xi^T | Y_i\right)\left(B^T K_{i+1} B + B^T L_{i+1} + H_{i+1} - \left(K_{i+1} B + \frac{1}{2} L_{i+1} \right)^T A_i \left(K_{i+1} B + \frac{1}{2} L_{i+1} \right)\right) E\left(\xi | Y_i\right)$$

Finally

$$W_i\left(y_i\right) = y_i^T K_i y_i + 2 y_i^T L_i E\left(\xi | Y_i\right) + E\left(\xi^T | Y_i\right) H_i E\left(\xi | Y_i\right) + Z_i$$

which proves the assertion.

Example 1. Let the linear system be described by the state equation

$$y_{j+1} = y_i - B u_j + \sigma w_{j+1} \tag{22}$$

The performance criterion with random horizon is

$$\inf_{u \in U} E\left\{ \sum_{i=0}^{\tau - 1} u_i^T R u_i + y_\tau^T Q y_\tau \right\} \tag{23}$$

where the horizon of control has a binomial distribution with success probability $0 \le p \le 1$. We decompose the above problem to next form

$$\inf_{u \in U} E\left\{ \sum_{i=0}^{N-1} \left[u_i^T R_i u_i + y_i^T Q_i y_i \right] + y_N^T Q_N y_N \right\} \tag{24}$$

where $R_i = R\sum_{k=i+1}^{N}\binom{N}{k}p^k(1-p)^{N-k}$ and $Q_i = Q\binom{N}{i}p^i(1-p)^{N-i}$ for $i = 0, 1,...,$ N - 1. Thus, the optimal control of linear system (22) for the auxiliary task (23) contains in follow

Corollary 2. If $\det\left[R_i + B^T G_{i+1} B\right] \neq 0$ for $i = 0, 1, 2,..., N - 1$. where

$$G_i = Q_i + G_{i+1} - G_{i+1}^T B\left[R_i + B^T G_{i+1} B\right]^{-1} B^T G_{i+1} \quad \text{and} \quad G_N = Q_N \tag{25}$$

then the optimal control is

$$u_i^* = \left[R_i + B^T G_{i+1} B\right]^{-1} B^T G_{i+1} y_i \tag{26}$$

and the value of performance criterion (24) is equal $W_0(y_0)$, where

$$W_N(y_N) = (y_N - a)^T G_N (y_N - a) \tag{27}$$

$$W_i(y_i) = (y_i - a)^T G_i (y_i - a) + \sum_{j=i+1}^{N} tr\left(\sigma^T G_j \sigma\right) \tag{28}$$

Remark 3. The formulas (25)-(28) can be used to optimal control of linear system (22) with deterministic horizon N also. In this case it is suffice to put $P(\tau=N)=1$ and $P(\tau=j)=0$ for $j = 0,...,N$-1.

Corollary 3. For both random and deterministic horizons we have:

1. the optimal controls of system (22) are given by (26)
2. the values of Bellman functions are given by equations (27) - (28)
3. the values of performance criterion are

$$W_0(y_0) = y_0^Y G_0 y_0 + \sum_{j=1}^{N} tr\left(\sigma^T G_j \sigma\right)$$

where the matrices G_i and R_i, $i = 0, 1,..., N$-1 are given:

- for a random horizon

$$G_i = Q\binom{N}{i}p^i(1-p)^{N-i} + G_{i+1} - G_{i+1}^T C\left[R_i + C^T G_{i+1} C\right]^{-1} C^T G_{i+1}$$

$$G_N = p^N Q$$

$$R_i = R\sum_{k=i+1}^{N}\binom{N}{k}p^k(1-p)^{N-k}$$

- for a deterministic horizon

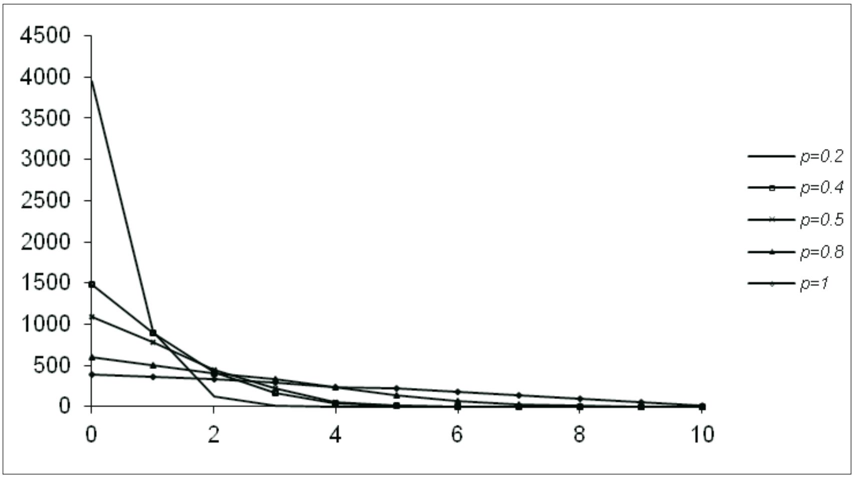
$$G_i = G_{i+1} - G_{i+1}^T C\left[R + C^T G_{i+1} C\right]^{-1} C^T G_{i+1}$$

$$G_N = Q;\ R_i = R.$$

Example 2. Let the linear system be described by a state equation (22) and performance criterion has a form (23). We assume that the random horizon τ has a binomial distribution with probability $p = 0.5$ and system can be controlled up to 10 times. Let $Q = \begin{bmatrix} 1.54 & 0.32 \\ 0.32 & 1.12 \end{bmatrix}$, $R = \begin{bmatrix} 0.87 & 0.13 \\ 0.13 & 0.35 \end{bmatrix}$, $B = \begin{bmatrix} 1.5 & 0.5 \\ 0.5 & 2.1 \end{bmatrix}$, $B = \begin{bmatrix} 1.5 & 0.5 \\ 0.5 & 2.1 \end{bmatrix}$ and the initial position is $y_0 = \begin{pmatrix} 100 \\ 20 \end{pmatrix}$. To determine the optimal control we must consider the auxiliary task (24) for $N = 10$.

Table 1. Simulations of the system states, controls and values of Bellman functions.

i	Random horizon τ (p = 0.5)			Deterministic horizon N = 5		
	y_i	u_i	W_i	y_i	u_i	W_i
0	(100;20)	(17.76;2.81)	1086.72	(100;20)	(12.87;-0.91)	755.67
1	(72.30;5.46)	(16.32;0.21)	786.99	(80.99;15.61)	(12.86;-0.88)	611.46
2	(49.63;-0.94)	(16.41;-0.39)	507.92	(62.58;11.87)	(12.91;-0.76)	475.18
3	(26.14;-8.31)	(10.86;-3.62)	268.73	(42.69;6.34)	(12.68;-0.88)	320.11
4	(12.66;-6.06)	(6.09;-2.92)	82.63	(22.60;0.71)	(12.01;-1.29)	163.04
5	(4.84;-2.17)	(2.49;-1.15)	12.46	(5.69;-1.85)	—	47.07
6	(2.62;-0.14)	(1.31;-0.2)	2.91	—	—	—
7	(0.28;0.56)	(0.08;0.23)	0.198	—	—	—
8	(-0.2;0.91)	(-0.21;0.43)	0.064	—	—	—
9	(-1.14;-0.61)	(-0.54;-0.19)	0.031	—	—	—
10	(-0.82;-1.01)	---------------	0.0026	—	—	—



Picture 1. Graphs of Bellman W_j functions for the different probabilities of success *p*.

If the control horizon is known (we can assume $p = 1$, see remark 3) or the success probability p is greater then we can reduce the energy of consumption and the object more accurate hit to the target. In case where horizon is random ($p \neq 1$)additionally we incur costs associated with the ignorance of horizon.

The simple simulation (see Table 1) shows that we can not replace a random horizon τ by a deterministic horizon $[E\tau] = [Np]$ (operator [.] means rounding to the integer part). Since in the case with random horizon the controls are much higher at the beginning, while in the case with deterministic horizon the controls are uniformly distributed in time.

5. Conclusion

In this article, the problem of optimal control of discrete-time stochastic system with unknown parameters and random horizon was introduced and the laws of the control were worked out. The random horizon was modeled by random variable at finite number of events. The described problem was reduced to the tasks of optimal control with finite horizon, where the aim of control (performance criterion) was modify suitably. The simple examples show, that the optimal control for the task with deterministic horizon must not use directly for the task with random horizon. Also the random horizon can not replace by the expected value of random horizon because the controls in cases with known and unknown horizons are completely different.

The above described system can be used directly to management of operation risk (to control the investment outlays in order to reduce losses caused by human factor), to control manufacturing process with the aim to take into account the production cycle, to control queuing systems.

Różnice pomiędzy sterowaniami liniowo kwadratowymi dla ustalonego i losowego horyzontów

Streszczenie

W niniejszym artykule rozważany jest problem sterowania adaptacyjnego z losowym horyzontem, gdzie cel sterowania polega na optymalizacji wskaźnika jakości, który jest formą kwadratową. Losowy horyzont modelowany jest za pomocą zmiennej losowej o skończonej liczbie zdarzeń elementarnych. Powyższe zagadnienie zostało zastąpione problemem optymalnego sterowania z ustalonym horyzontem. Podano prawa optymalnego sterowania systemem liniowym oraz proste przykłady. Ponadto podano różnice pomiędzy zadaniami z losowym i ustalonym horyzontami.

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